Pattern encoding on the Poincaré Sphere

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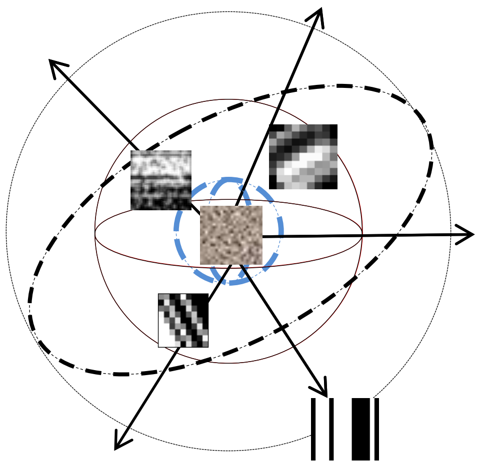
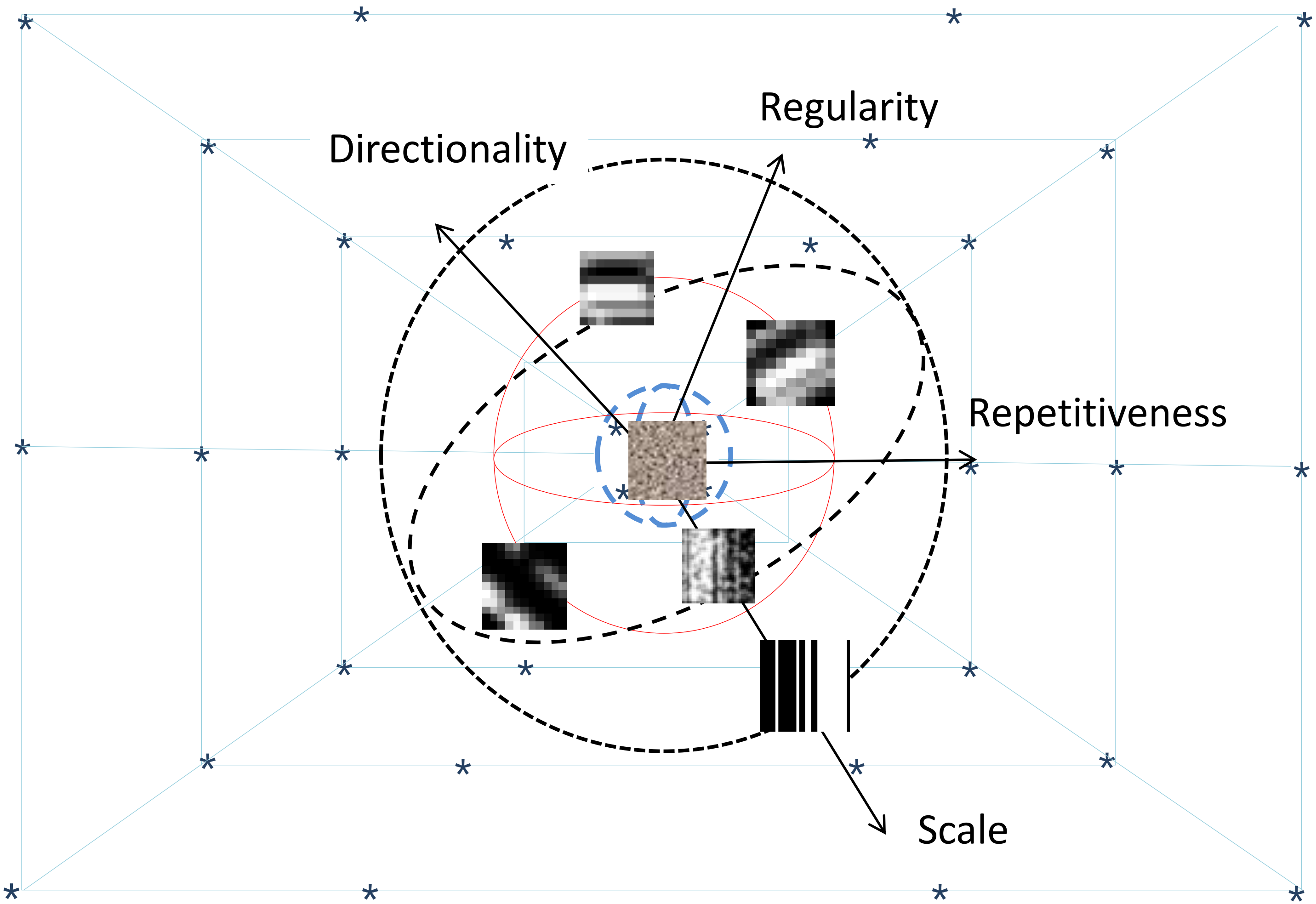
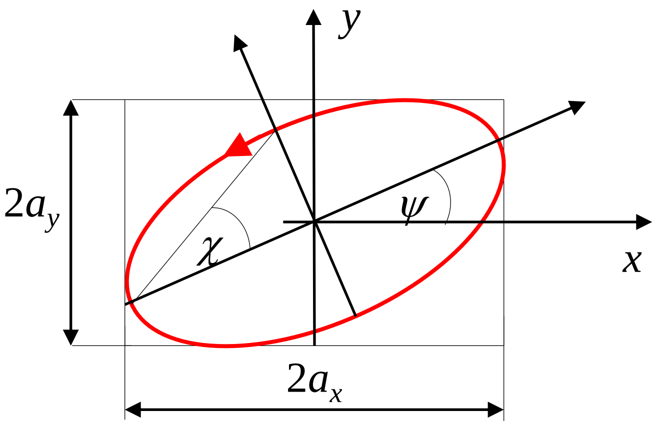


SAHD, UCL-Duke 2014

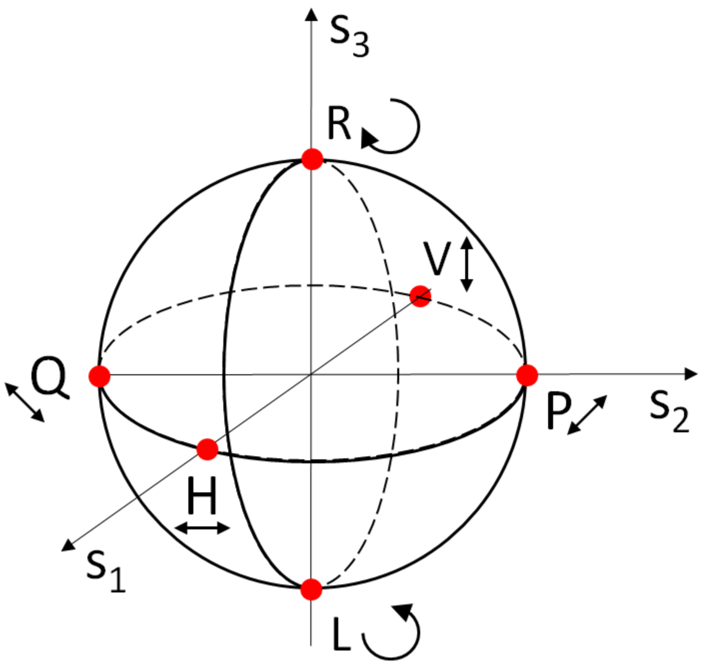
A graphical tool for pattern encoding



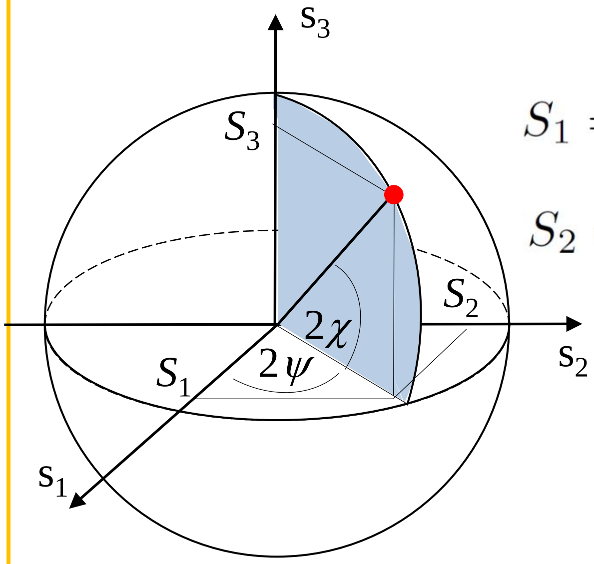
Inspiration: Encoding Polarization States



$$\left(\frac{E_x}{a_x}\right)^2 + \left(\frac{E_y}{a_y}\right)^2 - 2\frac{E_x}{a_x}\frac{E_y}{a_y}\cos\delta = (\sin\delta)^2$$



Poincaré sphere (Henri Poincaré, 1892)



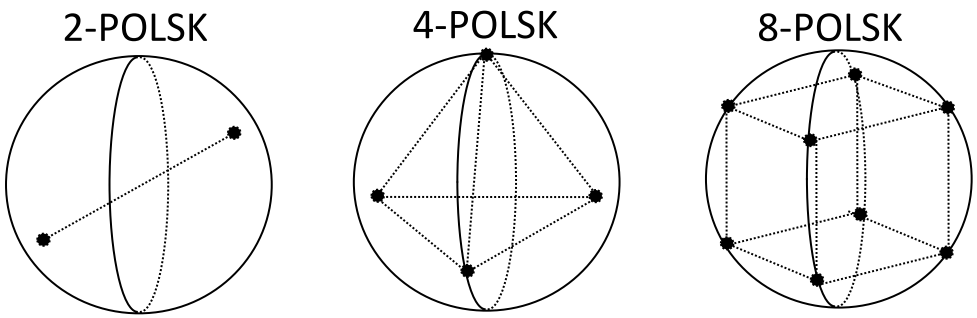
$$S_1 = S_0 \cos(2\chi)\cos(2\psi)$$

$$S_2 = S_0 \cos(2\chi)\sin(2\psi)$$

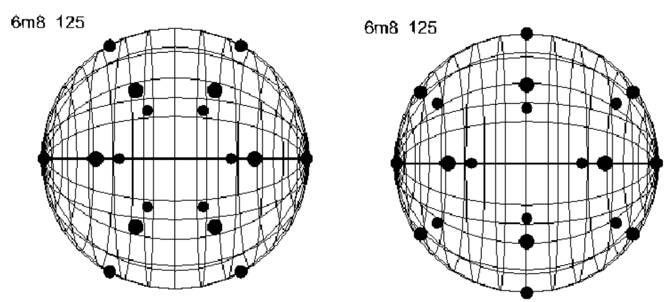
$$S_3 = S_0 \sin(2\psi)$$

Applications in communications

Optical communications with **POLarization Shift Keying** (**POLSK**) modulation **[S. Benedetto and P. Pogiolini, 1992]**

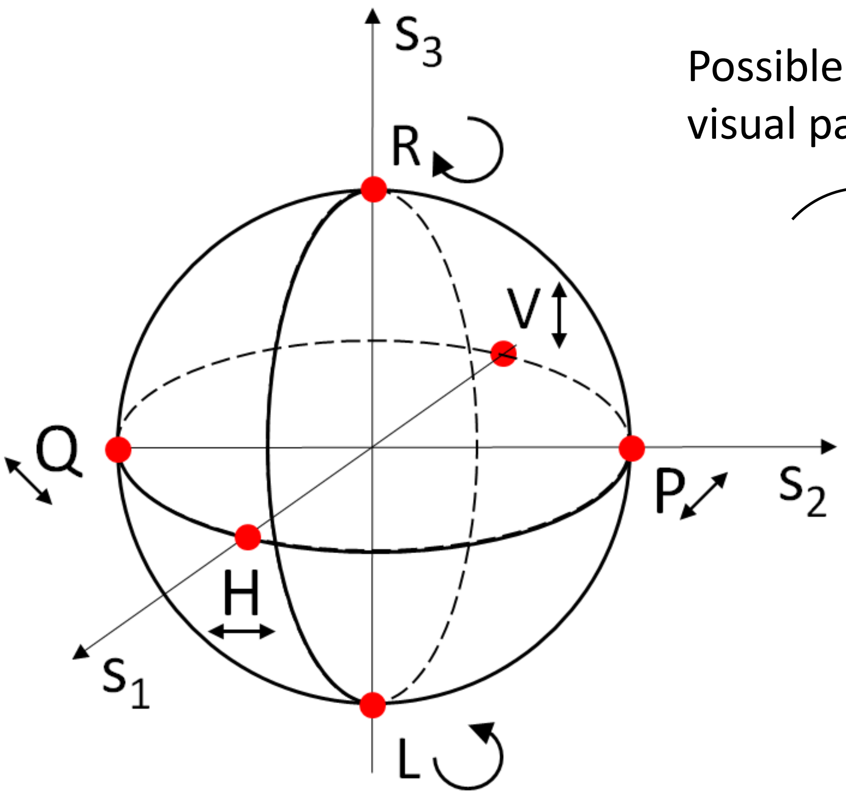


Spherical codes and lattice coding **[N.J.A. Sloane, 1981];**
[J.H. Conway, R.H. Hardin and N.J.A. Sloane, 1996];
[A.R. Calderbank, R.H. Hardin, E.M. Rains, P.W. Shor and N.J.A. Sloane, 1999]



Examples of 6-D constellations extracted from the E_6 lattice **[AP, V. Pizurica, V. Šenk, 1998]**

