Impact of APD Mismatch in Optical Code Division Multiple Access Systems using Perfect Difference Codes

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Abstract: Evaluation of error probability based on the assumption that the gains of APDs used in upper and lower branches of optical code-division multiple-access (OCDMA) systems are matched provides an approximation which may in practice be an overestimate or underestimate of the actual probability of error. In this paper, we analyze the impact of APD mismatch in a synchronous OCDMA system based on the perfect difference codes (PDC). The results demonstrate that the performance degradation caused by the mismatch in branches, substantially degrades the bit error rate necessitating the requirement for close matching of the branches.

1. Introduction.

Optical-CDMA (O-CDMA) allows very flexible access of the large communication bandwidth available in optical fibre networks with a capability to conceal the data content. This is possible with simple reconfiguration of codes at the transmitters and receivers. Many implementations of fibre optic CDMA systems have been investigated and presented in the literature under the name of Optical CDMA in the last fifteen years [1-5].

Evaluation of error probability based on the assumption that the gains of APDs used in the upper and lower branch of the desired user's receiver are matched provides an approximation which may in practice be an overestimate or underestimate of the actual probability of error [2-4]. We examine how the mismatch in APD affects the synchronous optical CDMA (SOCDMA) system performance proposed in [2]. The system performance, with consideration of shot noise, thermal noise, avalanche photodiode (APD), bulk and surface leakage currents, and APD gain mismatch is investigated.

The remainder of this paper is organized as follows. In section 2, the structure of the transmitter and receiver is described. Section 3 presents the analysis of APD mismatch gain. Section 4 discusses the numerical results obtained for the BER and system capacity. Finally, the conclusion is given in section 5.

2. System Description.

Each subscriber is assigned a unique code. Each active user transmits a signature sequence of k laser pulses (representing the destination address), over a time frame if mark "1" is transmitted. However, if the data bit is space "0", no pulses are transmitted during the time frame [2, 3].

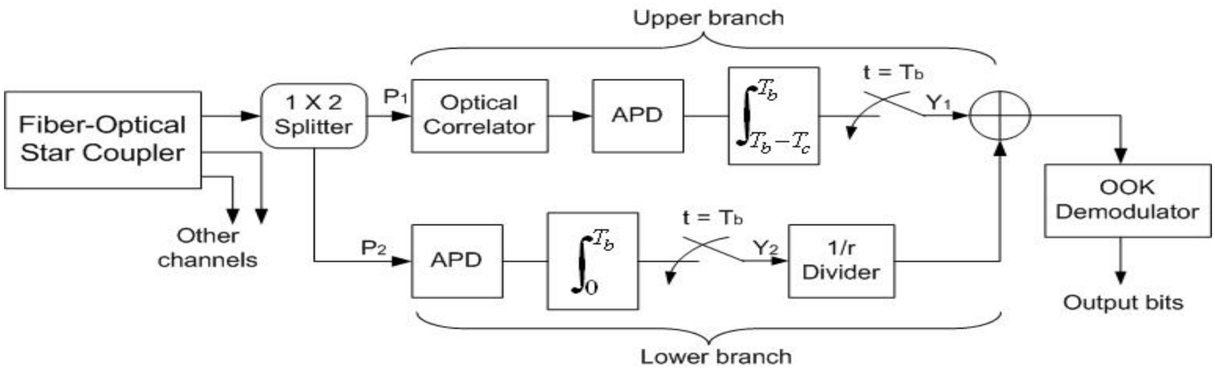

Figure-1: The receiver Structure

Fig.1 shows the block diagram of the desired user's receiver, which is designed by exploiting the property of PDC $(v, k, \lambda = 1)$, where k, v and λ are the code weight, code length, and exact cross correlation between any two distinct codes respectively, for which the cross correlation between any two distinct codes is exactly one [2]. The received signal P is split into two unequal powers per pulse P_1 and P_2 using a 1x2 optical splitter, which are fed to the upper and lower branch, respectively. The upper branch is used to detect the desired user's signal while the lower branch is used to eliminate the multi-user interference (MUI). P_1 is correlated with the PDC (signature code) that characterizes the desired user. The correlator output is then photodetected with gain G_1, integrated, and sampled. The sampler output, Y_1, contains the desired signal, interference from other users and noise. The lower branch has an APD with gain G_2, an integrator, a sampler, and a divider. After integration over the bit duration and sampling the output signal Y_2 is then divided by r, which is a constant and its optimal value is $r = \sqrt{k^2 - k + 1}$ given in [2]. The output signal Y which is obtained by subtracting Y_2 / r from Y_1 is then fed into the ON-OFF keying (OOK) demodulator. If Y is less than the constant threshold value of the OOK demodulator the output bit is 0, otherwise it is 1.

