Sidelobe Weighting Schemes for Small Phased Arrays

A.J.Willis
QinetiQ Ltd.

Abstract Taper weights that yield low sidelobe beams for phased arrays may be obtained by sampling an appropriate low sidelobe continuous weight function. As the number of phased array elements decreases, the number of sampling points also decreases. When there are only a small number of sample points the resultant beam exhibits sidelobes that differ from the intended design level. Two different continuous weight function sampling schemes are illustrated in this paper, and the subsequent effect on the beam pattern observed. An alternative approach, developed by the author, that calculates the sum beam explicitly and uses this as a basis to calculate weights to generate a desired sidelobe level is also presented.

1. Introduction

One of the most common amplitude weighting schemes, applied to control antenna sidelobes, is due to Taylor [1,2]. In [2], Taylor gave a formula for calculating the radial weight function for a continuous circular aperture. Two parameters are required to define the weight function: η, the sidelobe level required, and ň (nbar), which determines the boundary of the region of uniform sidelobes.

2. Sampling the Taylor Weight Function

Generating weights for phased arrays can be achieved by sampling the Taylor radial weight function. Two different sampling processes will be examined. As an example of a small phased array, consider a hexagonal arrangement of nineteen elements as shown in Figure 1.

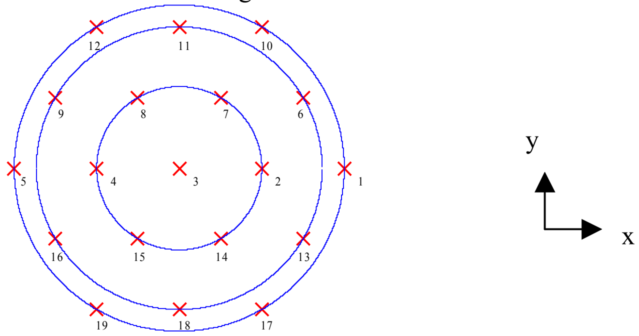

Figure 1: Hexagonal array of nineteen elements.

2.1 Sampling to the Edge of the Weight Function

The blue circles in Figure 1 show the three different radial distances present in the nineteen element hexagonal array. If the radius of the inner circle is d, then the outermost elements lie a distance 2d from the central element (number 3). The maximum distance from the array centre may then be sampled at the edge of the weight function, as shown in Figure 2. This sampling scheme relates to how weight functions are sampled in FFT time / frequency processing.

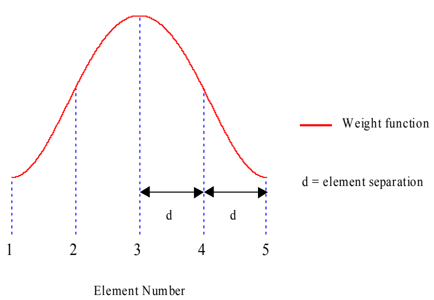

Figure 2: Edge of Weight Function Sampling Scheme

When this sampling scheme is applied to a -30 dB sidelobe, nbar = 4 Taylor weight function for the nineteen element hexagonal array, the sum beam in Figure 3 is obtained. Azimuth and elevation plots of the beam are shown alongside a full beam pattern.