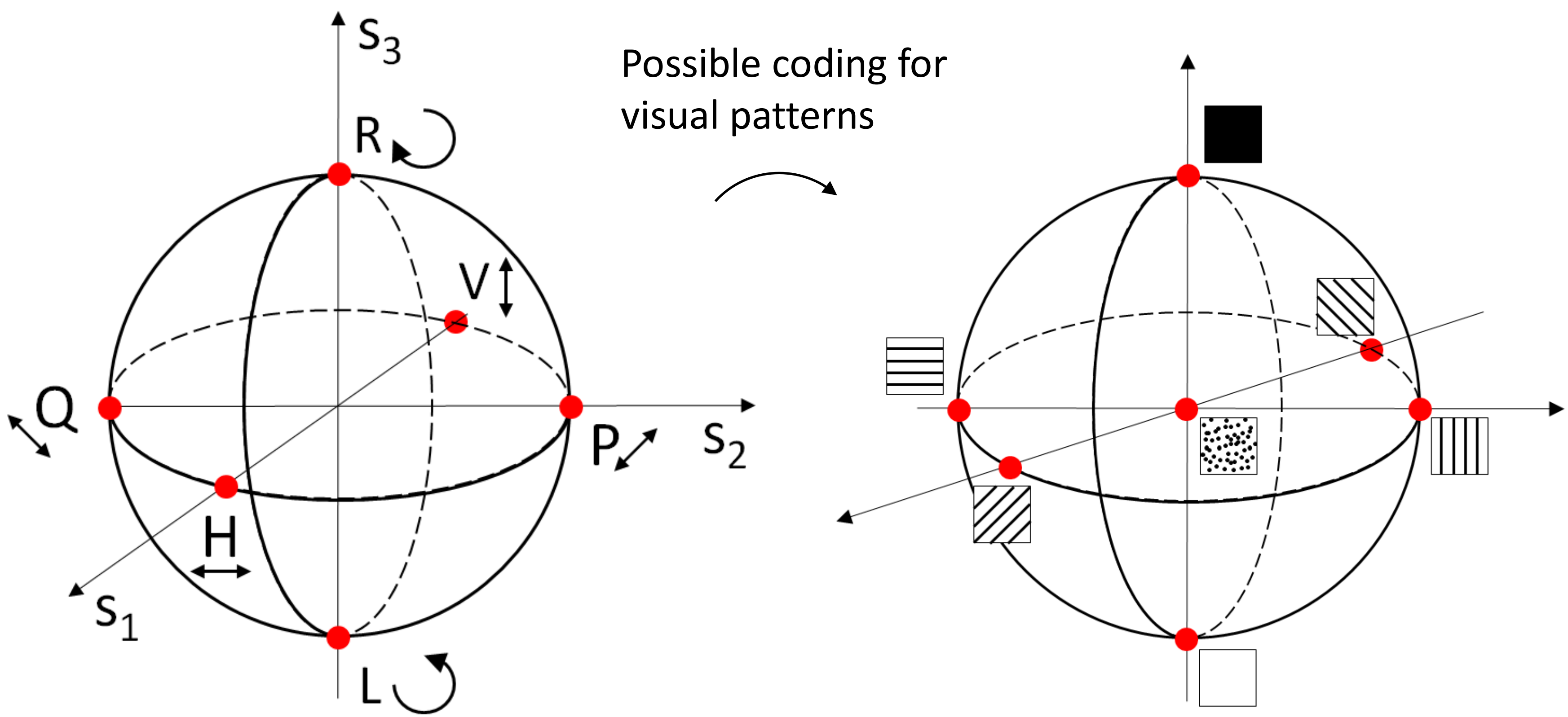
Visual patterns on the Poincaré sphere?

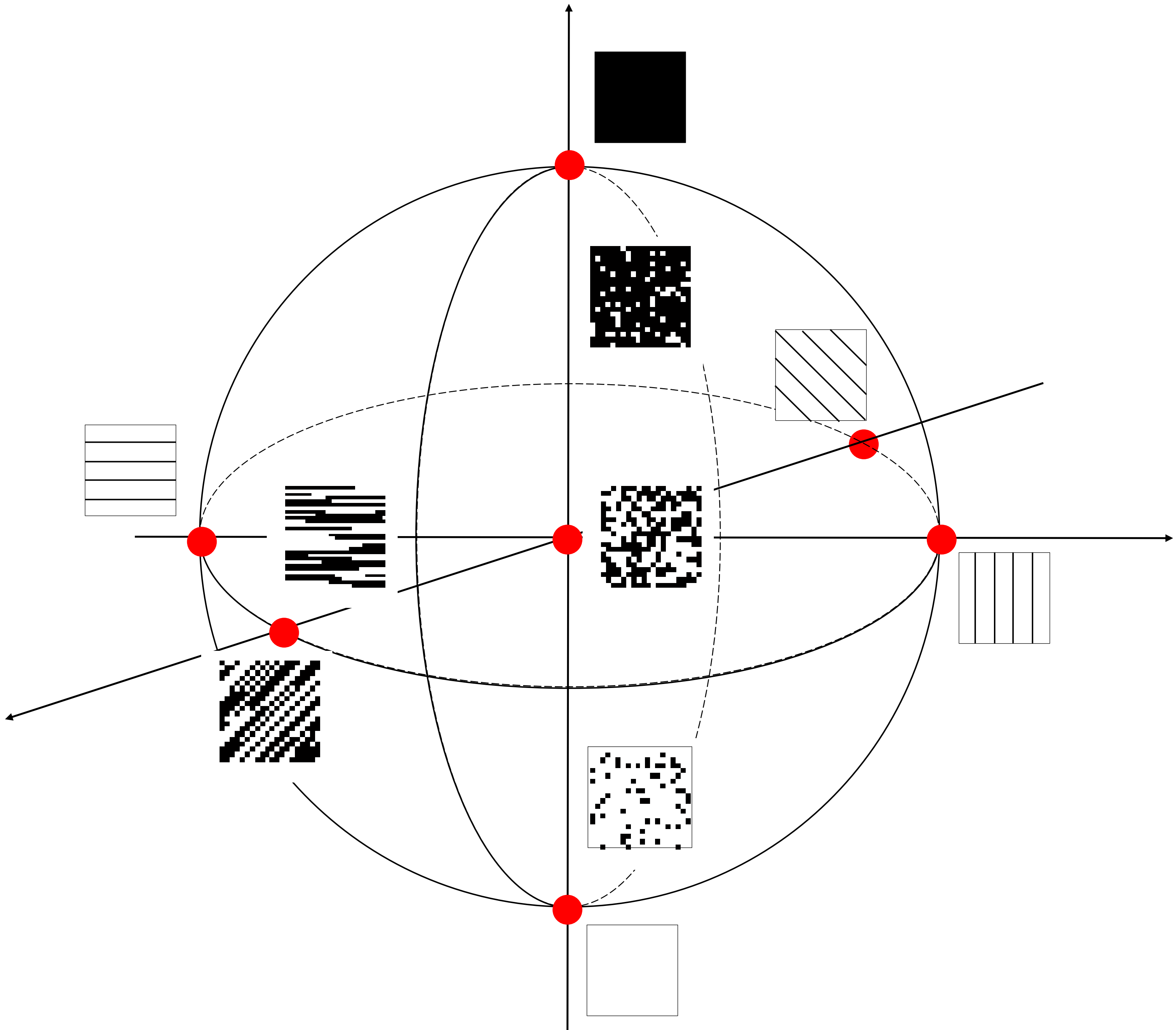


Possible coding for visual patterns

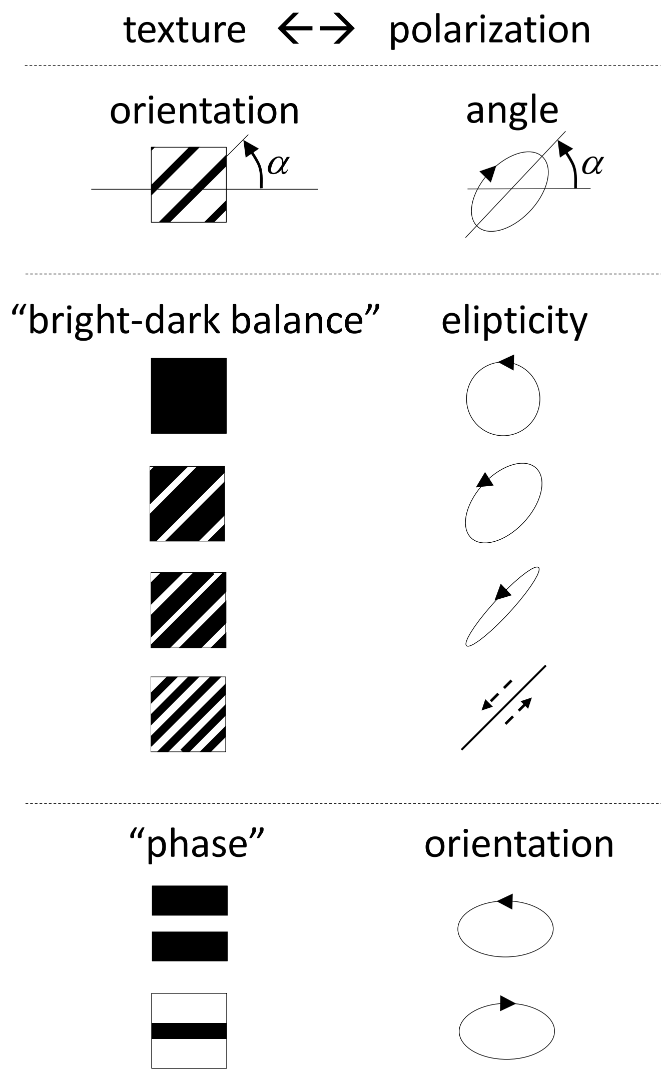
?

Visual patterns on the Poincaré sphere?

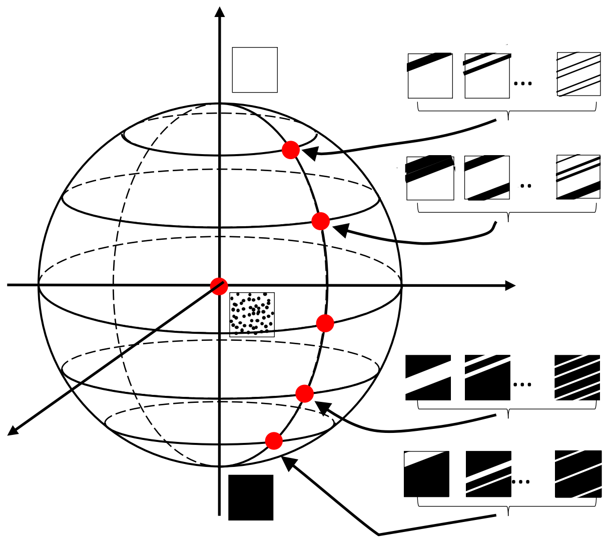




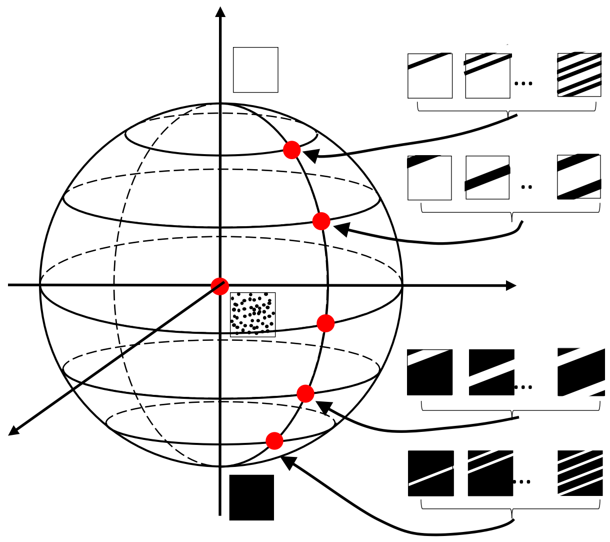
Some parallels with polarization encoding



Two examples with different formulations of the elevation angle

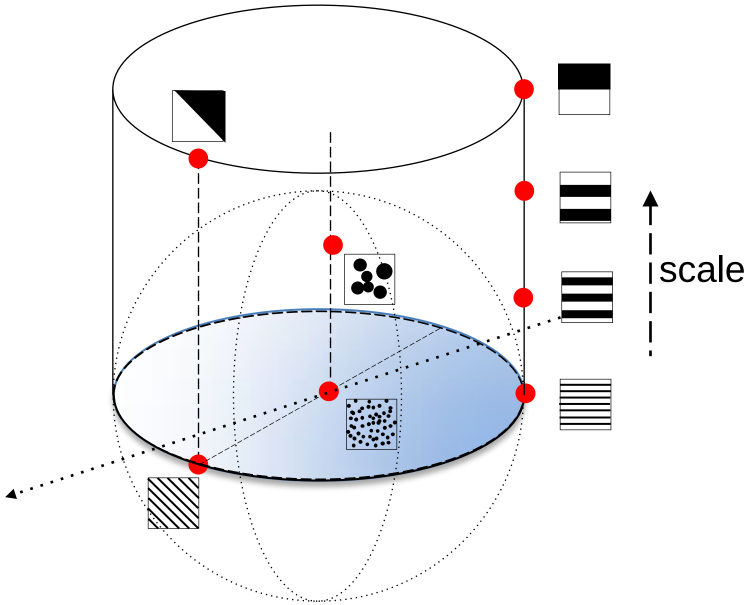


Formulation 1



Formulation 2

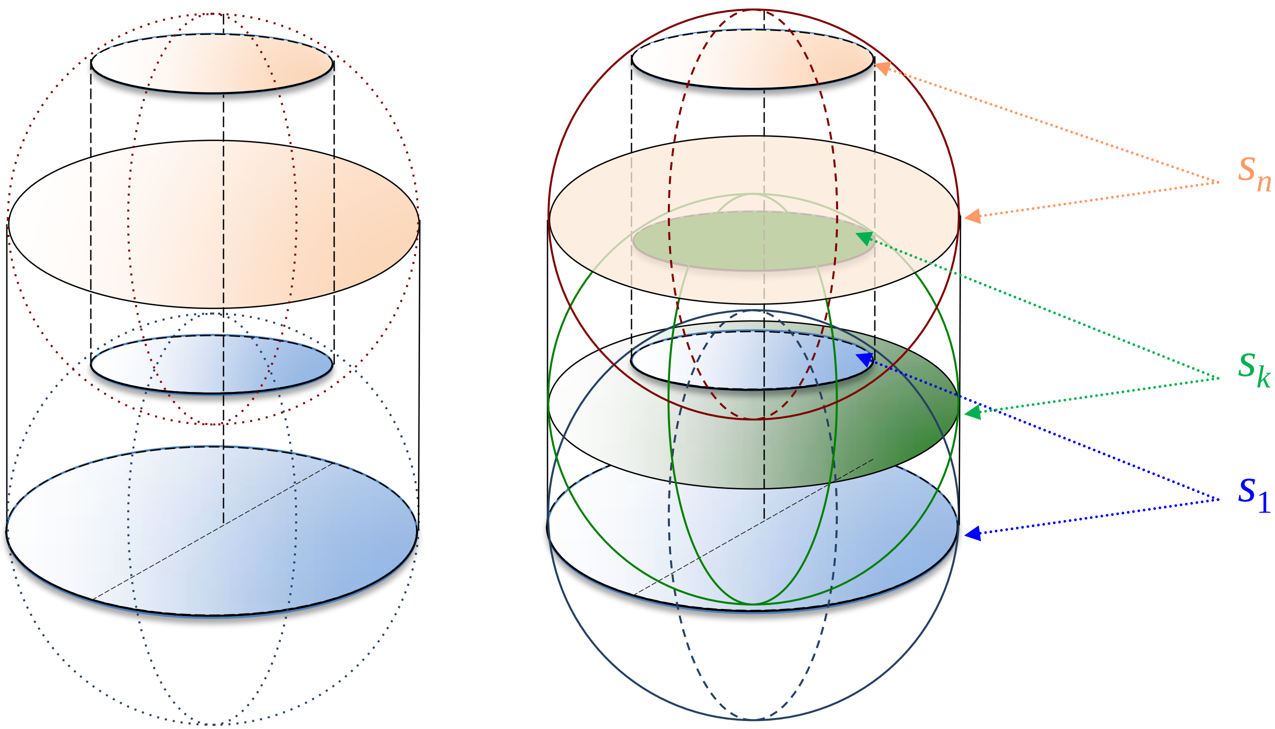
Scale cylinder



Take an intersection of the sphere with any plane parallel to the equatorial plane

