# Performance of Space-time Block Coding Using Estimated Channel Parameters

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**ABSTRACT:** The paper presents a method of estimating channel parameters in wireless communications using space-time block coding (STBC). Moreover, in this paper, the concept of STBC is reviewed, the system model is described, the estimation method is derived, and the system performances have been simulated using MATLAB.

## 1. Introduction

In recent years, space-time coding techniques have received much interest. The concept of space-time coding has arisen from diversity techniques using smart antennas. By using data coding and signal processing in both sides of the transmitter and receiver, space-time coding now is more effective than traditional diversity techniques. Mostly, traditional diversity techniques are receive diversities. In some cases it is difficult to use receive diversity because mobile units are supposed to be small lightweight pocket communicators. Therefore, use of transmit diversity in base stations appears an attractive method, as more complex base stations can be allowed[1]~[4].

Alamouti [5] proposed a simple but most effective transmit diversity technique which is called space-time coding because the source data are coded and transmitted through different antennas in different time slots. Initial and simple examples of implementation of space-time coding were given in [5], where two transmit antennas and two receive antennas were used. Tarokh, et al generalised the transmission scheme to an arbitrary number of transmit antennas, which can achieve the full diversity promised by the transmit and receive antennas [6],[7]. From their papers, we can find the key points of space-time coding. Firstly, the modulated symbols of source should be mapped into a transmit matrix of antennas and slots, or space and time, which has an orthogonal characteristic resulting in a simple method of detection. Secondly, the maximum likelihood detection algorithms can be used in the receiver to achieve full gain of diversity. Thirdly, more than one antennas could be used in both sides of the transmitter and receiver. It means that the radio links in space-time coding systems are multi-input multi-output (MIMO) radio channels, which have better characteristics in different propagation environments than traditional diversity techniques. According to [9], the space-time coding techniques can be divided into three categories: space-time coding, space-time Trellis codes and space-time block codes. It has been shown that space-time block codes offer a much simple way of obtaining transmit diversity without any sacrifice in bandwidth and without requiring huge decoding complexity compared with the other two categories. Therefore, space-time block codes are investigated in our paper. It should be noted that the decoding of spacetime block codes requires knowledge of channels at the receiver. In some of initial papers about space-time coding, the channel parameters are assumed known. Recently, channel parameter estimation was studied for OFDM systems with space-time codes in [9]. The work in our paper is on investigation and implementation of channel parameter estimation in space-time block coding (STBC) systems. An algorithm is proposed for channel estimation with STBC and computer programs are developed for implementing this algorithm.

The rest of the paper is organised as follows. In Section 2 the basic concept of space-time block codes is briefly described. In Section 3 the frame of radio links, which is to be studied in this paper, is presented. The algorithm of channel estimation is introduced and investigated in Section 4. The computer simulations of wireless communications with STBC are reported and results of the simulations are provided in Section 5. Finally, in Section 6, conclusions are given.

## 2. Space-time Block Codes

The basic principle of STBC is as the following: It is assumed that there are N transmit antennas and M receive antennas in a wireless communication system in which STBC is employed. The input source data bits are firstly modulated, and then carried into a space-time block encoder. Different modulation methods can be employed, such as PSK, QPSK, 8PSK and 16QAM [5],[7]. Mapping from the modulated symbols to a transmission matrix, which is completed by the STBC encoder, is a key step in STBC systems. The input symbols of the encoder are divided into groups of several symbols. The number of symbols in a group is according to the number of transmit antennas and mapping rule. A PxN transmission matrix means there are N transmit antennas and P time slots. Different symbol columns are transmitted through different antennas separately and different symbol rows in different time slots. For example, the encoded symbol of column i and row t should be transmitted through

the i<sup>th</sup> antenna in the t<sup>th</sup> time slot. Several rules of constructing transmission matrices have been proposed for STBC encoders [7]. For example,  $g_2$  represents a code which utilises two transmit antennas and is defined by

$$g_{2}(s_1,s_2) \rightarrow \left(\begin{array}{cc} s_1 & s_2 \\ & \\ -s_2^* & s_1 \end{array}\right)$$
(1)

where  $(s_1, s_2)$  is a group of input symbols. More STBC codes  $g_3$ ,  $h_3$  and  $h_4$ , were given in [7]. All of these codes are designed to be orthogonal. That means if c is an encoded matrix, then ,  $c^*c = \rho I$ , where I is a unit matrix and  $\rho$  is a constant. Simple detection methods can result from this characterastic of STBC. Since N symbols are transmitted in P time slots, the rate R of the code is defined as R=N/P. For example, the rate of  $g_2$  is one, half is for the  $g_4$ . This parameter presents the efficiency of transmission of the codes. Basically, constant channel parameters over PT periods are assumed in STBC transmission systems, where T is a symbol period. Let  $h_{ij}$  be the path fading parameter of complex from transmit antenna i to receive antenna j and  $c_i(t)$  the symbol transmited by antenna i in slot t, then the received signal from antenna j at time slot t,  $r_i(t)$ , is given by

