On the Behaviour of Non-linearly Distorted OFDM Signals

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Abstract: In this paper, we present novel and unintuitive results related to the behaviour of non-linearly distorted OFDM signals. Particularly, we show that the error probability performance of such signals can in fact decrease as the number of OFDM sub-channels is increased.

1. Introduction

In recent years, the wireless industry has shown great interest in OFDM systems. Indeed, such systems have been adopted in a variety of wireless applications such as digital audio broadcasting (DAB) and digital video broadcasting (DVB). Moreover, OFDM has also been proposed for indoor wireless networks and fixed wireless broadband access [1,2]. This growing interest was due to the realisation that OFDM is an efficient scheme to convey information in a frequency selective fading channel without requiring complex equalisers [3]. Moreover, OFDM uses the available bandwidth efficiently and can be efficiently generated and detected using the IFFT and the FFT, respectively [4].

The main drawback of OFDM is its high peak-to-average power ratio (PAPR) and consequently its vulnerability to non-linear distortion [5-10]. Note that high PAPR imply large amplitude fluctuations that will occasionally hit the non-linear region of elements in the OFDM communication system.

In the open literature, it has always been assumed that the higher the number of OFDM sub-channels and hence the higher the PAPR the larger the error probability of non-linearly distorted OFDM signals [5-10]. In this paper, we show however that the error probability performance of such signals can in fact decrease as the number of OFDM sub-channels is increased.

2. Statement of the Problem

The analysis of the behaviour of non-linearly distorted OFDM signals will be carried out with respect to the model shown in Figure 1. In this figure, $s(t)$ is the complex envelope of the transmitted OFDM signal, $r(t)$ is the complex envelope of the received OFDM signal, $n(t)$ is the additive white complex Gaussian noise (with power spectral density N_0) and the input-output characteristics $f(\cdot)$ of the bandpass non-linearity is represented by a "baseband" power series [11], i.e.,

$$f(z) = \sum_{k=1}^{\infty} c_{2k-1} z |z|^{2k-2} \tag{1}$$

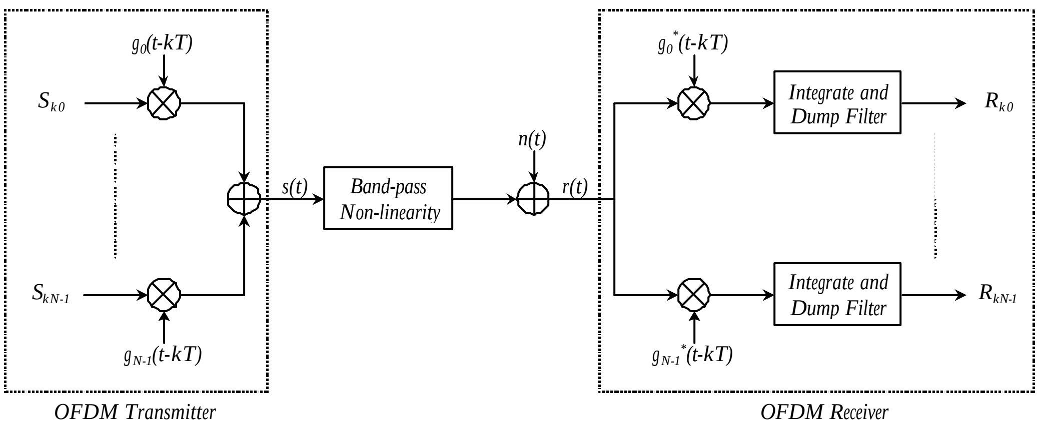


Figure 1: Model of a simple non-linear OFDM communication system.

S_{kn} is the complex transmitted symbol in time slot k and sub-channel n and R_{kn} is the complex received symbol in the same time slot and sub-channel; $g_n(t-kT)$ denotes the complex waveform used to convey the information in time slot k and sub-channel n and $g_n^*(t-kT)$ denotes its complex conjugate. Finally, the integrate and dump filter integrates from $kT+T_{CP}$ to $(k+1)T$, where T is the OFDM symbol duration and T_{CP} is the OFDM cyclic prefix. Note that this model assumes that the OFDM transmitter and the OFDM receiver are perfectly synchronised.

