Quasi Orthogonal Space Time Block Codes with Feedback And Transmit Beamformers

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Abstract: Space-time block coding is an efficient method to mitigate channel fading by providing transmit diversity. However, orthogonal codes providing full diversity while attaining full code rate do not exist for more than two transmit antennas for complex valued constellations. Quasi-orthogonal space-time block codes achieve full code rate at the expense of diversity gain. In this study, we propose two feedback methods to achieve full diversity and full code rate for quasi-orthogonal codes. In the first method signals radiated from various antennas are rotated by phasors according to feedback from the receiver. The second method is based upon antenna weighting/selection possibly with permutations for the quasi-orthogonal codes. It is shown that the performance gap between the transmit beamformer and the quasi-orthogonal codes can be narrowed down from 8 dB to 3 dB with only two bit feedback. It is also shown that in the presence of severe feedback error, antenna weighting appears to perform better than antenna selection.

1 Introduction.

Diversity is an effective method to overcome the destructive effects of fading and to increase the performance and the capacity of transmission over wireless channels. Two widely used types are the transmit and receive diversity. Due to the physical limitations at the mobile side, it is not favourable to employ receiver diversity. Instead, Space Time Block Codes (STBCs) [2], [3] have been proposed to effectively obtain transmit diversity without channel state information (CSI) knowledge at the transmitter. At the receiver, maximum likelihood (ML) detection is achieved by linear operations providing full diversity.

Unfortunately, STBCs providing both full diversity and full code rate do not exist for more than two transmit antennas for complex valued constellations (e.g. Alamouti's code [3]), as proved in [2]. In [4], quasi-orthogonal (QO) STBCs have been introduced as a new family of STBCs. These codes achieve full code rate at the expense of reduced diversity.

In [7], and independently in [9], a closed loop feedback scheme has been proposed to orthogonalise the QO-STBCs by rotating the transmitted signals from certain antennas in a prescribed way. In [7] and [8], it is further shown that with only two bit feedback as facilitated in UMTS-FDD and for a slow fading channel, a diversity order of four is achieved.

However, there is still a 6 dB gap between the performance of this feedback scheme and the performance of both transmit and receive beamformer type diversities. In [6], an antenna weighting algorithm is proposed for orthogonal STBCs, particularly for Alamouti's block code for two transmit antennas. In this study, we extend this algorithm to QO-STBCs and show that the transmit antenna weighting/selection method exploits certain structural property of the QO-STBCs which narrows down this performance gap while retaining minimal amount of feedback (2 or 3 bits).

2 Problem Statement.

For simplicity one receive antenna is considered, generalisation to multiple receive antennas is straightforward. In [4], Alamouti's well known STBC, [3], is extended to four antennas to provide the QO-STBC as

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix} \tag{1}$$

where $x_1, x_2, x_3 and x_4$ are the symbols sent over four symbol intervals. At i-th interval the i-th row is transmitted from the transmit antennas. The received signal $\mathbf{y} = [y_1\ y_2\ y_3\ y_4]^T$ is represented by

$$\mathbf{y} = \mathbf{X}\mathbf{h} + \mathbf{n} \tag{2}$$

where $\mathbf{h} = [h_1 \;\; h_2 \;\; h_3 \;\; h_4]^T$ and $\mathbf{n} = [n_1 \;\; n_2 \;\; n_3 \;\; n_4]^T$, $(\cdot)^*$ and $(\cdot)^T$ denote complex conjugate and transpose operators, respectively. The term h_k represents the fading channel weight between the k-th transmit antenna and the receive antenna, and n_k denotes the noise at the k-th symbol period. Both $\{h_k\}$ and $\{n_k\}$ are independent identically distributed complex valued circularly symmetric zero mean Gaussian random variables with variances unity and σ_n^2, respectively.

By complex conjugating certain rows of $\mathbf{y}$ as given in [3], [5], we obtain the following representation

$$\hat{\mathbf{y}} = \mathbf{H}\mathbf{x} + \hat{\mathbf{n}} \tag{3}$$

where $\mathbf{x} = [x_1 \; x_2 \; x_3 \; x_4]^T$ is the data vector and the channel matrix is

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3^* & h_4^* & -h_1^* & -h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix} \tag{4}$$

After applying channel matched filtering, we obtain

$$\mathbf{z} = \mathbf{H}^H\mathbf{H}\mathbf{x} + \mathbf{H}^H\hat{\mathbf{n}} \tag{5}$$

where $(\cdot)^H$ stands for conjugate transpose.