$$\mathbf{r}_{j}(t) = \sum_{i=1}^{N} h_{ij} c_{i}(t) + \eta_{j}(t)$$
(2)

where j=1,...,M, t=1,...,P, and noise samples  $\eta_j(t)$  are the independent samples of a zero-mean complex Gaussian random variable. Let S be defined as the set of all possible symbol groups,  $s=\{s_1,s_2,...,s_N\}$ . The receiver computes the optimum maximum likelihood decision metric

$$d_{m} = \sum_{t=1}^{P} \sum_{j=1}^{M} \left| r_{j}(t) - \sum_{i=1}^{N} h_{ij} c_{i}(t) \right|^{2}$$
(3)

over the set S and decides in favor of the symbol group that minimizes the metric  $d_m$ . The ML decoding rule in Equation (3) can be further simplified according to the orthogonality of STBC encoding [7],[9]. It is clearly indicated in Equation (3) that the knowledge of channel parameters,  $h_{ij}$ , are required for ML decoding. These parameters are considered to be known in references [5], [7] and [9].

## 3. System Description

The architecture of a radio transmission system using STBC with channel parameter estimation, which is studied in this paper, is given in Fig. 1.





The user data of source are transmitted in the frame structure. At the beginning of a frame, a training or pilot sequence, of which the bits are known, is transmitted for channel parameter estimation at the receiver. The bursts of source data are groupped as several blocks for space-time block coding. The bit number in a group depends on the number of transmit antennas and the coding method of STBC.

Let vector  $a=[a_1,...,a_K]^T$  denote the transmitted data group. After modulation, the transmitted symbol vector,  $s=[s_1,...,s_L]^T$ , is formed and carried into the STBC encoder. In the encoder, s is mapped to transmission matrix c of PxN,

$$\mathbf{c} = \begin{bmatrix} \mathbf{c}_1(1)\dots\mathbf{c}_N(1) \\ \dots \\ \mathbf{c}_1(\mathbf{P})\dots\mathbf{c}_N(\mathbf{P}) \end{bmatrix}$$
(4)

The symbols of column i of matrix c are transmitted through antenna i and those of row t are transmitted in time slot t. Let MxN matrix h be channel parameters of MIMO channels and MxP

matrix r be the signals received, then Equation (2) can be rewritten in matrix form as:

 $r = hc^{T} + n$ 

where

$$\mathbf{h} = \begin{bmatrix} \mathbf{h}_{11} \dots \mathbf{h}_{N1} \\ \dots \\ \mathbf{h}_{1M} \dots \mathbf{h}_{NM} \end{bmatrix}$$
(6)

(5)

$$\mathbf{r} = \begin{bmatrix} r_1(1) \dots r_1(P) \\ \dots \\ r_M(1) \dots r_M(P) \end{bmatrix}$$
(7)

and  $\eta$  is an MxP noise matrix, the elements of which are independent samples of a zero-mean complex Gaussian random variable. The element  $h_{ij}$  of h is the parameter of the channel from transmit antenna i to receive antenna j. In r,  $r_j(t)$  is the signal received at receive antenna j in time slot t. As in [5] and [7], the channel is assumed to be a flat fading channel. In the receiver, the received symbol vector,  $\hat{s}$ , is recovered from r by the STBC decoder according to Equation (2) or using some simplified methods [7]. Then, after demodulation, the received user data,  $\hat{a}$ , can be obtained. The channel parameters which are needed by decoding are obtained by the channel estimator. In the training period, the training sequence, being the same as the transmitted training sequence, is applied to the modulator and STBC encoder in the receiver, then into the estimator. In the burst period, the receiver in the receiver, then algorithm of channel parameter estimation is analysed in the next section.

