Parallel Concatenated Gallager Codes using Euclidean and Projective Geometry LDPC codes

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Abstract

We present a class of Parallel Concatenated Gallager Codes (PCGC), using LDPC codes based on the type I and II Euclidean (EG) and Projective Geometries (PG). Simulation results are shown for this particular PCGC that outperform the LDPC codes based on EG and PG codes without concatenation in Additive White Gaussian Noise (AWGN) by improving its waterfall BER performance.

I. Introduction

Low-density parity-check (LDPC) codes were first introuced by Gallager [1] as a family of block codes characterized by a sparse parity-check matrix, with each row in the matrix representing the coefficients in a parity-check equation. Gallager showed that random LDPC codes are asymptotically good for binary symmetric channels (BSC).

Turbo codes were introduced in 1993 by Berrou, Glavieux and Thitimajshima [2] as a class of concatenated codes. Recursive encoders, large pseudorandom interleavers and iterative decoding schemes are the essential elements that produce the outstanding performance provided by turbo codes.

H. Behairy and S. Chang presented in [3],[4] a procedure to concatenate two LDPC codes, using the turbo principle, with the advantage of avoiding the use of a pseudorandom interleaver, leading to a higher performance obtained by mixing 2 LDPC codes with different column weight.

Y. Kou, S. Lin and M. Fossorier presented in [5] a construction of LDPC codes based on Euclidean and Projective Geometries (EG and PG). The EG and PG LDPC codes, created by cyclically shifting each row of the parity check matrix, have good performance and converge faster than random LDPC codes. Besides, the transpose of any EG or PG LDPC matrix, defines a new parity check matrix with the same properties (size and performance) and are known as the type I and II EG/PG LDPC codes. These codes present a sharp waterfall error performance which improves as the length of the LDPC code increases.

We present the performance of PCGC using EG and PG LDPC codes as inner and outer codes in an Additive white Gaussian Noise (AWGN) channel. This mix takes advantage of the best characteristcs from the turbo concept applied to LDPC codes (PCGC) and the well structured EG and PG LDPC codes. We assume that the waterfall error performance will improve by concatenating these codes, which is not easily obtained when the length of the LDPC code is relatively small, and can be used for real time applications. The sum-product algorithm has been proved to be the best algorithm to decode the LDPC codes, and was used in this study.

II. EG and PG Parallel Concatenated Gallager Codes

Two distinct codes (type I and II EG and PG LDPC codes) are used in parallel concatenation to build a PCGC as shown in Fig. 1, where x denotes the systematic information bits, while V1 and V2 are the parity bits generated by the type I and type II (EG or PG LDPC) encoders, respectively. The information bits go direct to both encoders without the use of an interleaver. The

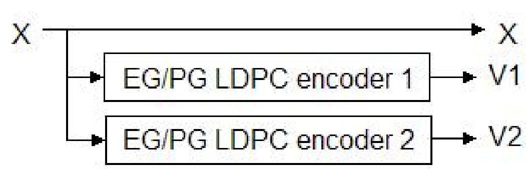

Fig. 1. EG/PG LDPC PCGC encoder.

characteristics of the EG and PG codes are completely defined in [5]. The constraint they present, is that the size of the parity check matrix can not be modified to fit a specific length, without modifying the cyclic definition of the parity check matrix. This implies that a limited number of rates can be obtained. Accordingly, the PG(273,191) was choosen for this paper; it will use 191 information bits to produce 82 parity bits from each encoder, having a total of 355 bits to transmit, with a rate $R = 0.538$ which is the closest that can be obtained to 0.5 in order to compare results with convolutional codes and punctuared turbo codes. Besides, the weight of the rows and columns is a function of the length, so that the parity check matrices always present a row and column weight higher than 3, and once again, can not be modified without losing the cyclic characteristic of this code. The EG/PG(m,s) LDPC codes used for this paper, have m=2, which produce a square LDPC matrix entirely defined by cyclic shifts of any of its rows.

III. The Decoder

The decoding process of PCGC follows the turbo decoding principle without using the interleaver. Each EG/PG LDPC decoder computes the a posteriori probability as described in [6] using the sum-product algorithm with modifications to accommodate the a priori information. Let $U \in (+1, -1)$, $y0$ corresponds to the systematic information bits, and $y1$ and $y2$ denote the received sequences corresponding to the parity bits of the first and second EG/PG LDPC codes, respectively. The entire process of the information shared between decoders is shown in Fig. 2. This implies the use of soft information shared between decoders. At

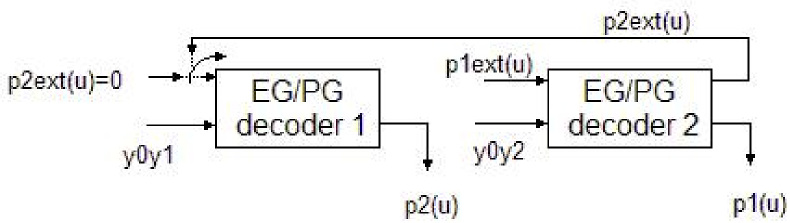

Fig. 2. EG/PG LDPC PCGC decoder.

the beginning of the decoding process, the first decoder computes the a posteriori probability $p2(u)$ using the received sequences $y0$ and $y1$ with no a priori information, but it will produce the extrinsic information $p1ext(u)$ which will be used by the second decoder as a priori information. The process of exchanging information between decoders continues until one or both decoders converge to a valid codeword, or a maximum of number of iterations is reached.