Any point on the resulting circle is a projection of a 4D line
→ The circle extends to a **scale cylinder**

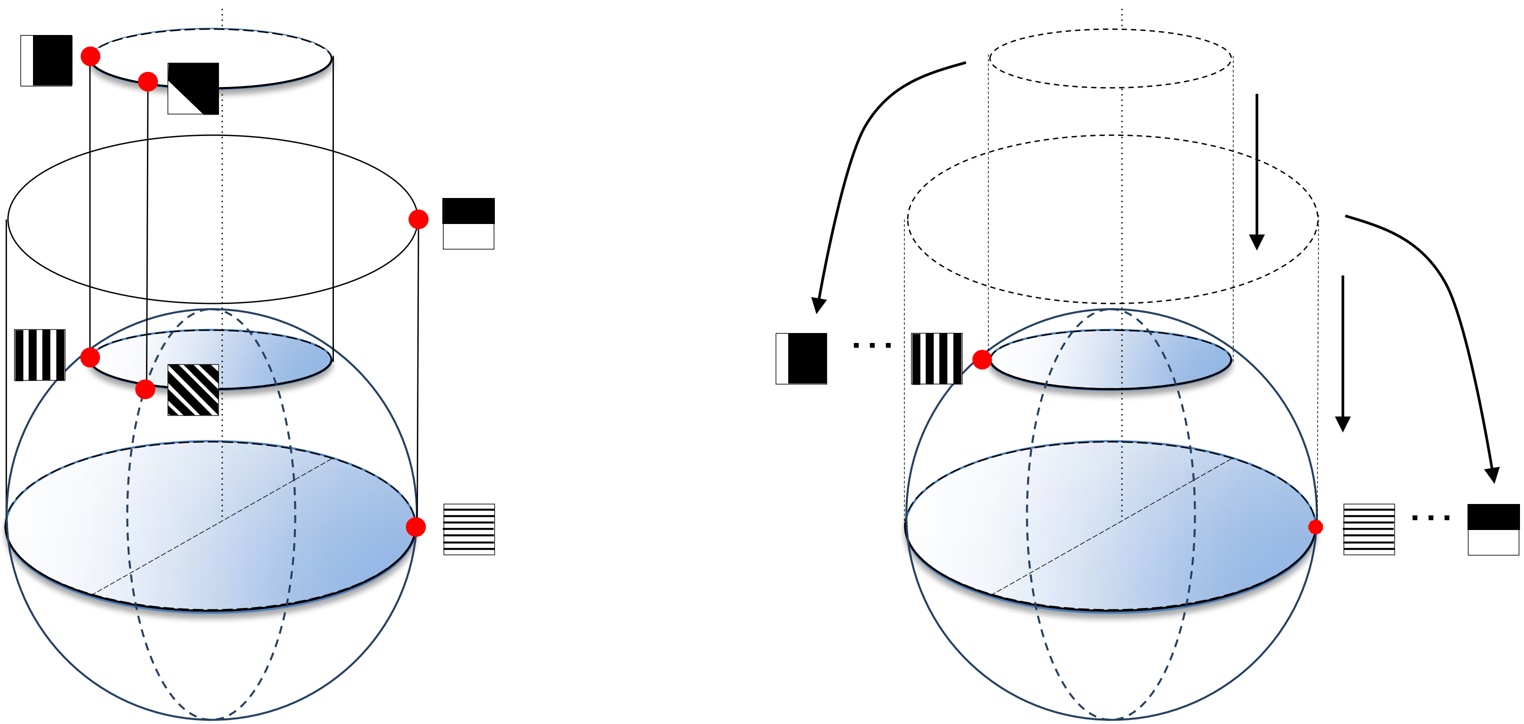
Scale hypersphere



Take from each scale cylinder a cross section at distance s_k from its base → All the resulting circles make a new sphere for scale s_k

The resulting spheres constitute a **scale hypersphere**

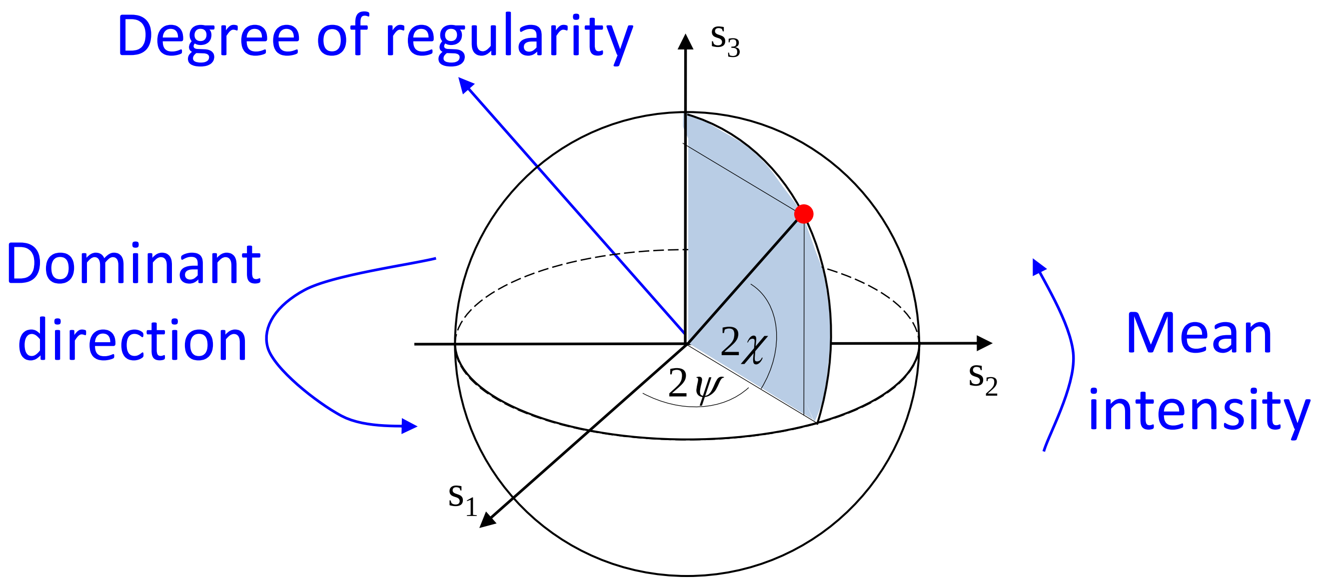
Unfolding and packing together the scales



Pool the scale cylinder out → unfold the scales in 4th dimension

Make the scale cylinder collapse → project 4D space on a 3D space where each point corresponds to a variety of scales

Constructing a toy example

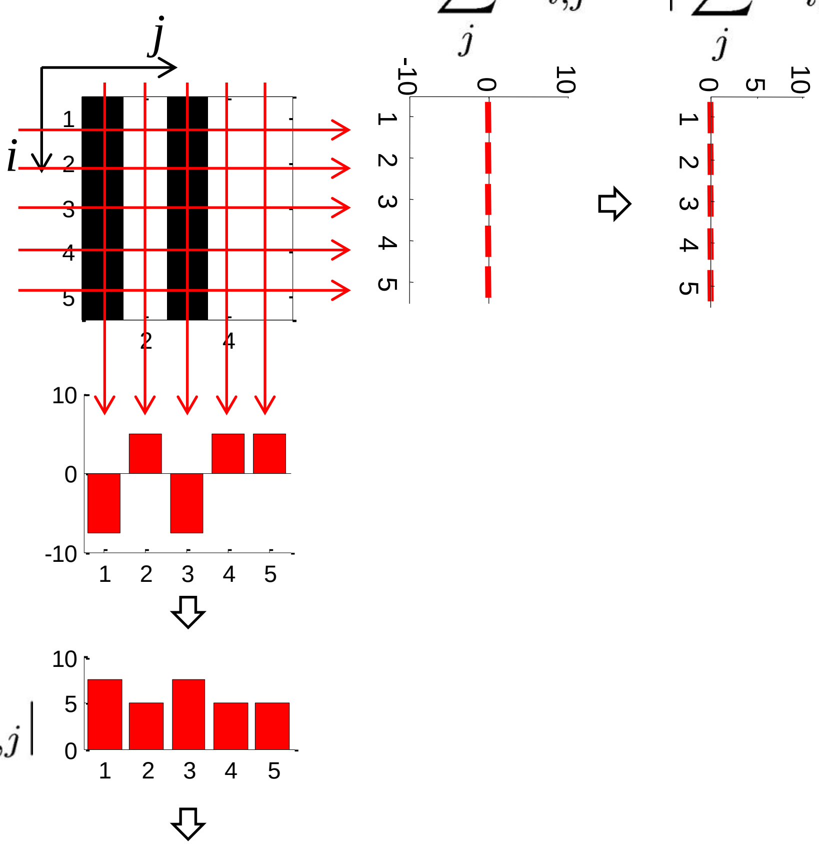


Dominant direction estimation: idea

Consider a zero mean image patch $\mathbf{w} = \{w_{i,j}\}, \; w_{i,j} = I_{i,j} - \mu_I$

E.g. ■ -1.5 □ 1

$\sum_j w_{i,j}$ $\quad |\sum_j w_{i,j}|$



$$R_h = \sum_i |\sum_j w_{i,j}| = 0$$

$\sum_i w_{i,j}$

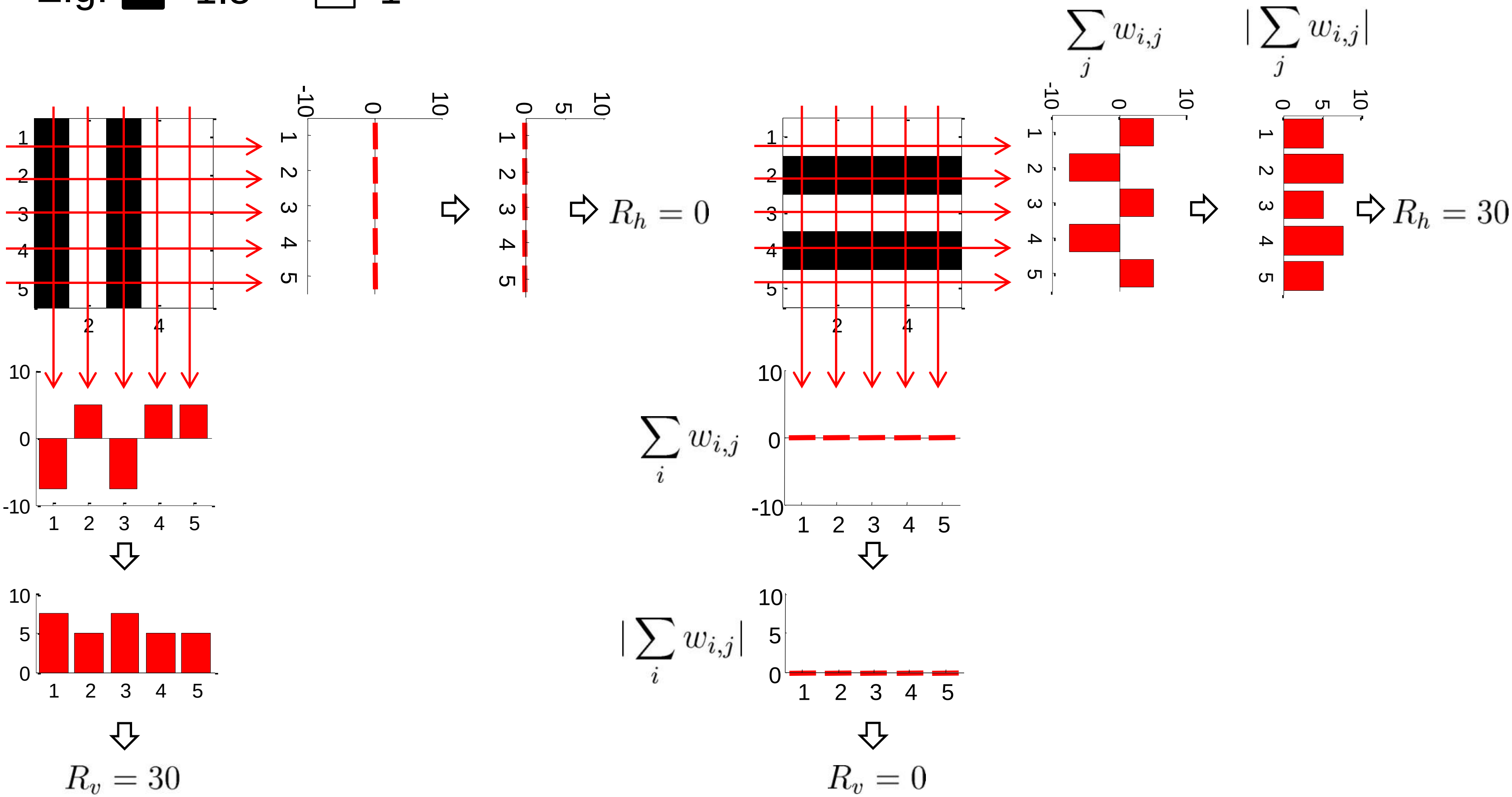
$|\sum_i w_{i,j}|$

$$R_v = \sum_j |\sum_i w_{i,j}| = 30$$

Dominant direction estimation: idea

Consider a zero mean image patch $\mathbf{w} = \{w_{i,j}\}$, $w_{i,j} = I_{i,j} - \mu_I$

E.g. ■ -1.5 □ 1



Design a direction estimation method based on the ratio of R_v and R_h.