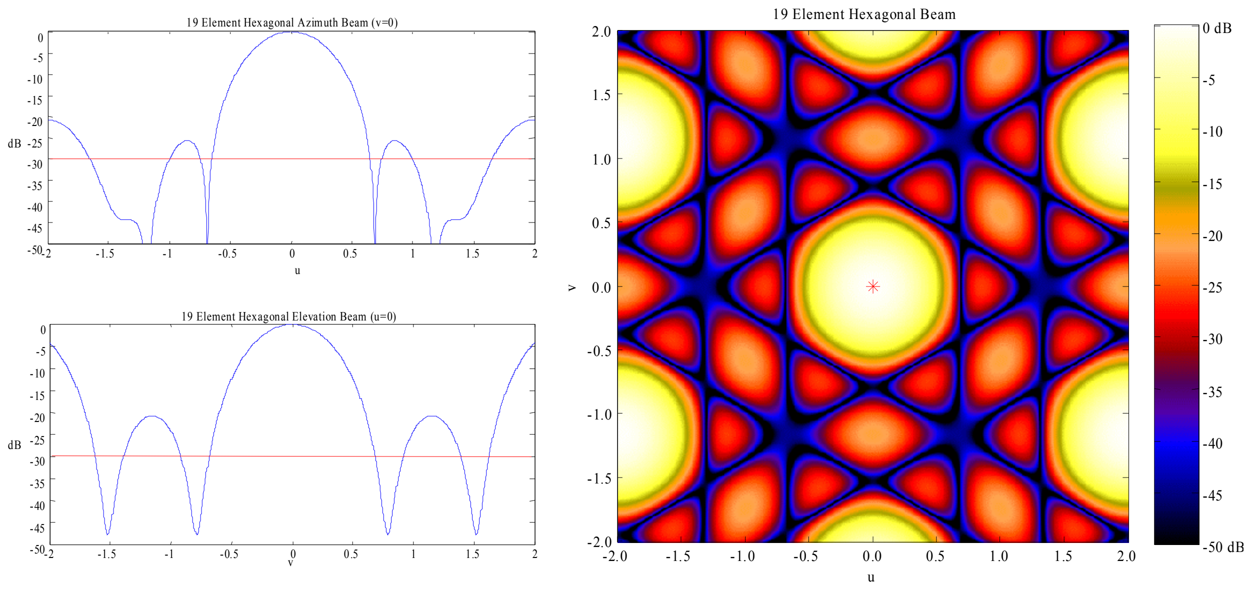

Figure 3: -30 dB Beam Pattern Obtained Through Edge Sampling Scheme

The full beam pattern is plotted using u,v co-ordinates. This co-ordinate system maps all possible directions in the hemisphere forward of the array within a unit circle. Points outside this unit circle enable us to “see” the sidelobe structure that will appear if the beam is subsequently electronically steered. In terms of azimuth and elevation,

$$u = \sin(\text{azimuth})\cos(\text{elevation}) \qquad 1)$$

and

$$v = \sin(\text{elevation}) \qquad 2)$$

From Figure 3, it can be seen that the maximum sidelobe level is around –21 dB, not the –30 dB level requested. The increase in sidelobe level is attributable to the low of number of samples of the weight function taken and a poor choice of sampling points. Even with a large array, the first sidelobe will not be at the correct level using this sampling scheme.

2.2 Ksienski Sampling

Ksienski [3] suggested the basis of an alternative sampling scheme. If each array element is considered as representing a cell of diameter d, then the maximum radius of the array (in this example) becomes 2.5d instead of 2d. With this new maximum radius, the sample positions on the weight function are adjusted as in Figure 4.

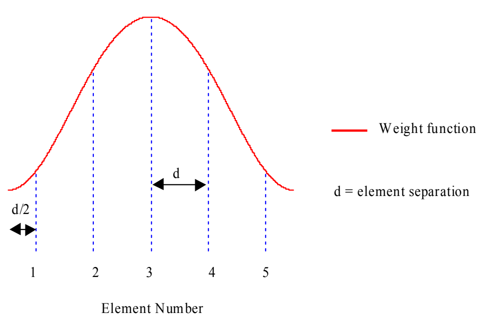

Figure 4: Ksienski Weight Function Sampling Scheme

When this sampling scheme is applied to a -30 dB sidelobe, nbar = 4 Taylor weight function for the nineteen element hexagonal array, the sum beam in Figure 5 is obtained. Azimuth and elevation plots are shown alongside the full beam pattern.