3. APD Mismatch Analysis.

We assume that the number of active users is N and that there are N_1 interfering users. Furthermore, without loss of generality, it is assumed that the first user is the desired user and that b_0 is the desired bit. The average photon arrival rate λ_1 per pulse at the input of the optical correlator in the first branch is given by $\lambda_1 = \eta P_1 / hf$. Given N_1 and the desired bit $b_0 = 1$, the mean and variance of output Y_1 after the sampler in the first branch can be expressed as [2]:

$$\mu_{y1,1} = G_1 T_c \left[\left(\frac{k + N_1 - 1}{k} \right) \lambda_1 + \frac{I_b}{e} \right] + \frac{T_c I_s}{e} \tag{1}$$

$$\sigma_{y1,1}^2 = G_1^2 F_{e1} T_c \left[\left(\frac{k + N_1 - 1}{k} \right) \lambda_1 + \frac{I_b}{e} \right] + \frac{T_c I_s}{e} + \sigma_{th}^2 \tag{2}$$

where, G_1 is the average APD gain of upper APD, e is the electron charge, I_b is the APD bulk leakage current, I_s is the APD surface leakage current, and T_c is the chip duration. The excess noise factor and the variance of the thermal noise is given by (3) and (4), respectively.

$$F_{e1} = K_{eff} G_1 + \left(2 - \frac{1}{G_1} \right) (1 - K_{eff}) \tag{3}$$

$$\sigma_{th}^2 = \frac{2 K_B T_r T_c}{e^2 R_L} \tag{4}$$

Here, K_{eff} is the APD effective ionization ratio, K_B is the Boltzmann's constant, T_r is the receiver noise temperature, and R_L is the receiver load resistance.

Similarly, given N_1 and $b_0 = 0$, the mean and variance of Y_1 is given by (5) and (6), respectively.

$$\mu_{y1,0} = G_1 T_c \left[\left(\frac{N_1}{k} \right) \lambda_1 + \frac{I_b}{e} \right] + \frac{T_c I_s}{e} \tag{5}$$

$$\sigma_{y1,0}^2 = G_1^2 F_{e1} T_c \left[\left(\frac{N_1}{k} \right) \lambda_1 + \frac{I_b}{e} \right] + \frac{T_c I_s}{e} + \sigma_{th}^2 \tag{6}$$

In the second branch, the average photon arrival rate λ_2 per pulse at the input of the APD is given by $\lambda_2 = \eta P_2 / hf$. Given N_1, the mean and variance of the output Y_2 can be expressed by (7) and (8), respectively.

$$\mu_{y2} = G_2 T_c \left[N_1 k \lambda_2 + \frac{v I_b}{e} \right] + \frac{v T_c I_s}{e} \tag{7}$$

$$\sigma_{y2}^2 = G_2^2 F_{e2} T_c \left[N_1 k \lambda_2 + \frac{v I_b}{e} \right] + v \left(\frac{T_c I_s}{e} + \sigma_{th}^2 \right) \tag{8}$$

After subtracting Y_2 / r from Y_1, we obtain the mean of Y as given by (9) for $b_0 = 1$ and $b_0 = 0$, respectively.

$$E(Y) = \begin{cases} \mu_{Y1} = T_c \left[G_1 \left(\frac{k + N_1 - 1}{k} \right) \lambda_1 - \frac{G_2}{r} N_1 k \lambda_2 \right] + \frac{T_c I_b}{e} \left[G_1 - \frac{G_2 v}{r} \right] + \frac{T_c I_s}{e} \left(1 - \frac{v}{r} \right) \\ \mu_{Y0} = T_c \left[G_1 \left(\frac{N_1}{k} \right) \lambda_1 - \frac{G_2}{r} N_1 k \lambda_2 \right] + \frac{T_c I_b}{e} \left[G_1 - \frac{G_2 v}{r} \right] + \frac{T_c I_s}{e} \left(1 - \frac{v}{r} \right) \end{cases} \tag{9}$$