Dominant direction estimation: idea

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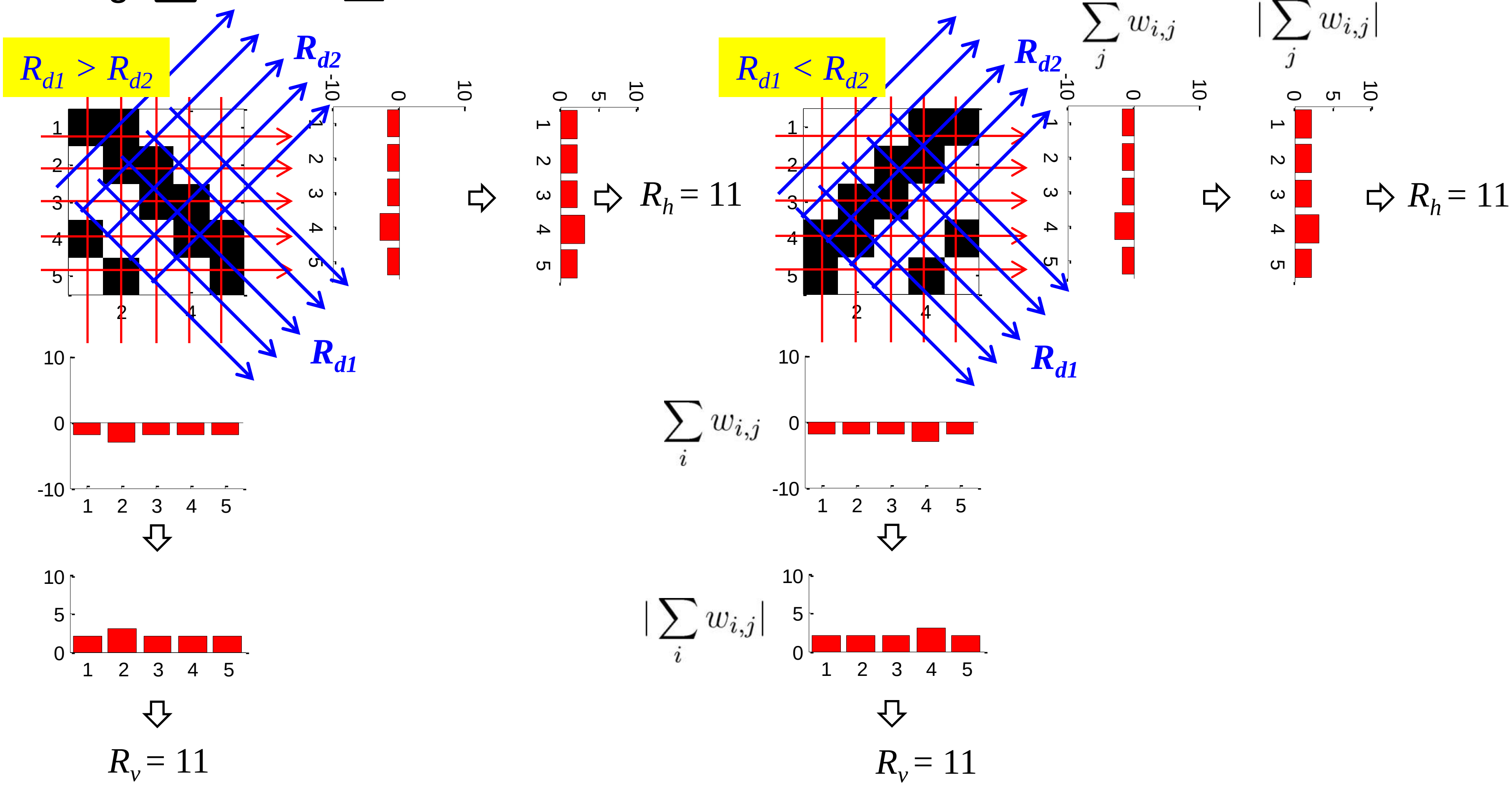
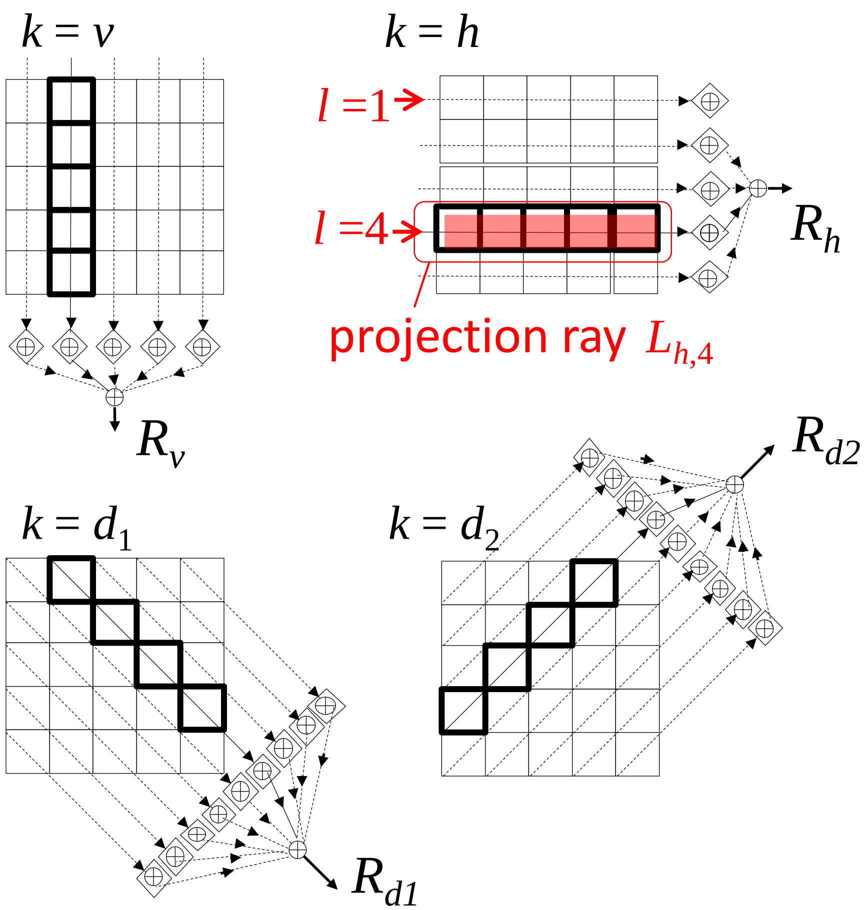
E.g. ■ -1.5 □ 1



Two diagonal projectors are sufficient to remove mirroring ambiguity .

Dominant direction estimation: method



Absolute value of the sum of elements, normalized by their number

$\mathbf{w} = \{w_{i,j}\}$ **zero-mean** image patch

$$U_{k,l} = \frac{1}{|L_{k,l}|} \sum_{(i,j) \in L_{k,l}} w_{i,j}$$

$$R_k = \frac{1}{\rho_k} \sum_{k=1}^{\rho_k} |U_{k,l}|$$

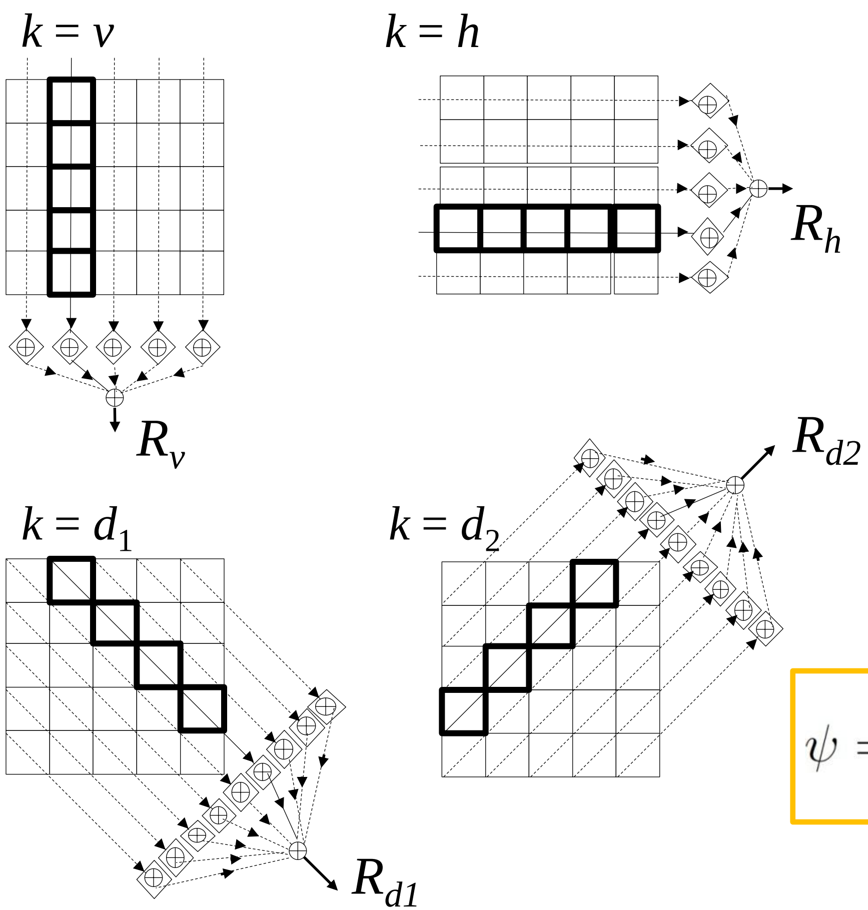
ρ_k: number of projection rays for direction k

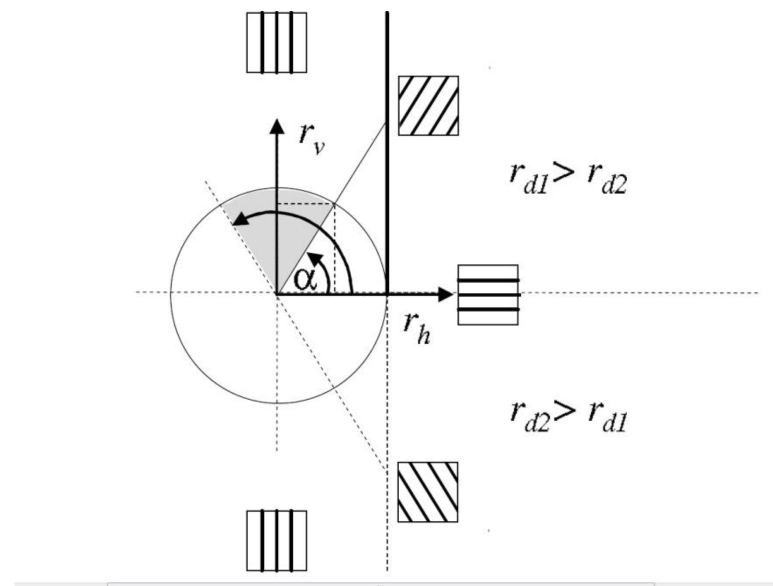
Normalization:

$$r_{h,v} = \frac{R_{h,v}}{(R_h^2 + R_v^2)^{\frac{1}{2}}}$$

$$r_{d_1,d_2} = \frac{R_{d_1,d_2}}{(R_{d_1}^2 + R_{d_2}^2)^{\frac{1}{2}}}$$

Dominant direction estimation: method



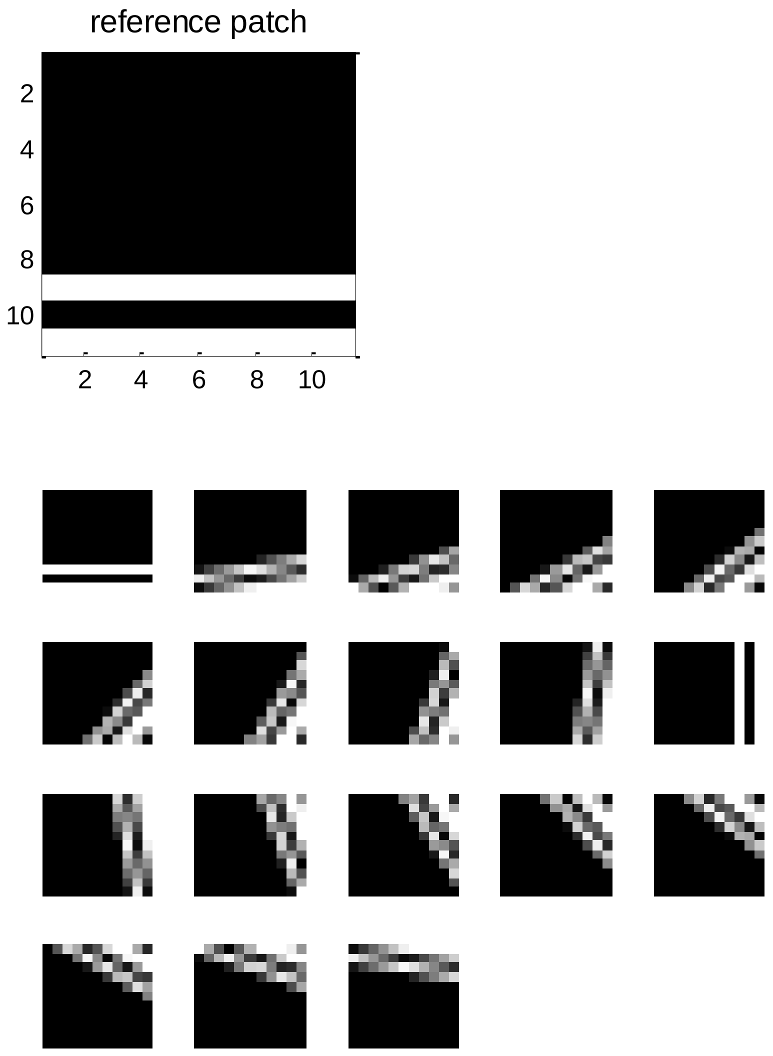


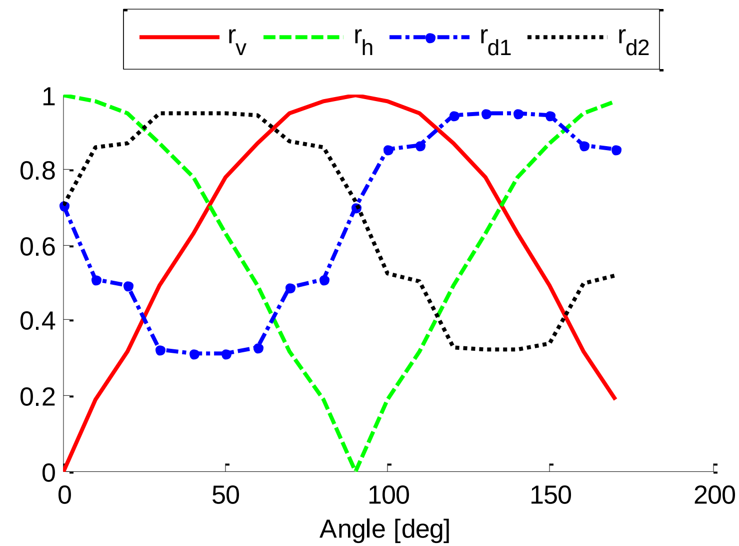
$$\psi = \arctan\left(\frac{r_v}{r_h}\right) + d_{corr} \cdot 2\left(\frac{\pi}{2} - \arctan\left(\frac{r_v}{r_h}\right)\right)$$