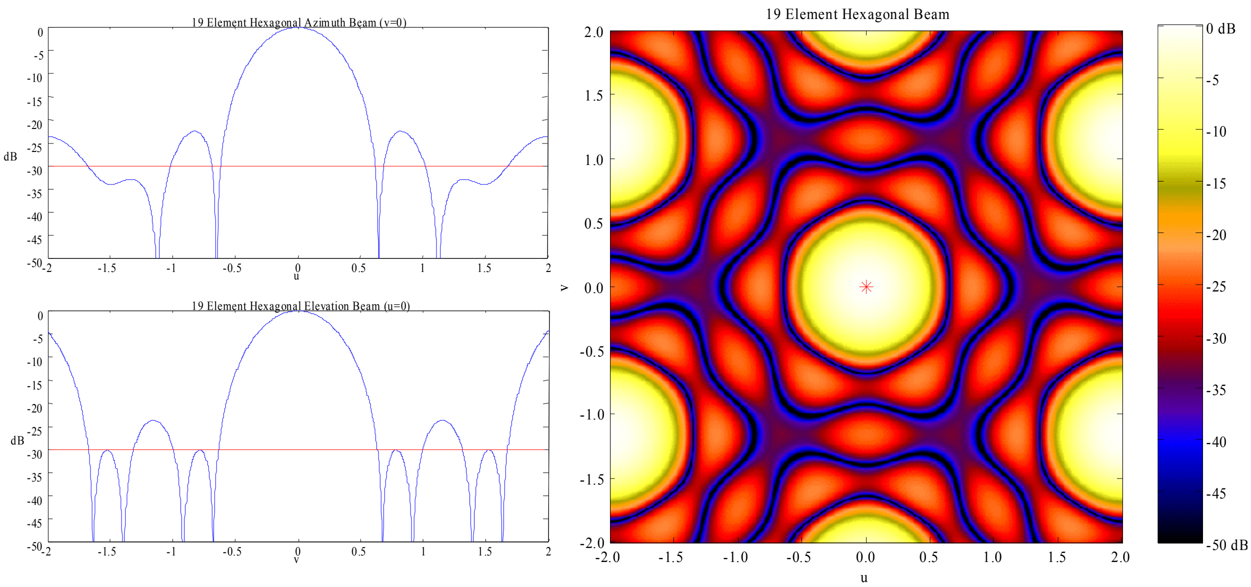

Figure 5: -30 dB Beam Pattern Obtained Through Ksienski Sampling Scheme

The beam pattern now shows more hexagonal structure, and the maximum sidelobe level has decreased to around –23 dB. The Ksienski sampling scheme has enabled sidelobes to be produced closer to the desired level of –30 dB, but the low number of sample points again means the desired sidelobe level is not met. In a large array the desired sidelobe levels are now achieved for all sidelobes.

3. Calculation of Small Array Taper weights

An alternative to sampling a continuous weight function is to calculate the weights analytically. This is achieved by breaking down the hexagonal array into 7 sub-arrays, each of whose contribution to the sum beam may be written down as shown in the table below. The central element is used as the phase reference point which enables the complex exponential phasor sum expression to be reduced to its real cosine components. For more information on this, and the expression for a rectangular sum beam, see [4].

Sub-Array	Type	Containing Elements	Sum Beam Contribution
1	Point	3	1
2	Linear	2,4	$2w_1*\cos(2U)$
3	Linear	1,5	$2w_3*\cos(4U)$
4	Linear	11,18	$2w_2*\cos(2V)$
5	Rectangular	7,8,14,15	$4w_1*\cos(U)*\cos(V)$
6	Rectangular	10,12,17,19	$4w_3*\cos(2U)*\cos(2V)$
7	Rectangular	6,9,13,16	$4w_2*\cos(3U)*\cos(V)$

Here w_1, w_2 and w_3 are the weights to be applied to elements at the three increasing radial distances from the centre of the array, and $U = \pi du$, $V = \sqrt{3}*(\pi dv)$ where d is the element spacing in wavelengths on the x-axis.

The sum beam is then formed from adding up all the beams from the individual arrays. Setting v and u to zero respectively forms the azimuth and elevation field strength equations:

Azimuth Field Strength =

$$(1+2w_2)+4w_1\cos(U)+2(w_1+2w_3)\cos(2U)+4w_2\cos(3U)+2w_3\cos(4U) \qquad (3)$$

Elevation Field Strength =

$$(1+2w_1+2w_3)+4(w_1+w_2)\cos(V)+2(w_2+2w_3)\cos(2V) \qquad (4)$$

Using cosine multiple angle formulae and letting cos(U) = x and cos(V) = y, equations (3) and (4) can be expressed as polynomials in x and y:

Azimuth Field Strength =

$$16w_3x^4 + 16w_2x^3 + 4(w_1 - 2w_3)x^2 + 4(w_1 - 3w_2)x + (1 - 2w_1 + 2w_2 - 2w_3) \qquad (5)$$

Elevation Field Strength =

$$4(w_2 + 2w_3)y^2 + 4(w_1 + w_2)y + (1 + 2w_1 - 2w_2 - 2w_3) \qquad (6)$$

Equations (5) and (6) can now be differentiated and set to zero and solved for x and y to find the location of maxima and minima (the sidelobes). The values for x and y can then be substituted back into (5) and (6) to give the field strength at these positions in terms of the unknown weights.