LDPC codes are defined by a bipartite graph. The concatenation of two LDPC codes can also be defined by a bipartite graph (fig. 3), in such a way that the variables corresponding to the information bits, are related to the checks in both parity checks, as well as the variables corresponding to the parity bits respectively.

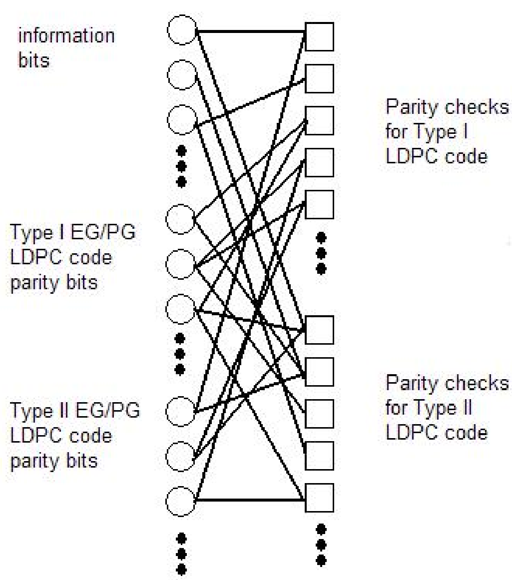

Fig. 3. Bipartite graph for a EG/PG LDPC PCGC.

IV. Comparison to other codes

EG/PG LDPC PCGC codes have the advantage over EG/PG LDPC codes of doubling the number of checks which results in a clear improvement in the performance. The exchange of extrinsic information between decoders makes the decoding process more reliable and can be used to fix a specific code rate. Using this approach, puntuaring or repetition of the parity bits will lead to a better performance of the decoding process compared to that when reducing or extending the EG/PG LDPC matrix, which produces changes on the performance and structured properties of these codes.

EG/PG LDPC PCGC codes have the advantage over PCGC codes of having a well structured LDPC matrix, that has been proved to give excellent performance at different lengths. Both codes (type I and II) can be defined by one row, which reduces the amount of memory required. The number of iterations required to converge is lower compared to the PCGC codes.

V. Simulation Results

We present simulation results for a PG(2,4) type I and II LDPC PCGC (355,191) $rate = 0.538$, compared with the PG(2,4) type I LDPC code (273,191) $rate = 0.6996$, with a binary convolutional code of $rate = \frac{1}{2}$ with generator polynomials 753-561 and constraint length 400, and with a PCGC (576,288) $rate = 0.5$. The PG LDPC PCGC outperforms both the PG(2,4) type I LDPC code by more than 0.5 dB for a BER of $1e^{-5}$ and the convolutional code by more than 2 dB for a BER of $1e^{-4}$. The PCGC outperforms the PC LDPC code when the Eb/No is lower than 3 [dB], but this seems to change at values higher values of Eb/No.

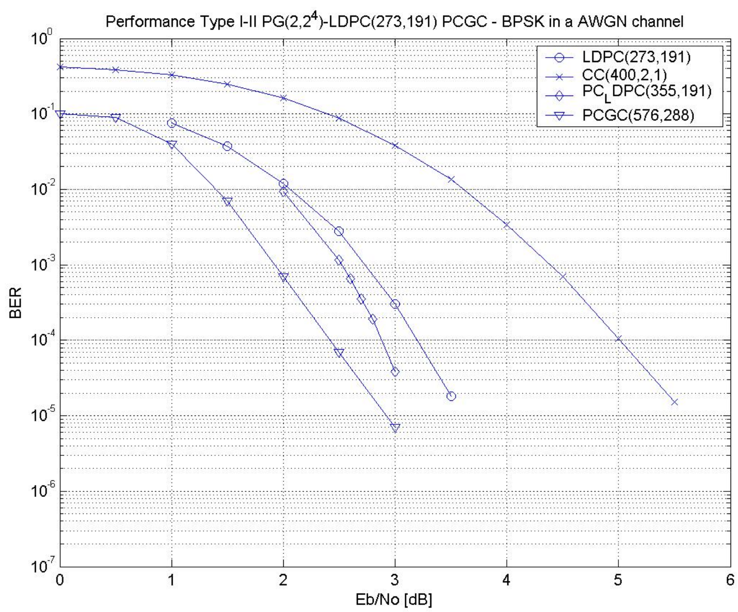

Fig. 4. Preformance of the Parallel Concatenated Type I and II PG(273,191) LDPC Code.

VI. conclusions

EG/PG LDPC codes are an atractive alternative to use with PCGC. They require a small amount of iterations to converge and their waterfall BER performance is increased, compared to that of a EG/PG LDPC code alone. This proves to be an alternative to increase the performance of small length codes, and requires a small amount of memory to define the LDPC matrices.

As analysed in [4], the concatenation of 2 codes, one having a column weight ≤ 2.5 and the other a column weight ≥ 2.5 that joined give a $rate = 1$, improve the performance of the decoder at low Eb/No and high Eb/No respectively. Further analysis should be made to test other types of well structured LDPC codes with the PCGC principle. One option is to consider the Geometry based designs of LDPC codes proposed by M. Zhang and J.M.F. Moura on [7]-[8]. This kind of structured codes have $girth \geq 8$ and variable column weight and length.

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