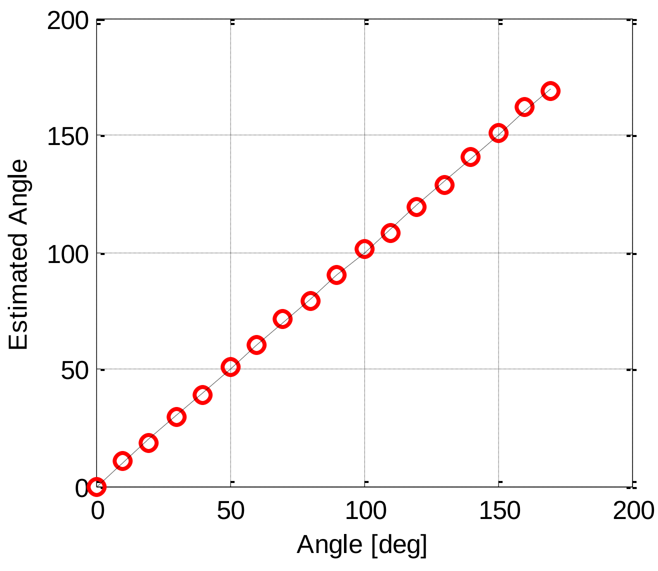
Absolute value of the sum of elements, normalized by their number

$$d_{corr} = \begin{cases} 0, & \text{if } \; r_{d1} \geq r_{d2}, \\ 1, & \text{otherwise.} \end{cases}$$