A mathematical software package such as Mathcad can then be used to generate values for the unknown weights subject to certain conditions using its solve block function. The following conditions were applied: 1) all sidelobes to be below –30 dB; 2) all weights to have a magnitude less than one; 3) sum of all weights to be maximised. Mathcad returned the values $w_1 = 0.753$, $w_2 = 0.340$, $w_3 = 0.170$, which generated a beam pattern as shown in Figure 6.

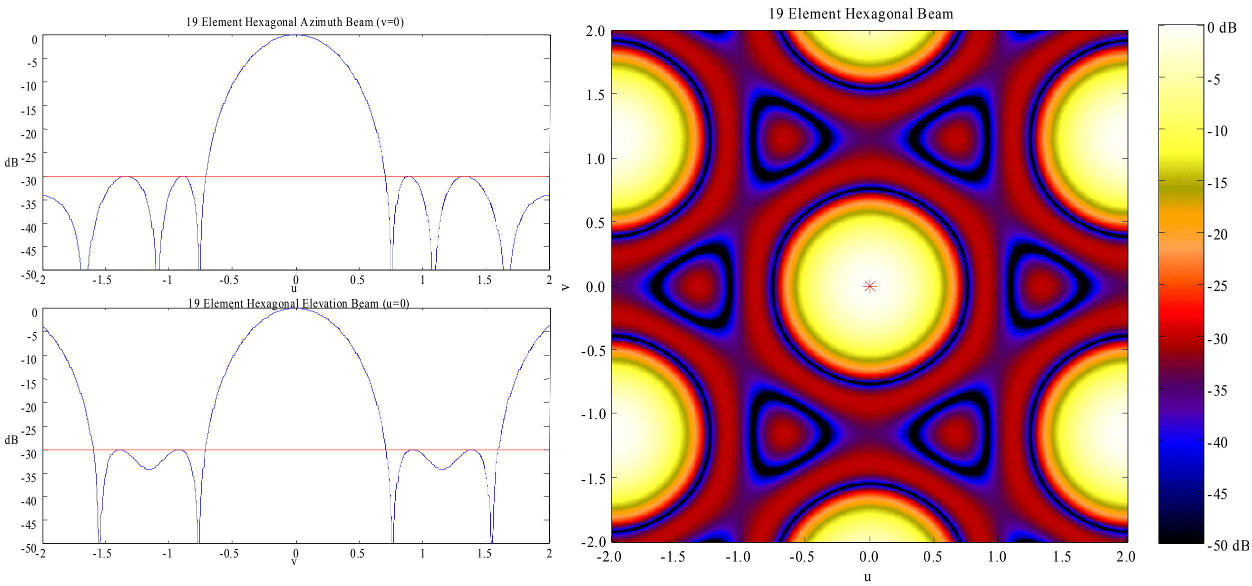

Figure 6: -30 dB Beam Pattern Obtained Through Explicit Weight Calculation

The azimuth and elevation plots clearly show all sidelobes are now at –30 dB or below as desired.

4. Conclusions

Sampling continuous weight functions for small phased arrays can yield sidelobe levels that differ substantially from the desired level. A sampling scheme due to Ksienski has been shown to yield sidelobes closer to the design level for small arrays, and achieves the desired sidelobes in larger arrays. An alternative scheme developed by the author has been used to analytically calculate weights for exact sidelobe levels.

5. References

[1] Taylor, T.T. – “Design of Line-Source Antennas for Narrow Beamwidth and Low Side Lobes”, IRE Transactions, Antennas and Propagation, January 1955, 16-28.

[2] Taylor, T.T. – “Design of Circular Apertures for Narrow Beamwidth and Low Sidelobes”, IRE Transactions on Antennas and Propagation, January 1960, 17-22

[3] Ksienski A, “Equivalence between continuous and discrete radiating arrays”, Canada Journal of Physics Vol 39 (1961) pp 335-349

[4] Fourikis, N – “Phased Array Based Systems and Applications”, Wiley 1997, ISBN 0471012122 pp 146 & 165.

The author is an EngD student at the Department of Electronic and Electrical Engineering, University College London.

The EGC domain of the MoD funded the work described in this paper.