We now wish to identify the various contributions to the symbol received in a particular time slot and sub-channel. To accomplish this task we first note that the complex envelope of the transmitted OFDM signal is given by

$$s(t) = \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} S_{kn} g_n(t-kT) \tag{2}$$

$$g_n(t) = \begin{cases} \frac{1}{\sqrt{T-T_{CP}}} e^{j\frac{2pn(t-T_{CP})}{T-T_{CP}}} & t \in [0,T] \\ 0 & t \notin [0,T] \end{cases} \tag{3}$$

and the complex envelope of the received OFDM signal is given by

$$r(t) = c_1 s(t) + c_3 s(t)s(t)s^*(t) + \cdots + n(t) \tag{4}$$

Finally, the symbol received in time slot k and sub-channel n is obtained by multiplying the received OFDM signal by the complex conjugate of the complex waveform used to convey the information in that time slot and that sub-channel and then integrating over the useful OFDM symbol duration, i.e., from $kT+T_{CP}$ to $(k+1)T$. Thus,

$$R_{kn} = \int_{kT+T_{CP}}^{(k+1)T} r(t) g_n^*(t-kT) dt \tag{5}$$

and by using (2) and (3) in (4) and the resulting expression in (5) then

$$R_{kn} = c_1 \sum_{n_1=0}^{N-1} S_{kn_1} \frac{1}{(T-T_{CP})^0} d(n_1-n) + c_3 \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} \sum_{n_3=0}^{N-1} S_{kn_1} S_{kn_2} S_{kn_3}^* \frac{1}{(T-T_{CP})^1} d(n_1+n_2-n_3-n) + \cdots + N_{kn} \tag{6}$$

where d?·?is the Kronecker delta product and

$$N_{kn} = \int_{kT+T_{CP}}^{(k+1)T} n(t) g_n^*(t-kT) dt \tag{7}$$

Note that the various contributions to the received symbol in time slot k and sub-channel n can now be identified on the right hand side of (6): the first set of terms yields an attenuated version of the complex transmitted symbol in time slot k and sub-channel n; the second set of terms yields the third-order intermodulation distortion; a third set of terms would yield the fifth-order intermodulation distortion; etc. and N_{kn} is a complex Gaussian random variable whose real and imaginary parts are uncorrelated zero-mean Gaussian random variables with common variance N_0.

The error probability performance of non-linearly distorted OFDM signals is ultimately controlled by the distribution of the intermodulation distortion [12,13]. It is important to note however that we are not generally concerned with the intermodulation distortion distribution in a particular sub-channel as given by the second, etc. set of terms on the right hand side of (6) but the intermodulation distortion distribution conditioned on a given transmitted symbol in that particular sub-channel (e.g., $-A$ or A in the BPSK case). Moreover, although the intermodulation distortion is a complex valued random variable, in the BPSK case we are not concerned with its two-dimensional distribution but with the one-dimensional distribution of its real part as explained in [12] or [13], where an analytic technique to determine the error probability of non-linearly distorted OFDM/BPSK signals is presented. An examination of the intermodulation distortion distribution and its implications is therefore provided in the next section.

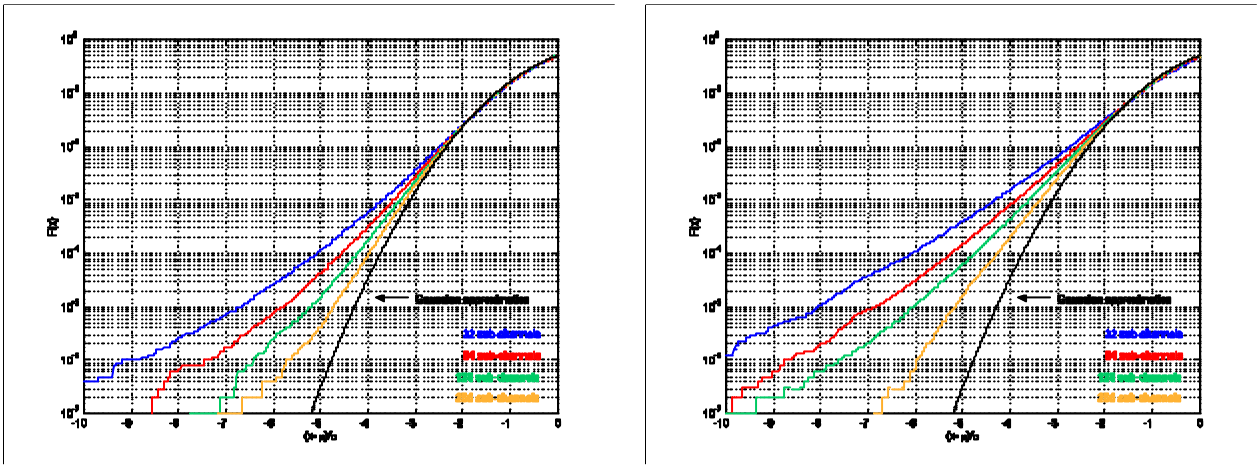
3. Simulation Results

In this section, we examine the intermodulation distortion distribution and its implications. We consider specifically a simple bandpass non-linearity for which its “baseband” power series representation consists of a linear and a cubic term.