Orthogonal STBCs have the following property $\mathbf{\Delta} = \mathbf{H}^H\mathbf{H} = \delta\mathrm{I}$ where δ is a code dependent constant and I is the identity matrix. Therefore, the vector can directly be used for estimation. However, a similar property does not hold for QO codes. For the code given in (1) we have

$$\mathbf{\Delta} = \mathbf{H}^H\mathbf{H} = \begin{bmatrix} \gamma & 0 & 0 & \alpha \\ 0 & \gamma & -\alpha & 0 \\ 0 & -\alpha & \gamma & 0 \\ \alpha & 0 & 0 & \gamma \end{bmatrix} \tag{6}$$

where $\gamma = |h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2$, and $\alpha = 2Re\left\{h_1^*h_4 - h_2^*h_3\right\}$.

It is seen that there is a coupling coefficient α between the estimates of the data symbols, nominally between x_1 and x_4 and between x_2 and x_3. This coupling decreases the diversity gain of the system. In fact the diversity order of the open loop QO code given in (1) is two which is the same as that of the two antenna Alamouti's scheme, which can also be verified by the results in [3] and [4]. Therefore our aim is to make α equal to zero.

3 Phase Feedback.

In [7], and [8], it has been shown that the coupling term can be eliminated by multiplying the first and second, or third and fourth antennas by proper phasors. Assume third and fourth antennas are multiplied over a code period by the phasors ϕ and θ, respectively. The new expression for α is [7], [8]

$$\alpha' = 2Re\left\{h_1^*h_4e^{j\theta} - h_2^*h_3e^{j\phi}\right\} \tag{7}$$

It can be shown that $\alpha' = 0$ has infinitely many solutions which makes the matrix Δ diagonal at $\theta = \arccos\left(\frac{|\lambda|}{|\kappa|}\cos\left(\phi + \angle\lambda\right)\right) - \angle\kappa$ provided that $\phi \in [0, 2\pi)$ when $|\lambda| < |\kappa|$, or $\phi \in [\pi - \xi - \angle\lambda, \xi - \angle\lambda]$ $\cup\,[-\xi - \angle\lambda, \pi + \xi - \angle\lambda]$ otherwise, where $\kappa = h_1^*h_4$, $\lambda = h_2^*h_3$, $\xi = \arccos\left(\frac{|\kappa|}{|\lambda|}\right)$, and, $|\cdot|$ and $\angle$ are the absolute value and angle (arctan) operators.

Note that, this two phase feedback scheme provides sufficient information on the CSI to make $\alpha = 0$. Therefore instead of full CSI feedback, we only need two phase variables in the range $[0, 2\pi)$. Quantising these phasors further reduces the amount of feedback which is studied in [7] and [8].

4 Antenna Selection.

Suppose that the transmitted signal is multiplied by a weighting matrix $\mathbf{W}$, which is drawn from a finite set $\Omega = \{\mathbf{W}_1, ..., \mathbf{W}_K\}$, based on the feedback, the received signal can be written as

$$\mathbf{y} = \mathbf{X}\mathbf{W}\mathbf{h} + \mathbf{n} \tag{8}$$

Here, we consider the special case of $\mathbf{W}$, the diagonal weighting as in [6]. At the receiver the quality of channels is measured in terms of the norms of individual channels and the two bit information to choose one of the following matrices is sent back to the transmitter,

$$\begin{aligned}
\mathbf{W_1} &= \left\{diag\{|a|^2, |a|^2, 1-|a|^2, 1-|a|^2\}\right\}^{1/2} \\
\mathbf{W_2} &= \left\{diag\{|a|^2, 1-|a|^2, |a|^2, 1-|a|^2\}\right\}^{1/2} \\
\mathbf{W_3} &= \left\{diag\{1-|a|^2, |a|^2, 1-|a|^2, |a|^2\}\right\}^{1/2} \\
\mathbf{W_4} &= \left\{diag\{1-|a|^2, 1-|a|^2, |a|^2, |a|^2\}\right\}^{1/2} \qquad (9)
\end{aligned}$$

where diag{.} is the diagonal matrix with the terms in the brackets as the diagonal entries, and $|a|^2 \leq 1$. The case $|a|^2 = 0.5$ corresponds to the no selection scenario in which all antennas transmit with equal power. In [6], analysis of $|a|^2$ is given in terms of bit error rate (BER) and the feedback error probability (P_e). QO code for four transmit antennas can be viewed as two independent Alamouti schemes, due to its grouping property. So in the first group, if $|h_1| \geq |h_4|$, the first and fourth antennas are multiplied by $|a|$ and $\sqrt{1-|a|^2}$, respectively, otherwise, by $\sqrt{1-|a|^2}$ and $|a|$, respectively. Similar approach also applies to second and third antennas. Therefore two feedback bits are required, one per group.