For the nominal case, that is $G_1 = G_2 = G$, the MUI is cancelled for the condition shown in (10).

$$\frac{N_1 \lambda_1}{k} = \frac{N_1 k \lambda_2}{r} \tag{10}$$

That is, $r = k^2 \lambda_2 / \lambda_1$ and hence $\lambda_2 / \lambda_1 = \sqrt{k^2 - k + 1} / k^2$ from which equation (11) follows.

$$P_1 = \frac{k^2 / r}{1 + k^2 / r} P \tag{11}$$

Then the mean and variances of Y are given by equations (12) and (13), respectively.

$$E(Y) = \begin{cases} \mu_{Y1} = GT_c \left[\left(\frac{k-1}{k} \right) \lambda_1 + \frac{I_b}{e} \left(1 - \frac{v}{r} \right) \right] + \frac{T_c I_s}{e} \left(1 - \frac{v}{r} \right) \\ \mu_{Y0} = GT_c \left[\frac{I_b}{e} \left(1 - \frac{v}{r} \right) \right] + \frac{T_c I_s}{e} \left(1 - \frac{v}{r} \right) \end{cases} \tag{12}$$

$$Var(Y) = \begin{cases} \sigma_{Y1}^2 = \sigma_{y1,1}^2 + \frac{1}{r^2} \sigma_{y2}^2 \\ \sigma_{Y0}^2 = \sigma_{y1,0}^2 + \frac{1}{r^2} \sigma_{y2}^2 \end{cases} \tag{13}$$

Consequently, the constant threshold value of the OOK demodulator is given by (14) as defined in [2].

$$\theta = GT_c \left[\left(\frac{k-1}{2k} \right) \lambda_1 + \left(1 - \frac{v}{r} \right) \frac{I_b}{e} \right] + \frac{T_c I_s}{e} \left(1 - \frac{v}{r} \right) \tag{14}$$

Given the number of simultaneous users $N = n$ and the assumption that $P_r(b_0 = 0) = P_r(b_0 = 1) = 1/2$ then the bit error rate can be derived as:

$$\begin{aligned} & P_r(error | N = n) \\ & = P_r(Y \geq \theta | N = n, b_0 = 0) \times P_r(b_0 = 0) + P_r(Y < \theta | N = n, b_0 = 1) \times P_r(b_0 = 1) \\ & = \frac{1}{2} \sum_{m=0}^{n-1} P_r(Y \geq \theta | N_1 = m, b_0 = 0) \times P_r(N_1 = m | N = n, b_0 = 0) \\ & + \frac{1}{2} \sum_{m=1}^{n} P_r(Y < \theta | N_1 = m, b_0 = 1) \times P_r(N_1 = m | N = n, b_0 = 1) \end{aligned} \tag{15}$$

where

$$P_r(Y \geq \theta | N_1 = m, b_0 = 0) = \frac{1}{2} erfc \left(\frac{\theta - \mu_{Y0}}{\sqrt{2\sigma_{Y0}^2}} \right) \tag{16}$$

$$P_r(Y < \theta | N_1 = m, b_0 = 1) = \frac{1}{2} erfc \left(\frac{\mu_{Y1} - \theta}{\sqrt{2\sigma_{Y1}^2}} \right) \tag{17}$$

$$\Pr(N_1 = m \mid N = n, b_o = 0) = \binom{n-1}{m} \frac{1}{2^{n-1}} \tag{18}$$

$$\Pr(N_I = i \mid N = n, b_o = 1) = \binom{n-1}{m-1} \frac{1}{2^{n-1}} \tag{19}$$

and, $erfc(\cdot)$ stands for the complementary error function, as defined in (20).

$$erfc(z) = \frac{2}{\pi} \int_z^{\infty} \exp(-u^2) du \tag{20}$$

4. Experiments & Results.

For our experiments, the gain of the upper branch's APD, which is used to extract the information of the desired user, is kept fixed and set to the typical value of 100 [2, 4]. The gain of the lower branch's APD is incremented and decremented in steps of 5%. It is assumed that the power received per pulse is fixed at $P = 10\mu W$. We use the value of r obtained from (10) which assumes that the gains of both APDs are perfectly matched. However, in practice this is not the case. As a result, the MUI is not completely cancelled, and consequently only a 5% difference between the gains of the two APDs causes a 50% reduction in the system capacity, as shown in Fig.2.

