Least-Squares Constant Modulus Algorithm based on Parallel Interference Cancellation

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Abstract: Blind source separation for mobile communications systems is considered in this paper. Adaptive blind source separation algorithms are likely to diverge when the available data are limited in length due to the rapid change of channel characteristics. We propose to deal with this problem in a manner that leads to a novel iterative blind source separation algorithm based on least-squares constant modulus algorithm (LSCMA) for instantaneous multiple-input multiple-output (MIMO) systems. The approach taken exploits the parallel interference cancellation to reduce blindly the effect of interference. The algorithm is robust and exhibits good performance as verified by computer simulations.

1 Introduction

Blind source separation is a fundamental problem in signal processing, with applications in many areas of communications, such as array signal processing and blind channel equalization. Adaptive blind algorithms based on some statistics of output signals can be used to separate the received signals [1]. It is known that adaptive algorithms require data of sufficient length to converge. If the channels change rapidly, adaptive blind algorithms may diverge. Iterative blind source separation techniques could serve as a possible solution to such problems. In this paper, we consider iterative blind source separation algorithms for instantaneous mixtures such as the BLAST system as proposed in [2] - [4].

A promising approach to the iterative blind separation problem is the least-squares constant modulus algorithm (LSCMA) [5]. Based on this algorithm, in [6] we presented an extension of LSCMA which included successive interference cancellation. Here we present an algorithm that takes advantage of parallel interference cancellation (PIC) to remove the degradation of interference from other transmit antennas. Simulation results show that our proposed algorithm exhibits better performance and is more robust to the length of data used.

2 System Model

In this paper, we consider a system with M transmitters and N receivers as shown in figure 1. The instantaneous multiple-input multiple-output observations collected over N_B consecutive symbol periods are

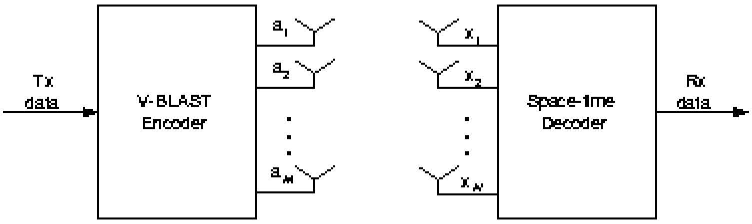

Figure 1: Vertical-BLAST system.

$$\mathbf{X} \;=\; \mathbf{HA} + \mathbf{V}, \tag{1}$$

where

X	$N \times N_B$ received signal matrix,
H	$N \times M$ channel matrix,
A	$M \times N_B$ transmitted signal matrix,
V	$N \times N_B$ i.i.d Gaussian noise matrix,

obtained from an uncoded vertical-BLAST (V-BLAST) system [3], [4]. In a rich multipath environment, the channel matrix H can be modelled by a matrix having independent identically distributed (iid), complex, zero-mean, unit-variance entries. We assume that H is unitary. If it is not unitary, the received signals can be prewhitened. Each component of **A** is drawn from the same constellation, i.e. QAM, and thereby has the same statistical properties. This problem is regarded as a problem of blind source separation (BSS) where the goal is to recover the transmitted signals **A** using only the received signals **X**.

3 The Proposed Algorithm

The proposed algorithm is based on LSCMA, which has been developed from a cost function represented in a form of the sum of squares error. The algorithm is derived by using Gauss's method, and exhibits good performance for a system consisting of one transmitter and two receivers [5]. Based on LSCMA and PIC, the proposed algorithm is hence named the parallel interference cancellation least-squares constant modulus algorithm (PIC-LSCMA) and is developed as follows. Denoting $\mathbf{w}_m$ the mth column vector of the $N \times M$ equalizer matrix **W**, the proposed algorithm is developed from the cost function

$$J_p(\mathbf{w}_m) = \sum_{k=1}^{N_B} (|\mathbf{w}_m^H \tilde{\mathbf{x}}_m(k)| - 1)^2 = \|\Phi(\mathbf{w}_m)\|_2^2, \tag{2}$$