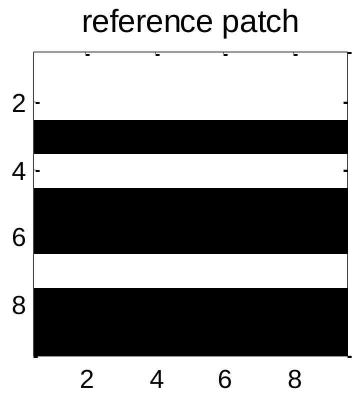
Dominant direction estimation: examples

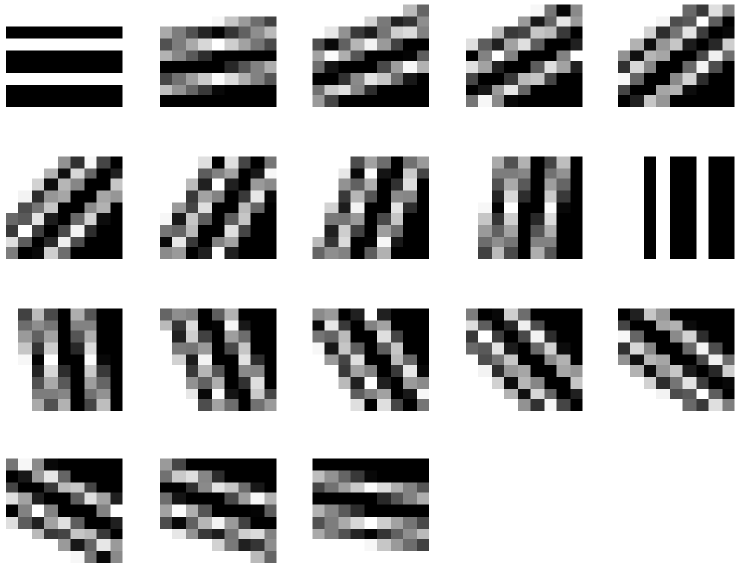


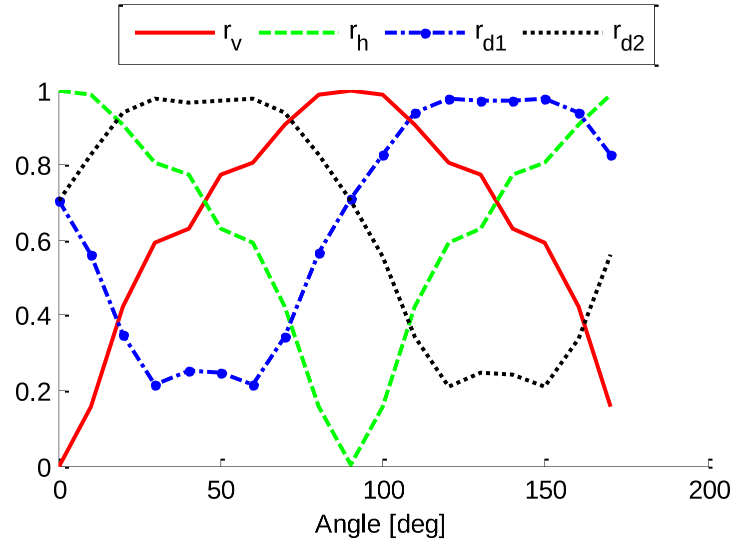


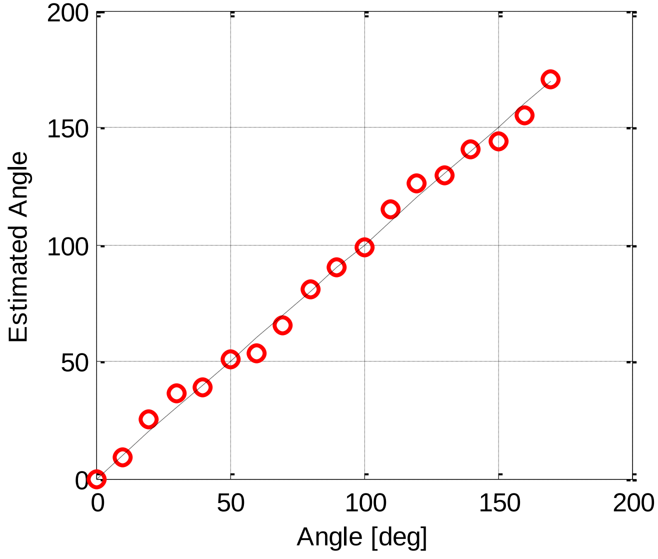


Dominant direction estimation: examples

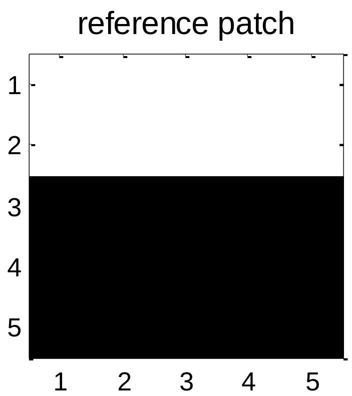


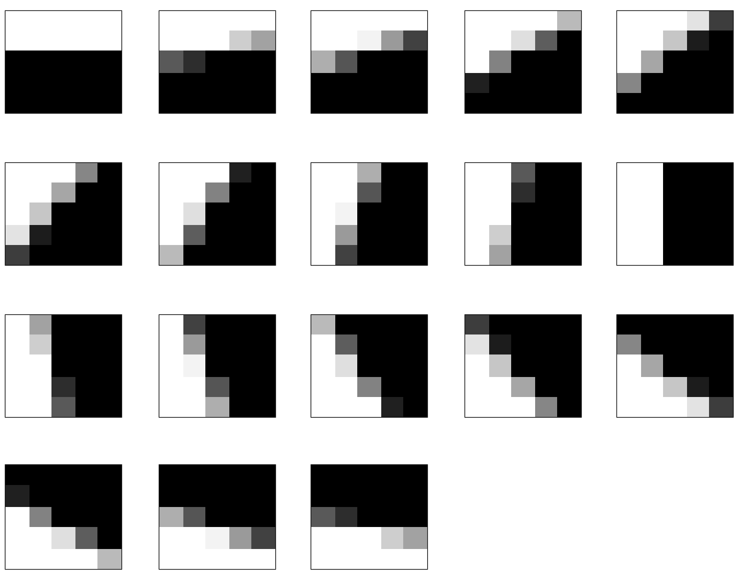


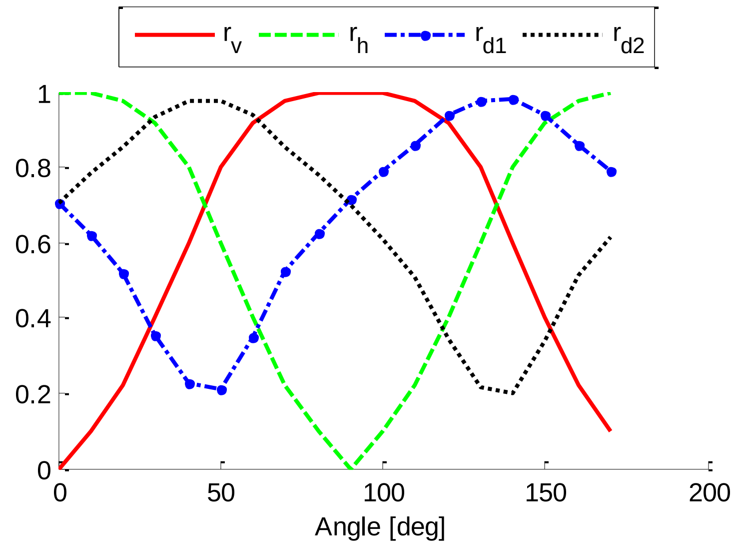


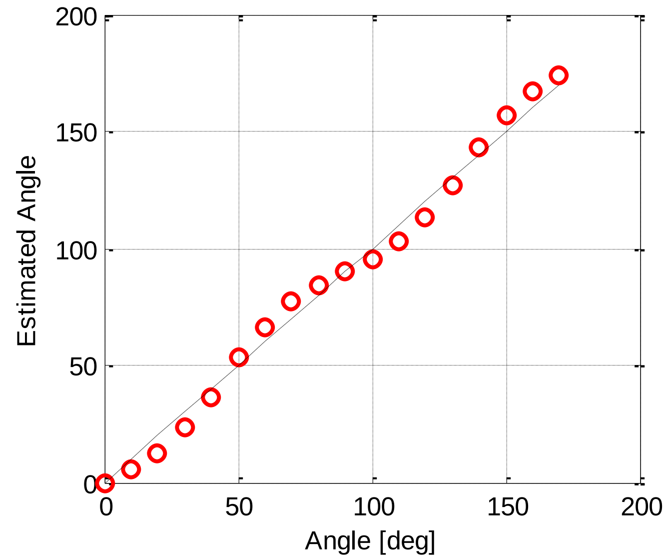


Dominant direction estimation: examples

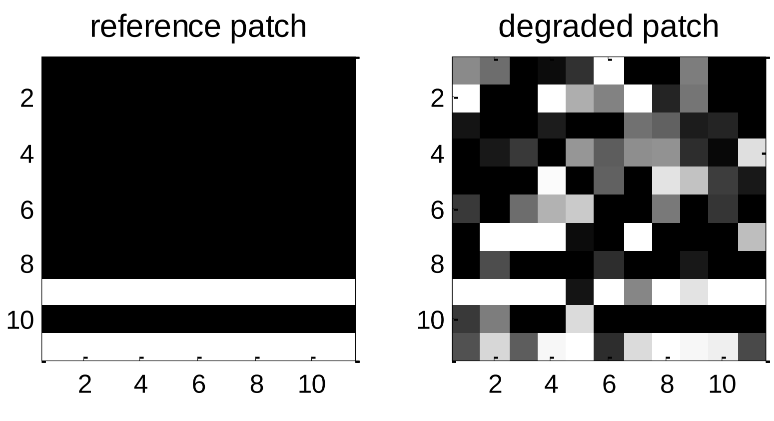


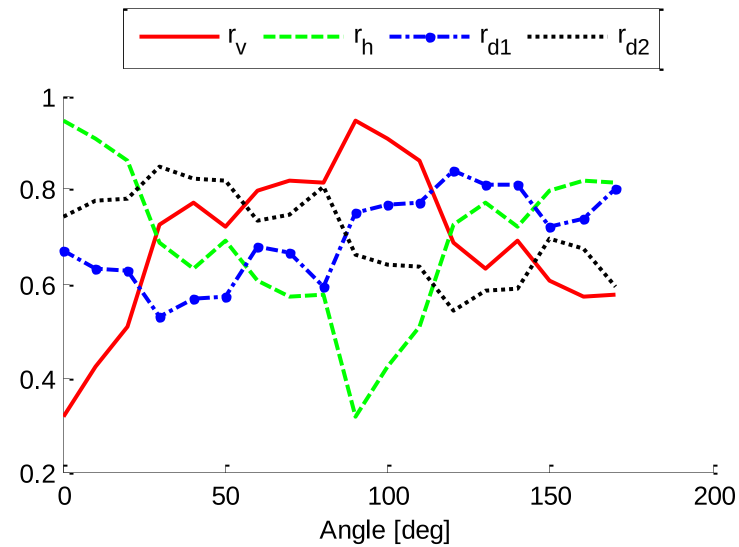


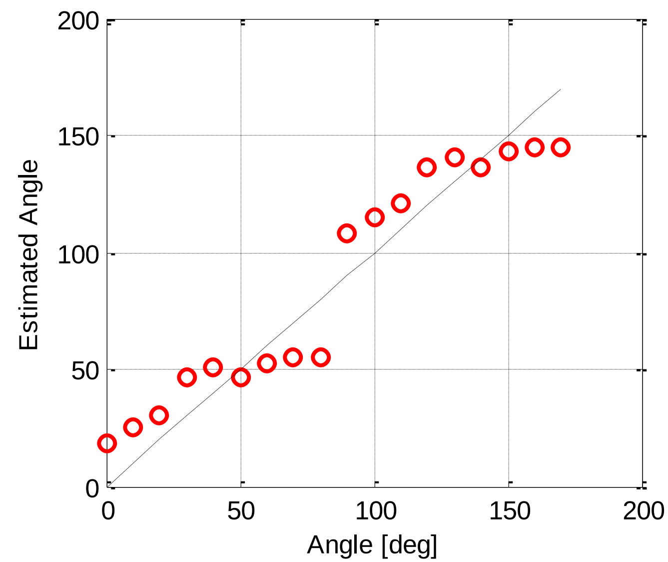




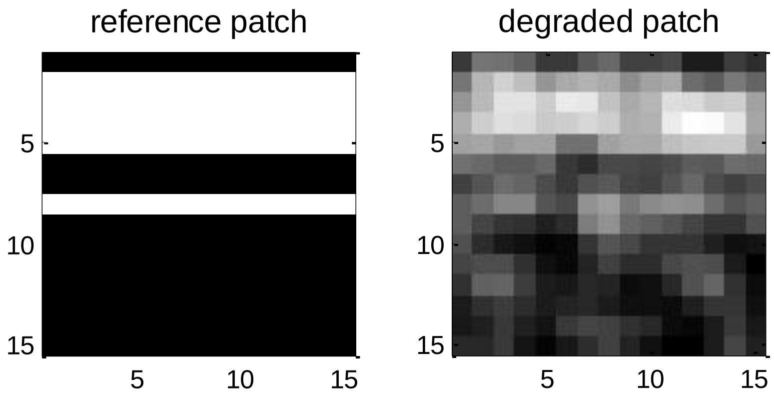
Dominant direction estimation: examples

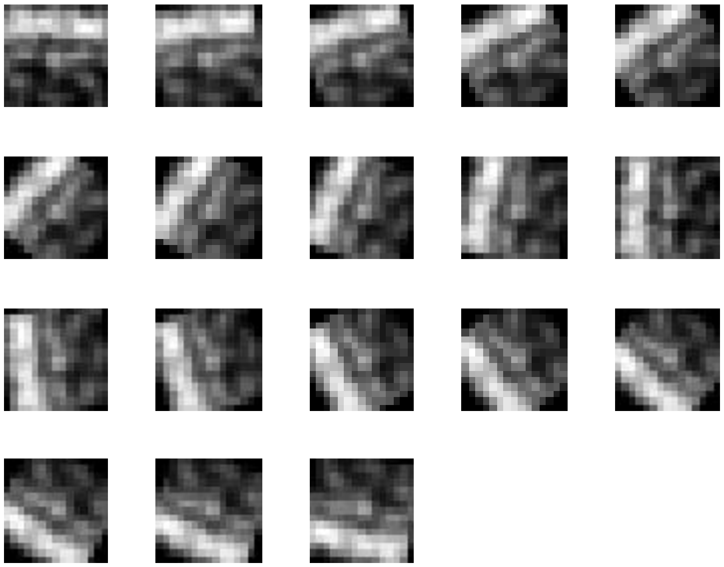


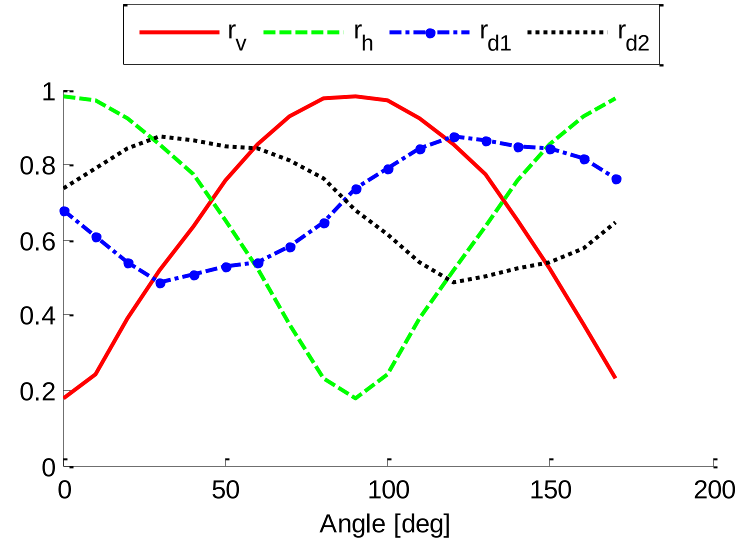


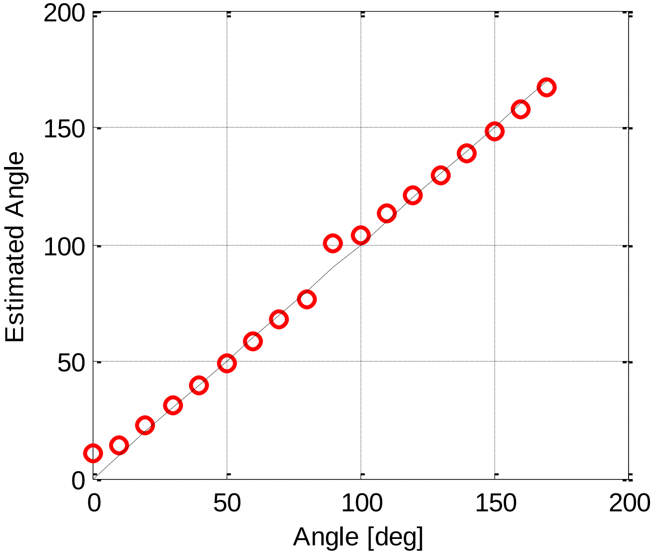


Dominant direction estimation: examples

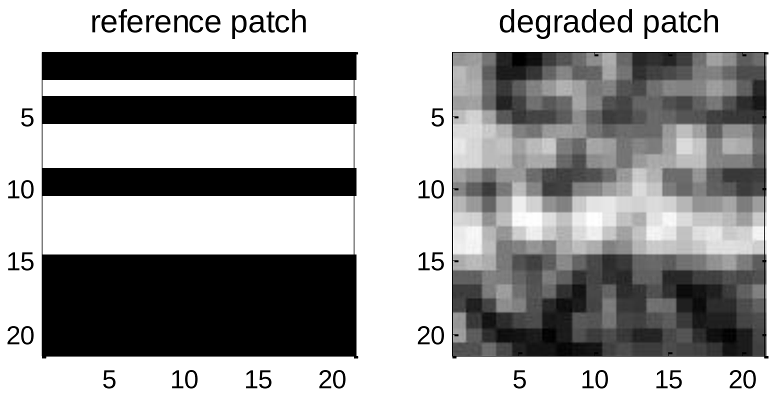


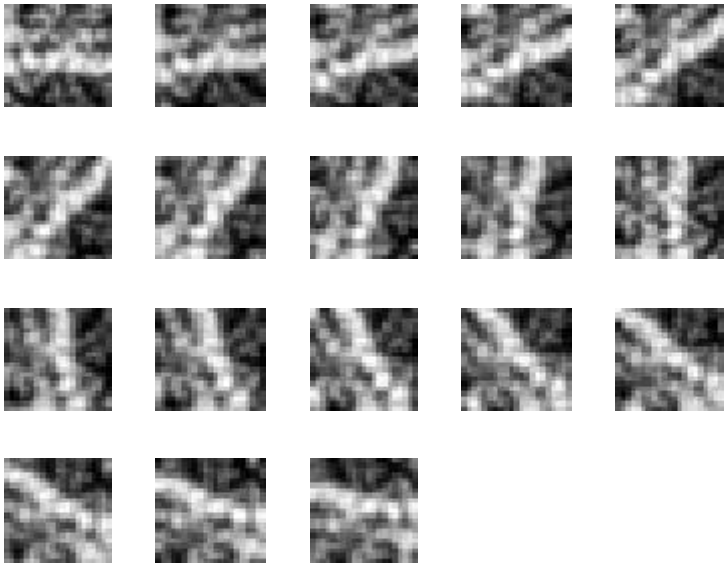


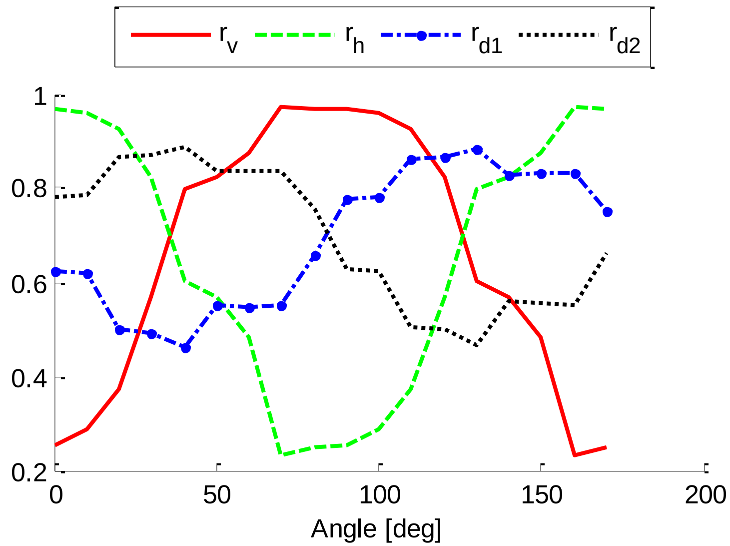


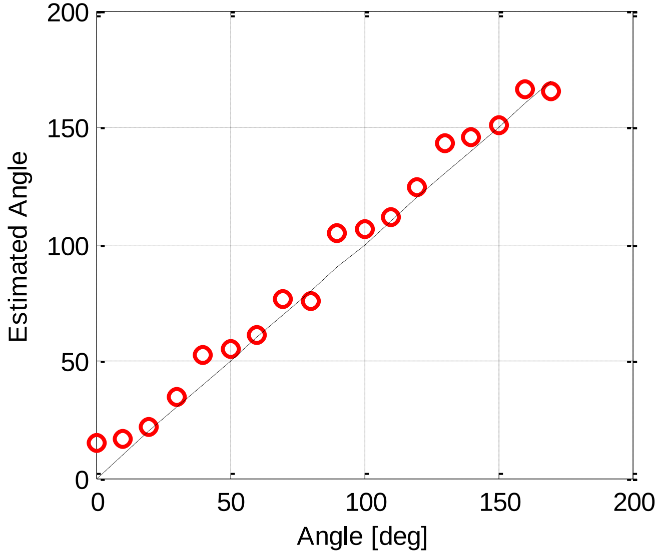


Dominant direction estimation: examples

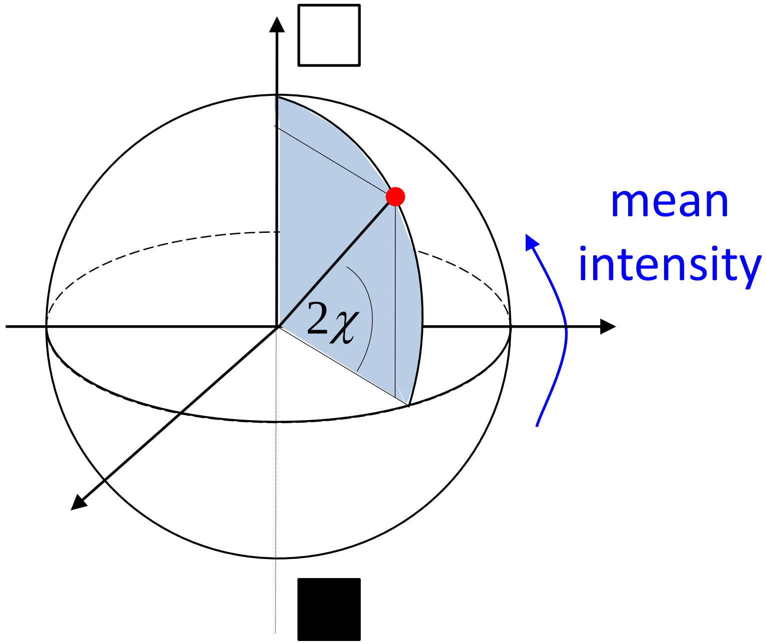








Encoding the level of grey

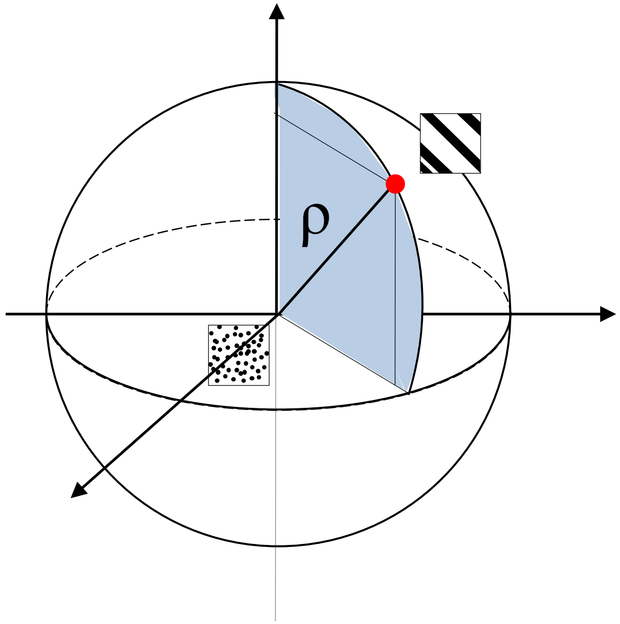


$$\Theta = 2\chi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Let T denote a normalized mean intensity of an image patch $I=\{I_{i,j}\}$

$$T = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} I_{i,j}}{255MN}\ , \quad 0 \leq T \leq 1,$$ and define $$\boxed{\Theta = 2\chi = (T - 0.5)\pi}$$

Encoding patch regularity



$$\rho \in [0, 1]$$

$$E_I = -\sum_j p_j log_2(p_j)$$

relative occurrence of grey level *j*

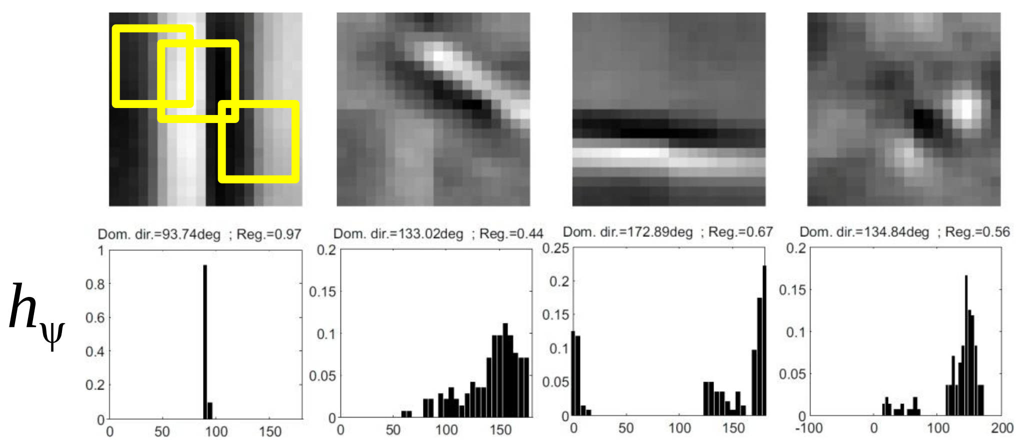
with 2 levels: $\max\{E_I\}=1$; with 256 levels: $\max\{E_I\}=8$

$$\rho_E = \min\left(1 - \frac{E_I - 1}{7}, 1\right)$$

Encoding patch regularity

Think of the degree of regularity as the degree of orientedness and examine local directional consistency (LDC).

Let ψ_i denote dominant orientation of a sub-block i and let h_ψ denote the histogram of $\psi=\{\psi_i \ldots \psi_i\}$.