### 4. Algorithm of Estimation

Let h<sub>ii</sub> be the estimated parameter of the channel from transmit antenna i to receive antenna j. It can be obtained

by minimising the mean square error (MSE) cost function[8]: 
$$D(\hbar_i) = \sum_{t=1}^{P} r(t) - \sum_{i=1}^{N} \hbar_i C(t) \Big|^2 j = 1, ..., M$$
 (8)

This is a problem of solving the extreme value of a multi-variable function. Hence,  $\hbar_{ij}$  can be obtained by solving the following equations:  $\frac{\partial D(\hbar_{ij})}{\partial D(\hbar_{ij})} = 0, \quad i=1,...,N, j=1,...,M \quad (9)$ 

tions: 
$$\frac{\partial D(hy)}{\partial h_{ij}} = 0$$
,  $i=1,...,N$ ,  $j=1,...,M$  (9)

Direct calculation yields that Equation (9) is equivalent to:  $\sum_{t=1}^{P} \left[ r_{j}(t) - \sum_{i=1}^{N} \hbar_{i} c(t) \right] c^{*}(t) = 0 , j = 1, ..., M$ (10)

where 
$$c_i^*(t)$$
 is the complex conjugate of  $c_i(t)$ . Equation (10) can be rewritten in matrix form as:  
 $R_ch_j=\tilde{r}_j$ ,  $j=1,...,M$  (11)  
where  $R_c$  is the sum of NxN correlation matrices of the signals transmitted in time slot t, that is,

$$R_c = \sum_{t=1}^{r} R_t \tag{12}$$

where

$$R_{t} = \begin{bmatrix} c_{1}(t)c_{1}^{*}(t) & \dots & c_{N}(t)c_{1}^{*}(t) \\ \dots & \dots & \dots \\ c_{1}(t)c_{N}^{*}(t) & \dots & c_{N}(t)c_{N}^{*}(t) \end{bmatrix}$$
(13)

 $\check{\mathbf{r}}_{j} \text{ is the sum of the vectors, } \check{\mathbf{r}}_{j}(t) = [r_{j}(t)c_{1}^{*}(t) \dots r_{j}(t)c_{N}^{*}(t)]^{T}, \text{ that is,} \qquad \check{\mathbf{r}}_{j} = \sum_{t=1}^{P} \check{\mathbf{r}}_{j}(t)$ (14)

and  $\hbar_j$  is the vector whose elements are the parameters of the channels from N transmit antennas to receive antenna j, that is  $h_j = [h_{1j}, h_{2j}, ..., h_{Nj}]^T$ . If the coefficient matrix  $R_c$  is non-singular,  $h_j$  can be calculated as:  $\hbar_j = R_c^{-1}\check{r}_j, \quad j=1,...,M$  where  $R_c^{-1}$  is the inverse matrix of  $R_c$ . (15)

 $\hbar_j = R_c^{-1}\check{r}_j$ , j=1,...,M where  $R_c^{-1}$  is the inverse matrix of  $R_c$ . (15) The key of this algorithm is that the matrix  $R_c$  is non-singular. We find that if the training sequence is treated by the modulator and STBC encoder with the same rules as for the user data, then  $R_c$  is always non-singular because of the orthogonality of STBC codes.

#### 5. Simulation and Results

The environment of simulation is MATLAB where basic functions are used and main model functions are developed by us, in C language. The main program is written as M-file and model functions are written as C

MEX-file. Simulations are conducted in complex baseband. The system model of simulation is shown in Fig. 1. A pseudo random sequence generator is used for producing source user data. The method of modulation chosen is QPSK. Two and four antennas are used in the transmitter, and one and two antennas in the receiver. Correspondingly,  $g_2$  and  $g_4$  of STBC codes are employed. The channel parameter matrices and noise matrices are generated separately by two model functions using corresponding algorithms. Modulation and demodulation, encoding and decoding, and channel estimation are also performed by relevant model functions, the algorithm of which are described in Sections 3 and 4. The system performances of STBC with channel parameter estimation were simulated. The results are shown in bit error probability (BER) and symbol error probability (SER) for different transmit and receive antenna combinations. The bit streams that are run in our simulations are long enough for confidence in the results. The performance curves from the simulations with known channel parameters are identical to those in [5]. In Figs. 2 and 3 we provide BER for transmission using two and four transmit antennas and one and two receive antennas. The results for channel parameters known and estimated are separately exhibited in each figure. It can be seen from fig.2 and fig.3 that at the bit error rates from  $10^{-5}$  to  $10^{-5}$  the average loss of performance for channel parameter estimation is only 2.5 ~ 3.0 dB, lower than that of known channel parameters.



Fig. 2 Bit error probability versus SNR for STBC with two or four transmit antennas and one receive antenna



Fig. 3 Bit error probability versus SNR for STBC with two or four transmit antennas and two receive antennas

### 6. Conclusions

In this paper, we have studied the estimation of channel parameters for radio links with space-time block codes and developed an algorithm of MIMO channel estimation. Literature has shown that significant gain can be achieved theoretically by using space-time coding in high data rate wireless radio systems. Estimation of channel parameters which are necessary for ML detection of STBC is important for practicability of space-time coding techniques. The simulated results have shown a good agreement of system performances by using channel parameters estimated and channel parameters known. Therefore, the proposed channel estimation method for STBC wireless communications is practicable and effective.

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