We have examined the intermodulation distortion distribution in the sub-channel on the edge of the band and in the sub-channel in the centre of the band for non-linearly distorted OFDM/BPSK signals with 32 to 256 sub-channels. Note that we strictly consider the intermodulation distortion in the sub-channel on the edge of the band, where the number of intermodulation products is lowest, and in the sub-channel in the centre of the band, where the number of intermodulation products is highest, because these represent the two extreme cases.

We have assumed that the complex transmitted symbols were independent and took any value belonging to a BPSK constellation $\{-A,A\}$ with equal probability. We have also assumed that the complex transmitted symbol in sub-channel n, where n corresponds to the sub-channel being examined, took the value A. Symmetry considerations suggest that the intermodulation distortion distribution given $-A$ or given A is identical.

Figure 2 shows the cumulative distribution function of the real part of the intermodulation distortion in the sub-channel on the edge of the band and in the sub-channel in the centre of the band. Note that the x-axis has been normalised to the mean μ and the standard deviation s of the distributions. Observation of this figure reveals that the intermodulation distortion distribution is characterised by very heavy tails for a low number of OFDM sub-channels and by not so heavy tails for a high number of OFDM sub-channels, eventually tending to a Gaussian distribution. This is the case both for the intermodulation distortion distribution in the sub-channel on the edge of the band and in the sub-channel in the centre of the band although the first appear to converge faster than the second to a Gaussian distribution.



(a) (b)

Figure 2: Cumulative distribution function $F(x)$ of the real part of the intermodulation distortion for OFDM/BPSK signals with 32 to 256 sub-channels distorted by the bandpass non-linearity. a) Sub-channel on the edge of the band; b) sub-channel in the centre of the band.

The nature of the intermodulation distortion distribution suggests an important result whose origin rests on the following argument: On the one hand, by increasing the number of OFDM sub-channels the number of intermodulation products also increases and hence the support of the intermodulation distortion distribution and its variance augment. On the other hand, by increasing the number of OFDM sub-channels the heaviness in the tails of the intermodulation distortion distribution diminishes. Since the first and the second effects imply respectively an increase and a decrease in the error rate of non-linearly distorted OFDM signals then it appears that a worst number of OFDM sub-channels may exist from the error rate point of view.

To verify this conjecture we have simulated the bit error rate (BER) for OFDM/BPSK signals with 8 to 256 sub-channels distorted by the bandpass non-linearity for different values of OBO and different values of E_b/N_0 [11].

Figure 3 shows the BER against number of OFDM sub-channels for OFDM/BPSK signals distorted by the bandpass non-linearity for different OBO and E_b/N_0 values. Observation of this figure reveals that there is indeed a worst number of OFDM sub-channels. It is interesting to note that this trend is particularly evident for high E_b/N_0 values for which the non-linearity contribution to the error rate dominates with respect to the noise contribution. It is also interesting to note that this trend is valid for the different OBO values and hence for different degrees of intensity of non-linear behaviour.

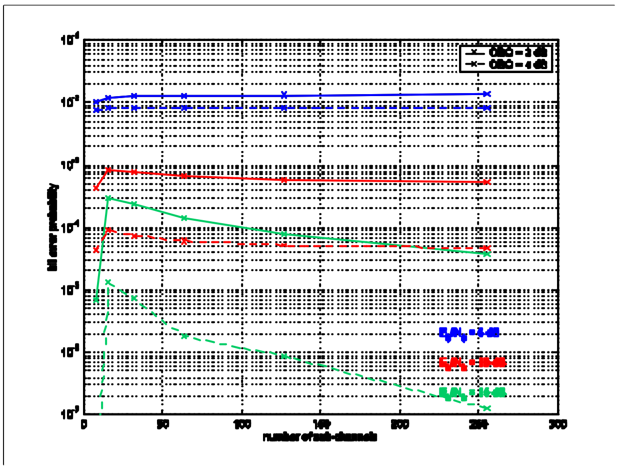


Figure 3: BER against number of OFDM sub-channels for OFDM/BPSK signals distorted by the bandpass non-linearity. Different curves correspond to different *OBO* and E_b/N_0 values.

4. Conclusions

In this paper, we have presented novel and unintuitive results related to the behaviour of non-linearly distorted OFDM signals. Particularly, we have shown that the error probability performance of such signals can in fact decrease as the number of OFDM sub-channels is increased.

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