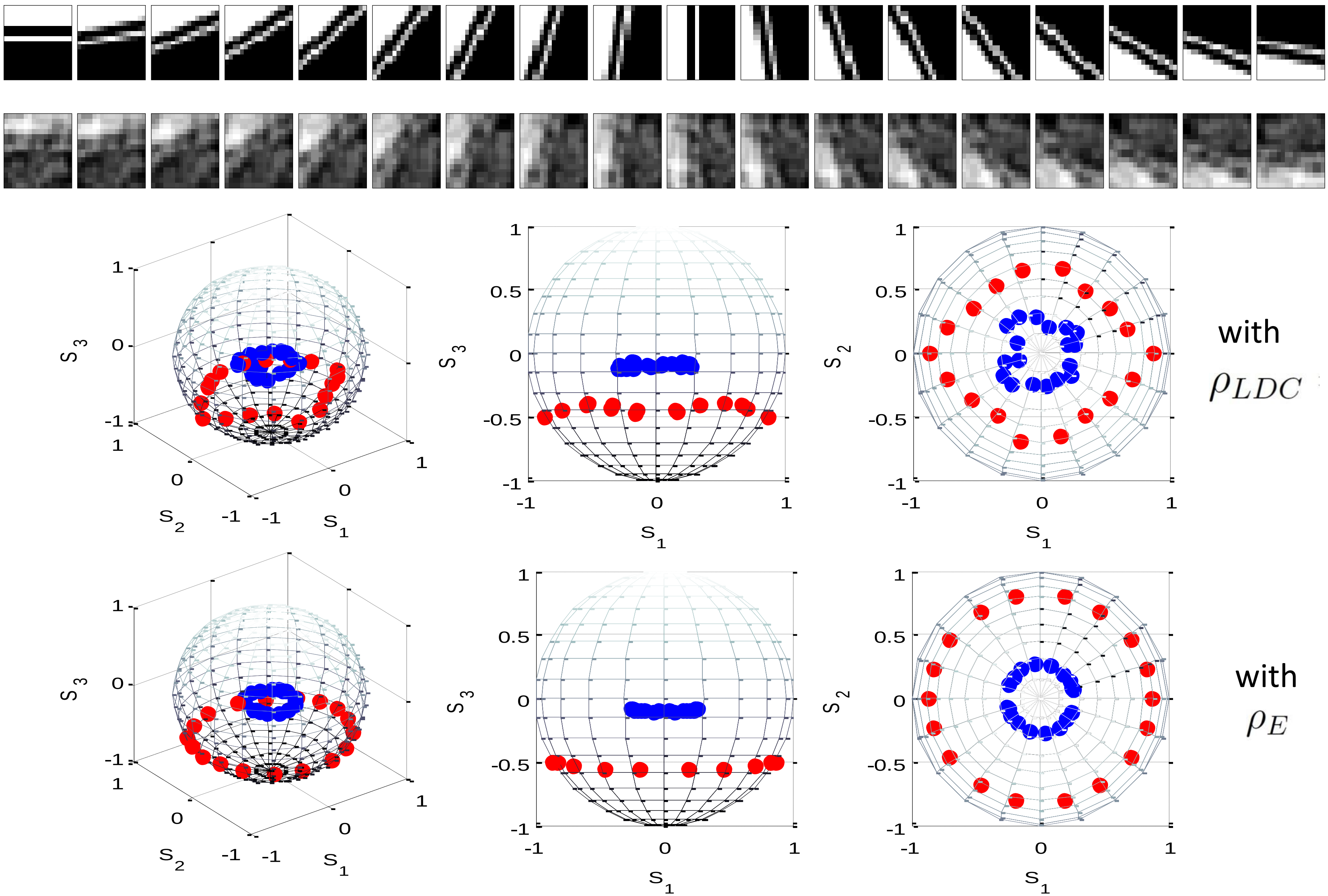


$$\rho_{LDC} = \frac{B-b}{B-1}$$

B – total number of bins in h_ψ

b – number of populated bins (with counts above a small threshold)

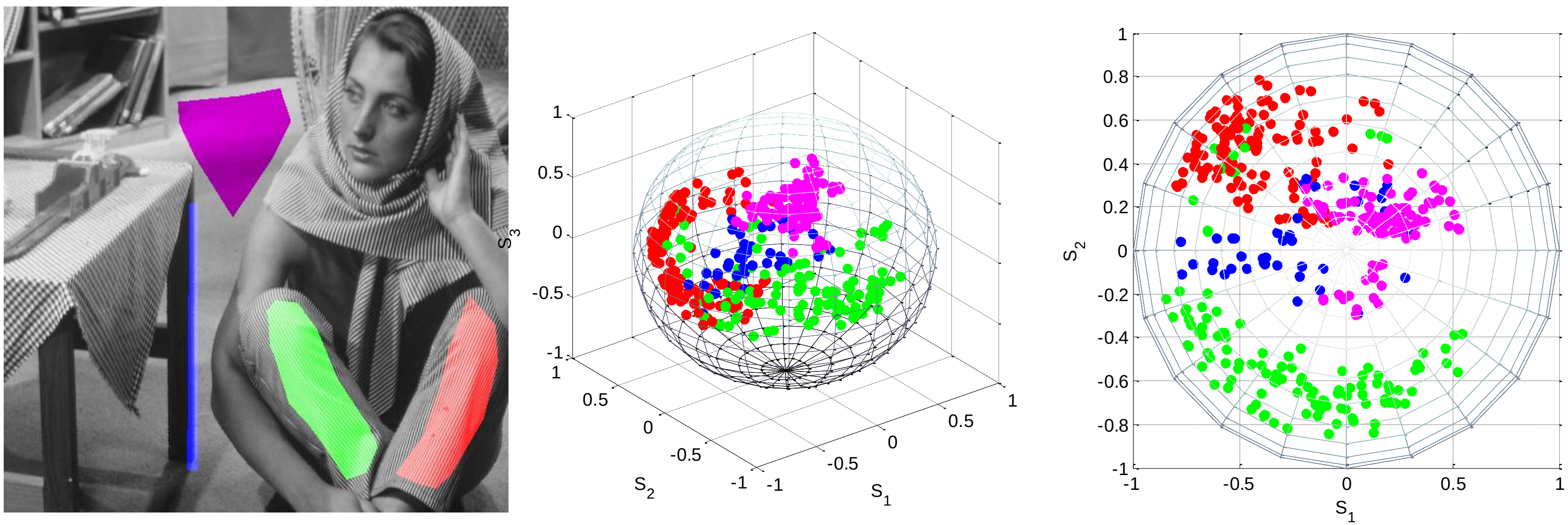
Patch encoding example



Some possible applications

- Patch clustering
- Analyzing learned dictionaries of image atoms
- Generating dictionaries of image atoms

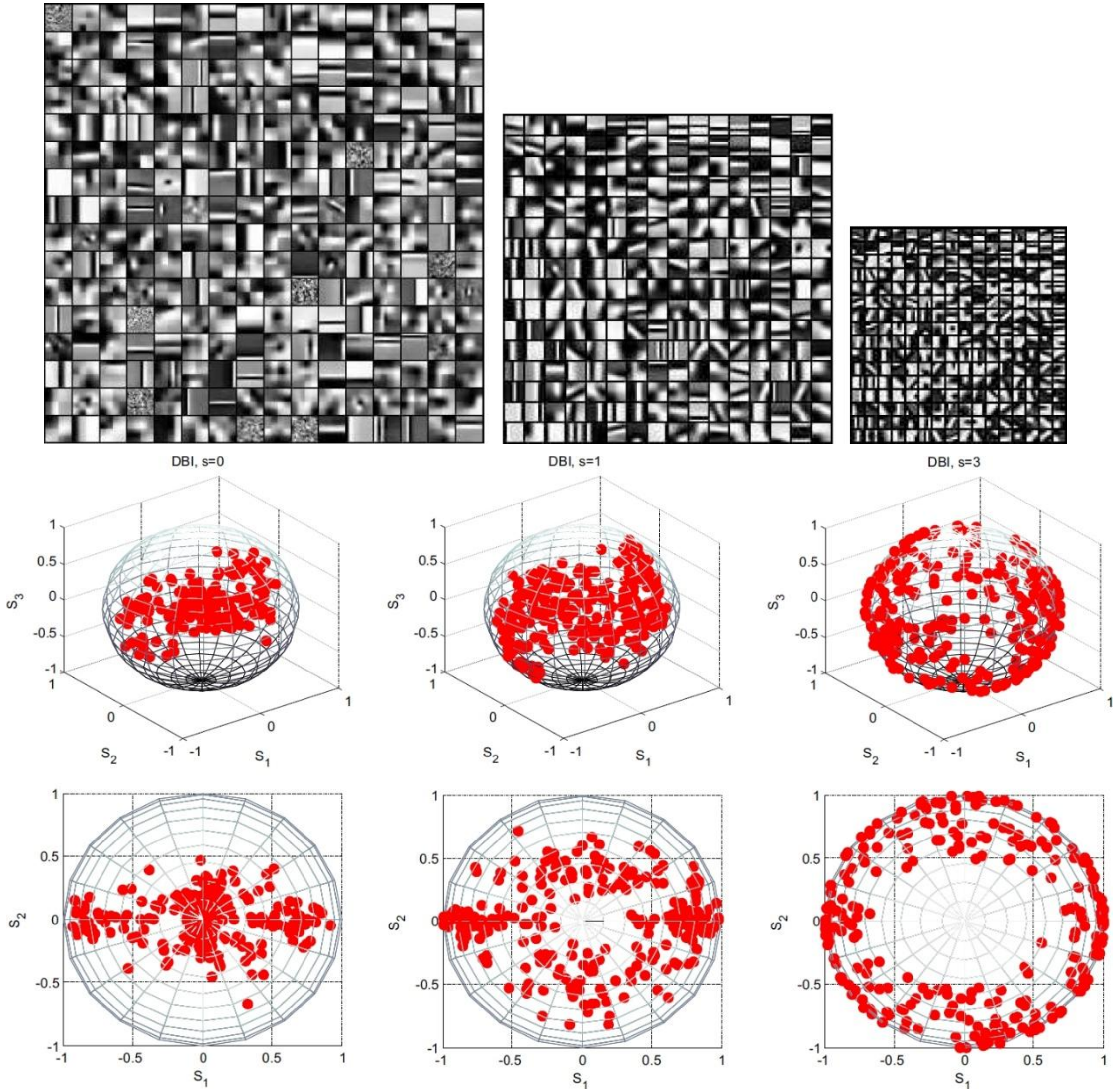
Applications: Patch clustering



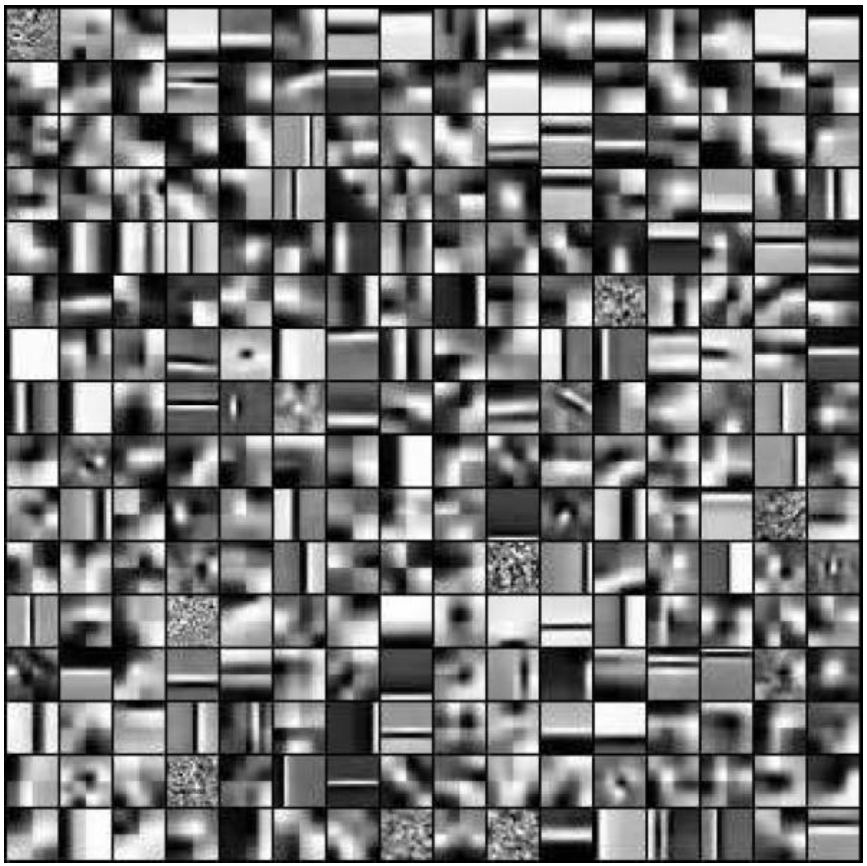
Random patches of size 16x16 taken from four image regions highlighted with the corresponding colors.