where $\tilde{\mathbf{x}}_m(k)$ is the kth column vector of $\tilde{\mathbf{X}}_m$ and $\Phi(\mathbf{w}_m) = |\mathbf{w}_m^H \tilde{\mathbf{X}}_m| - 1$. The parallel interference cancellation process is

$$\tilde{\mathbf{X}}_m = \mathbf{X} - \tilde{\mathrm{H}}_{\overline{m}} \Psi_{\overline{m}}, \tag{3}$$

where $\tilde{\mathrm{H}}_{\overline{m}}$ is obtained by discarding the mth column of the estimated channel matrix $\tilde{\mathrm{H}}$. Let $[\,\cdot\,]_{i,j}$ denote the (i,j)th element of a matrix (and similarly for the ith element of a vector). The (m,k)th element of Ψ is given by

$$[\Psi]_{m,k} = [\mathbf{y}_m]_k / |[\mathbf{y}_m]_k|, \tag{4}$$

where the mth row of Ψ is the estimated mth stream, and $\mathbf{y}_m = \mathbf{w}_m^H \tilde{\mathbf{X}}_m$ is the output of the mth equalizer. Similarly, $\Psi_{\overline{m}}$ is formed by discarding the mth row of Ψ.

Due to the limited number of data used in each data block, the sample average is used to estimate the channel matrix, which is given by

$$\tilde{\mathrm{H}}(i+1) \;=\; \frac{1}{N_B} \mathbf{X} \Psi^H(i), \tag{5}$$

where i is the iteration index. The proposed algorithm is then obtained by using Gauss's method. This method updates $\mathbf{w}_m$ by the offset Δ_m which minimizes

$$J(\mathbf{w}_m + \Delta_m) \approx \|\Phi(\mathbf{w}_m) + Q^H(\mathbf{w}_m)\Delta_m\|_2^2, \tag{6}$$

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Initialization: $\mathbf{R} = \mathbf{X}\mathbf{X}^H$, $\tilde{\mathbf{H}}(0) = \mathbf{I}$, $\mathbf{W}(0) = \mathbf{I}$, $[\Psi(0)]_{m,k} = [\mathbf{X}]_{m,k} / |[\mathbf{X}]_{m,k}|$
For each iteration $i$:
    1. for $m = 1, 2, ..., M$
        ▪ create $\tilde{\mathbf{H}}_{\overline{m}}(i)$ by discarding $m$th column of $\tilde{\mathbf{H}}(i)$
        ▪ create $\Psi_{\overline{m}}(i)$ by discarding $m$th column of $\Psi(i)$
        ▪ set $\tilde{\mathbf{X}}_m(i) = \mathbf{X} - \tilde{\mathbf{H}}_{\overline{m}}(i)\,\Psi_{\overline{m}}(i)$
        ▪ set $\mathbf{y}_m(i) = \mathbf{w}_m^H(i)\,\tilde{\mathbf{X}}_m(i)$
        ▪ update $\Psi$ as $[\Psi(i)]_{m,k} = [\mathbf{y}_m(i)]_k / |[\mathbf{y}_m(i)]_k|$
       end for
    2. update $\tilde{\mathbf{H}}$ as $\tilde{\mathbf{H}}(i+1) = \frac{1}{N_B}\mathbf{X}\Psi^H(i)$
    3. update $\mathbf{W}$ as $\mathbf{W}(i+1) = \mathbf{R}^{-1}\mathbf{X}\Psi^H(i)$
    4. if $||\mathbf{W}(i+1) - \mathbf{W}(i)|| / ||\mathbf{W}(i)|| > \xi$, go to step 1.
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Figure 2: Summary of the proposed algorithm.

and results in

$$\begin{aligned} \mathbf{w}_m(i+1) &= \mathbf{w}_m(i) - \Delta_m(i) \\ &= \mathbf{w}_m(i) - (R_Q(i))^{-1} Q(\mathbf{w}_m(i)) \Phi^H(\mathbf{w}_m(i)), \end{aligned} \tag{7}$$

where $Q(\mathbf{w}_m(i)) = \nabla_{\mathbf{w}_m(i)} J(\mathbf{w}_m(i))$ and $R_Q(i) = Q(\mathbf{w}_m(i)) Q^H(\mathbf{w}_m(i))$. By assuming that the transmitted signals are linearly independent, the update equation is therefore obtained as

$$\begin{aligned} \mathbf{w}_m(i+1) &= \mathbf{w}_m(i) - (\tilde{\mathbf{X}}_m(i)\tilde{\mathbf{X}}_m^H(i))^{-1}\tilde{\mathbf{X}}_m(i)(\mathbf{y}_m(i) - \psi_m(i))^H \\ &= (\tilde{\mathbf{X}}_m(i)\tilde{\mathbf{X}}_m^H(i))^{-1}\tilde{\mathbf{X}}_m(i)\psi_m^H(i). \end{aligned} \tag{8}$$