This problem can be compensated, if the gains of the two APDs are accurately measured under the prevailing operating conditions. Then the condition for a complete cancellation of MUI is defined by $r = k^2 \lambda_2 G_2 / \lambda_1 G_1$ instead of $r = k^2 \lambda_2 / \lambda_1$. As can be seen in Fig.2 for the compensated case, the performance of SOCDMA is almost similar to the case when the two branches are completely matched.

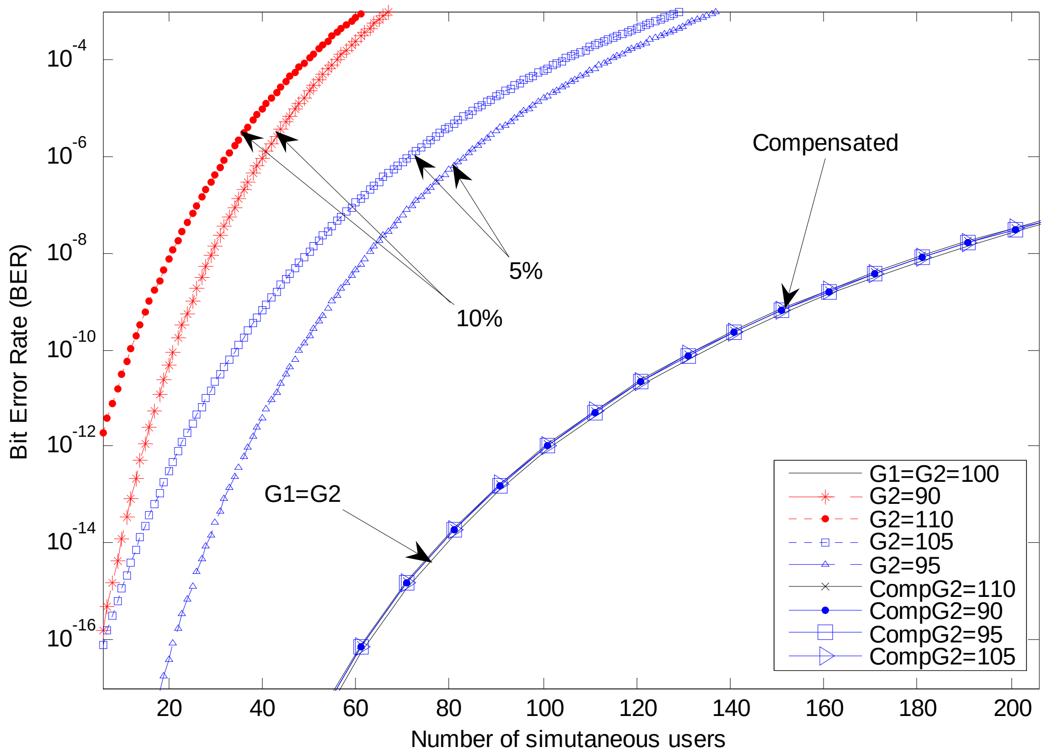

Figure-2: Bit error rate probability under the same receiver power per pulse $P = 10 \mu W$

5. Conclusions.

We have considered the problem of APD mismatch in the upper and lower branches of a synchronous OCDMA system based on perfect difference codes. Results show that a small mismatch in the gains of the APD's used in the upper and lower branch can seriously degrade the system BER performance. However, if the gains of both APDs are measured and taken into account when setting the ratio r then the system can perform satisfactorily. However, it should be noted that, there are other factors which prevent the accurate matching of branches such as I_b, I_s and electrical gain differences. These effects are the topics of further study by the authors.

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