In the above technique we considered two antenna groups (antennas 1 and 4) and (antennas 2 and 3), and chose one from each group resulting in four combinations. When $|a|^2 = 1$, we could rather choose the best two antennas from four antennas resulting in six possible combinations. This scheme requires three bits for the feedback.

5 Simulations and Results.

For our simulations, we considered QPSK constellations, and BER performance is generated against E_b/N_o where E_b denotes bit energy and $N_o/2$ is the double sided noise power spectral density. The linear ML algorithm is used for decoding as in [4]. The simulation results are given for scenarios with and without feedback. Infinite precision phase feedback is assumed, but only a slight degradation in performance can be seen with quantisation as shown in [7].

Figure 1 depicts the BER comparison of the proposed feedback schemes with open loop QO-STBC and transmit beamformers. The feedback error rate is assumed to be P_e=0.001, therefore $|a|^2$ is set to 1 for best performance. Since $|a|^2 = 1$ makes the coupling term α equal to 0, there is no need for a phase feedback (but note the improvement introduced by the phase feedback for high feedback error rates). For transmit diversity schemes, each antenna is multiplied by the complex conjugate of the corresponding channel coefficient and normalised to have the total transmit power unity. For a fair comparison, we also considered a two transmit antenna based beamformer where the best two antennas (according to the quality of the resulting channel) are chosen out of the four antennas. Also it should be noted that we used full CSI instead of quantised CSI for the transmit beamformers, as then these can be used as a benchmark. But degradation in performance should be expected with quantisation. But for QO-STBC, we assumed finite bit feedback with two and three bits. The difference between the two antenna transmit beamformer and three bit feedback QO-STBC (when channel coefficients are sorted according to their norms and the best two antennas are chosen) is approximately 2.8 dB at BER of 10^{-3}. In fact the performance gain between the three bit feedback and the two bit feedback is only 0.2 dB, which is not very significant considering the feedback overheads involved. The performance of QO-STBC with antenna selection is more than 3 dB better than that without feedback. Therefore, the performance gap between the QO-STBC and the transmit beamformer is reduced from 8 dB to 3 dB when feedback is employed.

Due to lack of space we do not give the performance curves of the antenna weighting case ($|a| < 1$). We investigated the performance for different feedback error rates, P_e=0.1, and P_e=0.001. The results revealed that for the low SNR region, the best selection for $|a|^2$ is 1 resulting in the antenna selection diversity algorithm. But at high SNR, the weighting algorithm is more attractive. It is interesting to note that the phase feedback provides robustness against the deviation in $|a|^2$, because the performance remains close to the minimum point over a wide range of $|a|^2$. The reason for this is due to the orthogonality that has already been achieved by the phase feedback. Also note that the BER performance for phase feedback is better than that without phase feedback for various values of $|a|^2 < 1$. When $|a|^2 = 1$, schemes with and without phase feedback provide identical BER performance because the coupling term α is zero in both cases.

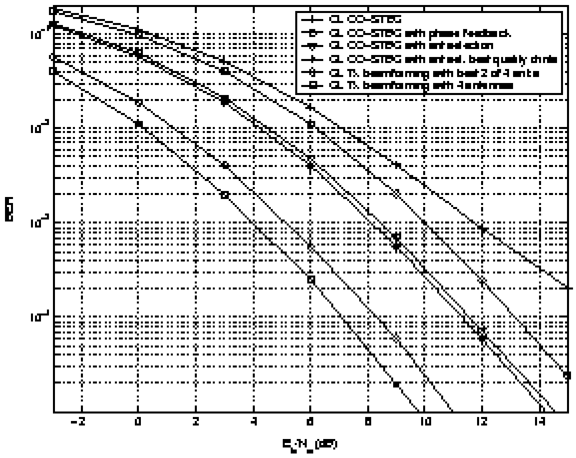

Figure 1: The BER performance comparison of the open loop quasi-orthogonal code, the transmit beamformer with 2 and 4 antennas, and the proposed phase feedback and the antenna selection feedback (i.e. $|a|$=1) schemes, $P_e = 0.001$

6 Conclusions.

In this study we proposed a simple feedback scheme for an antenna weighting/selection algorithm for QO-STBCs. These feedback techniques not only increase the diversity performance but also exploit the structural property to orthogonalise the QO-STBC. In the presence of severe feedback error, setting the antenna weight $|a|^2$ less than one provides better performance. Moreover, the phase feedback appears to relax the sensitivity of the optimum antenna weights. However, for low feedback error rate, antenna selection method (i.e. $|a|^2 = 1$) seems to perform better. Also in a low SNR environment, the antenna selection method always performs better regardless of the feedback error rate. It is also shown that the proposed scheme narrows down the performance gap between the QO-STBCs and the transmit beamformer techniques by more than 5 dB with only two bit feedback.

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