The algorithm is summarised in figure 2 where ξ is a small stopping parameter. This algorithm requires $O(NM(M-1)N_B)$ flops to perform the parallel interference cancellation and another $O(NMN_B)$ flops to update $\mathbf{W}$.

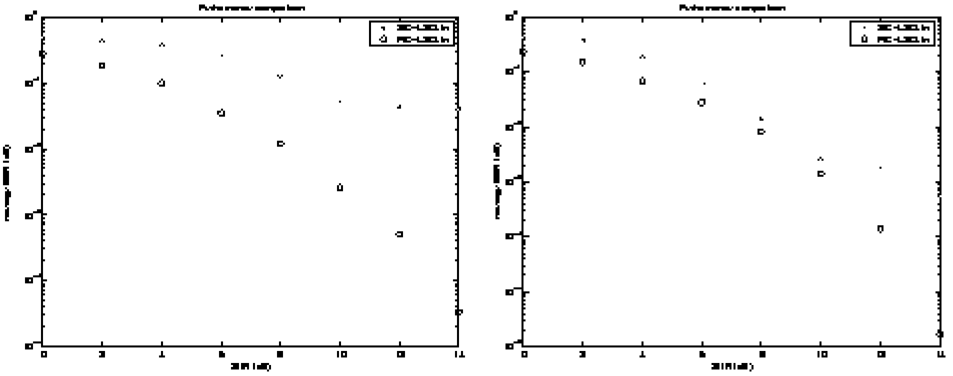
4 Simulation Results

Performance of the proposed algorithm is observed through simulations and compared with the successive interference LSCMA (SIC-LSCMA) proposed in [6]. A V-BLAST system consisting of $N = M = 3$ was considered with a random instantaneous unitary channel H. The transmitted signal was drawn from a 4-QAM constellation. The block size N_B used in simulations was chosen to be 50, 100 and 150. In addition, the stopping parameter ξ was set to 10^{-4}.

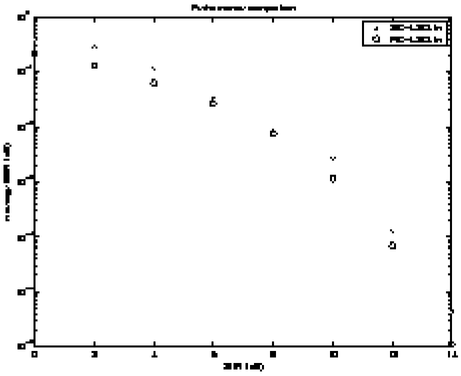
Figure 3 shows the performance comparison of PIC-LSCMA and SIC-LSCMA for $N_B = 50, 100$ and 150 respectively. In each case we see that PIC-LSCMA gives better performance than SIC-LSCMA. Comparing figures 3(a), 3(b) and 3(c), we note that PIC-LSCMA is convergent even for small data block size. The reason for these effects is that estimated signals from other transmit antennas are removed from the received signals before equalization is performed. As the estimated signals converged to the transmitted signals, the parallel interference cancellation removes the true interference from the received signals, and thereby the algorithm exhibits improved performance.

5 Conclusions

We have presented a novel iterative blind separation algorithm based on the least-squares constant modulus algorithm (LSCMA) and parallel interference cancellation. Simulation results



(a) N_B=50 (b) N_B=100



(c) N_B=150

Figure 3: Bit error rate (BER) comparison for $N = M = 3$ with different data block sizes.

have shown that the proposed algorithm exhibits good performance and is more reliable as compared with the algorithm based on the successive interference cancellation.

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