Characterisation of Semiconductor Laser FM Response

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Abstract: This paper describes characterisation of the FM response of semiconductor lasers by frequency to intensity conversion. The fundamental theoretical basis is outlined and experimental results are shown.

1 Introduction.

A common method of determining the frequency modulation (FM) response of a semiconductor laser is by frequency to intensity conversion, usually using interferometric methods as Mach-Zender, Michelson or Fabry-Perot based interferometers [1-3]. Common for all these methods is that they have a sinusoidal or approximately sinusoidal output intensity response to the input optical frequency. In this paper, a common theoretical basis, valid for all these methods will be outlined, and illustrated by experimental characterisation of the FM response for a typical DFB laser and a frequency stable external cavity laser.

2. Theory.

The sinusoidal response can be derived by adding the incoming optical field with a time-delayed version of itself:

$$E_{out} = a_1 e^{2\pi jf t + \phi} + a_2 e^{2\pi jf(t + \tau_d) + \phi}$$

(1)

\(\tau_d\) is the difference in time delay between the two paths. The detected power at the output is proportional to \(|E_{out}|^2\), and can be written:

$$I_{out} = \Gamma_1 I_0 + \Gamma_2 I_0 + 2I_0 \sqrt{\Gamma_1 \Gamma_2} \cos(2\pi f \tau_d)$$

(2)

\(\Gamma_1\) and \(\Gamma_2\) is the optical loss in each path and \(\iota\) is the coherence efficiency of the interaction between the two signals. Assuming a laser FM response of \(df/di\), we find the corresponding intensity response:

$$\left. \frac{dI_{out}}{di} \right|_{FM} = \frac{df}{dt} 4\pi \iota \tau_d I_0 \sqrt{\Gamma_1 \Gamma_2} \sin(2\pi f \tau_d)$$

(3)

If the frequency of the laser is changed by changing the injection current, we will also have component due to intensity modulation (IM):

$$\left. \frac{dI_{out}}{di} \right|_{IM} = -\frac{dI_0}{dt} \left[ \Gamma_1 + \Gamma_2 + 2\sqrt{\Gamma_1 \Gamma_2} \cos(2\pi f \tau_d) \right]$$

(4)

The FM response needs to be separated from the IM response. This is achieved by tuning either the optical frequency, or \(\tau_d\) to a point where the frequency sensitivity has a maximum value, \(\sin(2\pi f \tau_d) = \pm 1\). By subtracting the total intensity response taken at these two points, the AM response is cancelled and we get:

$$\left. \frac{dI_{out}}{di} \right|_+ - \left. \frac{dI_{out}}{di} \right|_- = -2 \frac{df}{dt} 4\pi \iota \tau_d I_0 \sqrt{\Gamma_1 \Gamma_2}$$

(5)
The FM response of the optical source can easily be extracted from this relationship. The accuracy of the determination of the FM response is dependent on how accurately the optical frequency, or \( \tau_d \), can be tuned to achieve \( \sin(2\pi f \tau_d) = 1 \). To investigate what impact a small tuning error has, we need to differentiate one of the measurement points with regard to tuning error:

\[
\delta \left( \frac{dI_{out}}{dt} \right) = \frac{df}{dt} \frac{d}{dt} \left[ \tan^{-1} \left( \frac{\sin(2\pi f \tau_d)}{1 + \sin(2\pi f \tau_d)} \right) \right] \frac{dI_{out}}{d\tau_d} + 4\pi \frac{dI_{out}}{dt} \sqrt{\tau_d} \left( \delta \tau_d + \delta f \tau_d \right)
\]

(6)

The relative measurement error is then found by dividing Eq.6 with half Eq.5. Assuming that the change in path length is much smaller than the total path length (\( \delta \tau_d \ll \tau_d \)), the relation can now be put into terms of phase error (\( \delta \phi = \tau_d \delta f + \delta \tau_d \)):

\[
\Lambda = \frac{(dI_{out}/dt) \delta \phi}{2\pi (df/dt) \tau_d I_{out}}
\]

(7)

This relation is needed to determine the required path delay.

3. Experiment.

In the experiments, a low-Q Fabry-Perot cavity was used as interferometer. Because of the low reflectivity of the Fabry-Perot cavity, the output signal mainly contains the zero and first order reflection roundtrips, and therefore the output response is approximately sinusoidal.

The first semiconductor laser to be investigated was a 16 quantum-well DFB laser. The static tuning response was 0.83 GHz/mA, and the slope of the L-I curve was 0.025 mW/mA at 65 mA bias current and 1.4 mW output power. Based on these values, a cavity round-trip length of 10 cm was chosen, corresponding to a round trip delay of 0.33 ns. The resulting FM response can be seen in Fig. 1. The continuous lines were obtained by using a network analyser, while the points represents discrete measurements made at lower frequencies. The minimum magnitude of the FM response is about 70 MHz/mA. Assuming that the intensity modulation response is relatively constant over the frequency range, 0.025 mW/mA, the maximum relative error of measurement can be calculated using Eq.7. For 0.1 rad phase setting error, the relative error of the measurement is 1.2%.

![Fig. 1: Measured FM response of 16 quantum well DFB laser. The left scale shows the magnitude, and the right scale shows the phase of the FM response.](image)
The second semiconductor laser to be investigated was an external cavity fibre-grating laser. The static tuning response was only 20 MHz/mA, and the slope of the L-I curve was 0.027 mW/mA at 50 mA bias current and 1.1 mW output power. The cavity round-trip length was increased to a maximum available 25 cm, corresponding to a round trip delay of 0.83 ns. The resulting FM response can be seen in Fig. 2. The minimum amplitude of the FM response is about 4 MHz/mA. Using the same assumptions as for the DFB laser, the relative error of the measurement is 11.7% for 0.1 rad phase setting error. In the FM response of both lasers, a dip can be observed in the amplitude with a corresponding transition of phase from 0 to $\pi$ between high and low frequencies. The explanation is that at low frequencies, the FM response is dominated by thermal effects with a red-shift. At higher frequencies, the FM response is dominated by carrier effects with blue shift. The dip occurs at the transition between the two effects.

4. Conclusion.

In this paper, a theoretical basis for characterisation of the FM response of semiconductor lasers by interferometric frequency- to intensity modulation conversion has been presented. It was found that the differential time delay had a critical importance for the accuracy of the measurements, and had to be increased for lasers with low FM response. Experimental results for the FM response of a standard DFB laser and a frequency stable external cavity laser were presented and it is shown that by increasing the differential time delay sufficiently, accurate characterisation of the FM response even for very frequency stable lasers is possible.

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References.