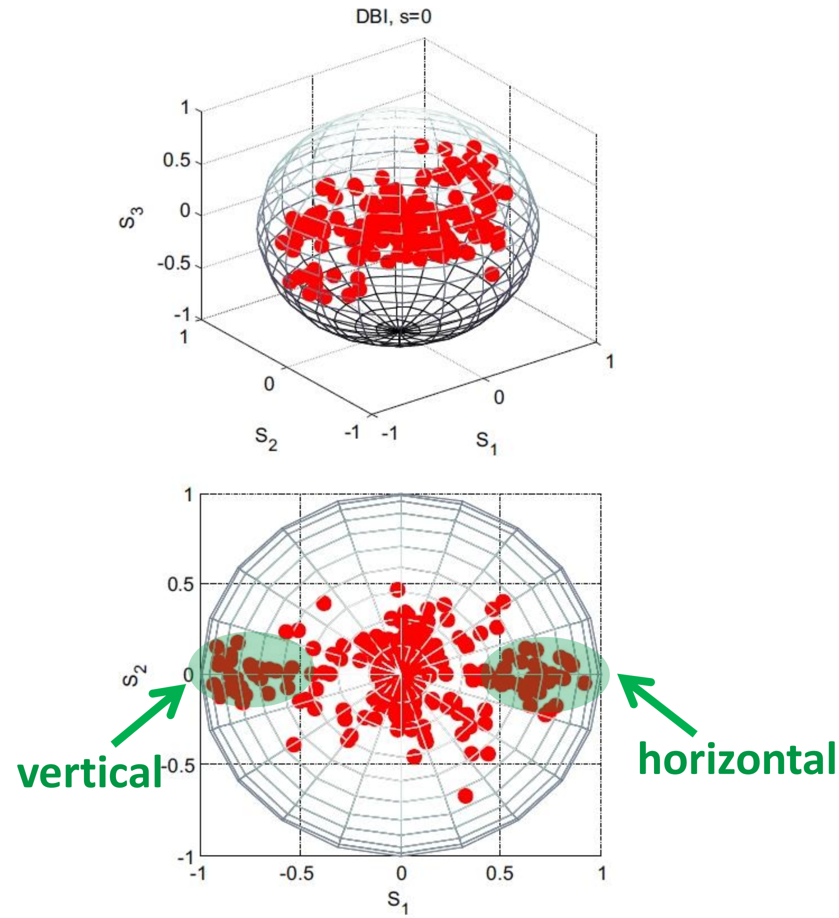
Applications: Dictionary analysis

Examples of multiscale dictionaries from **[Mairal, Sapiro and Elad, 2008]**



Dictionary analysis: Zoom In

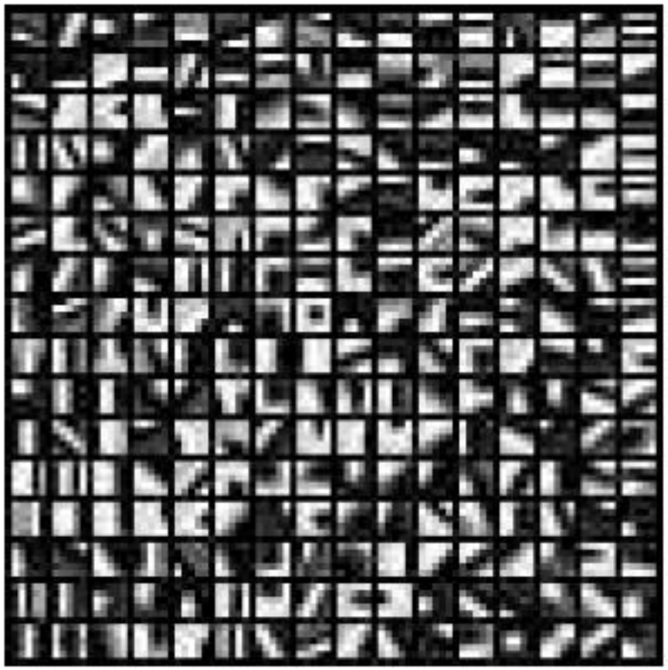




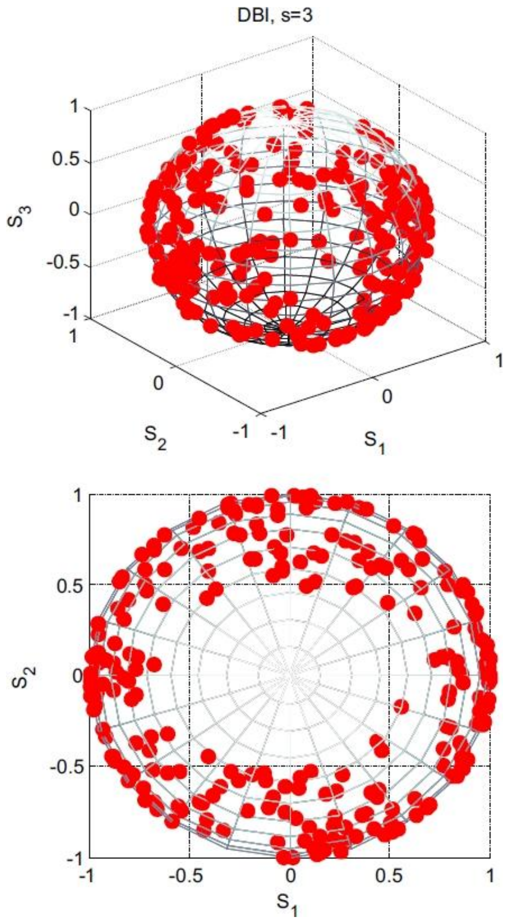
DB1, s=0 **[Mairal, Sapiro and Elad, 2008]**

Notice lack of diagonal highly oriented atoms – this is visible in the Poincaré representation!

Dictionary analysis: Zoom In

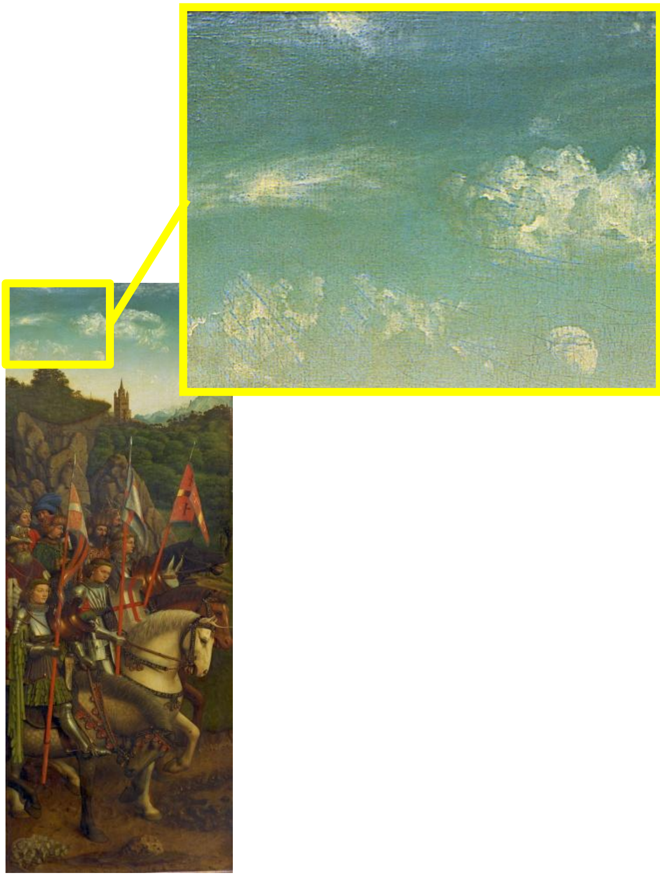


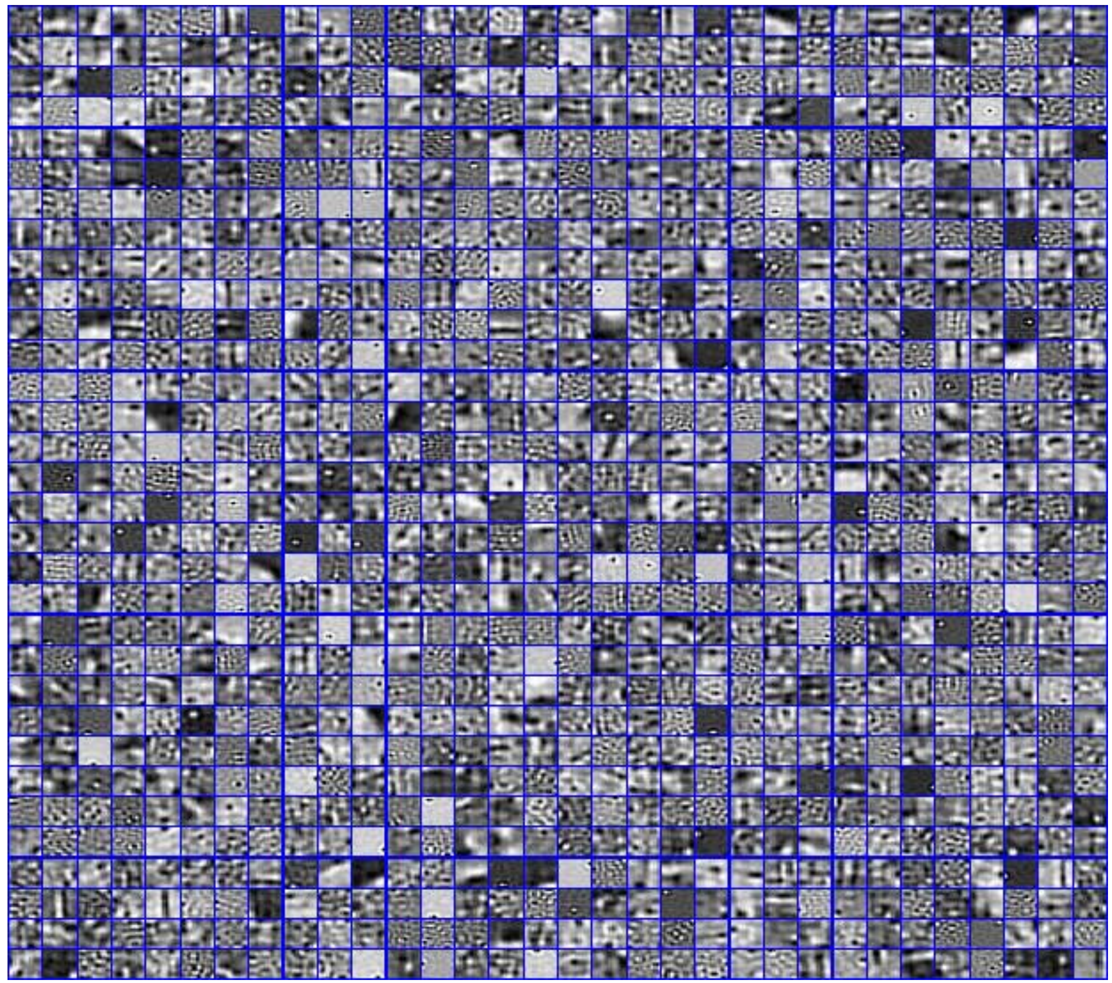
DB1, s=3 **[Mairal, Sapiro and Elad, 2008]**



Notice many diagonal atoms – reflected in the Poincaré code

Applications: Encoding image atoms





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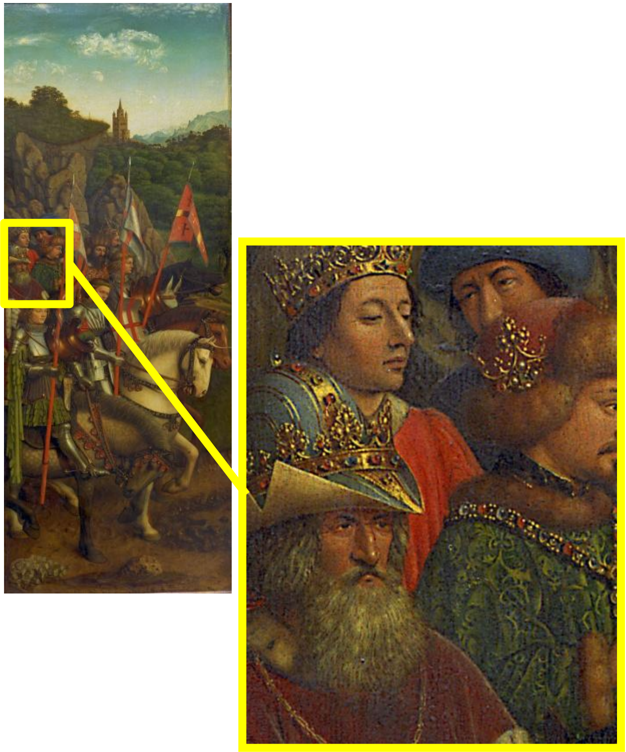
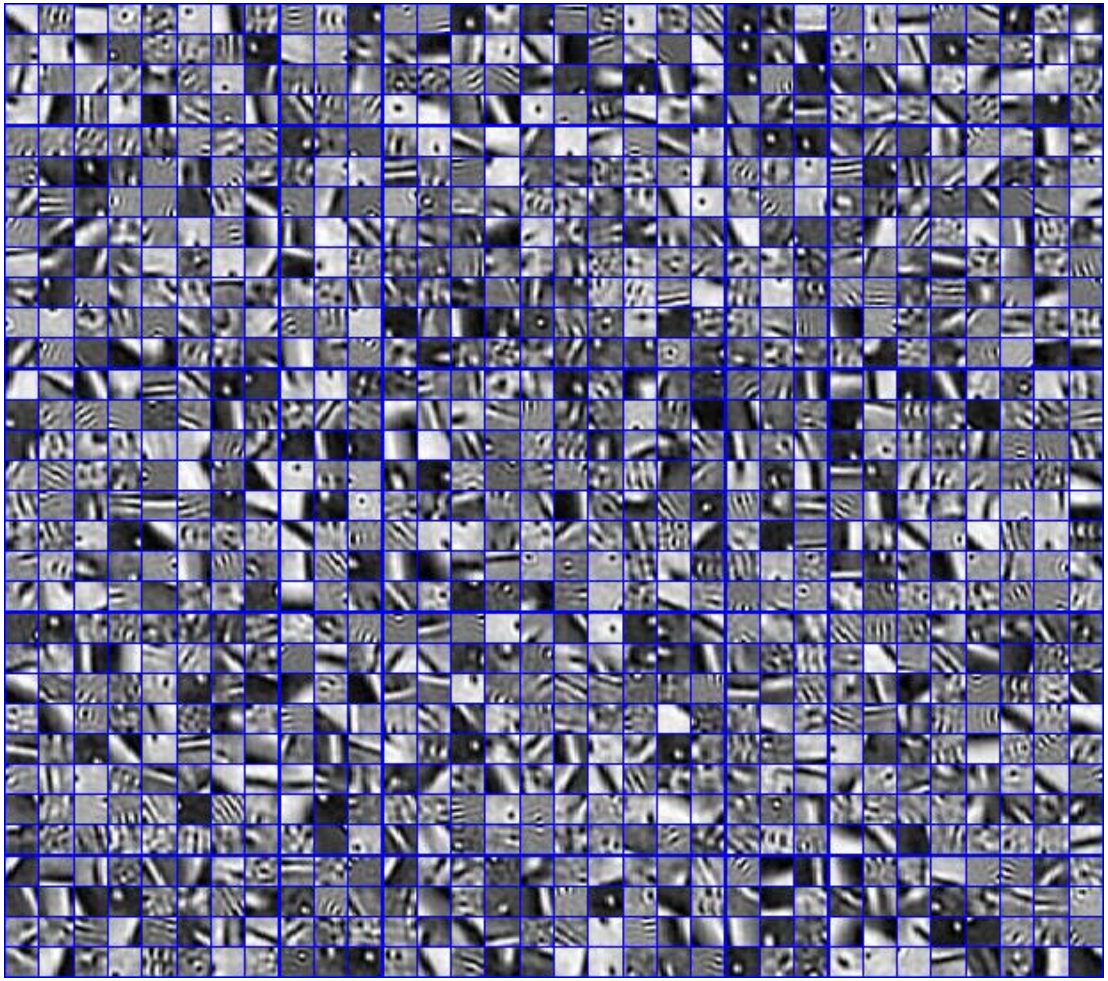
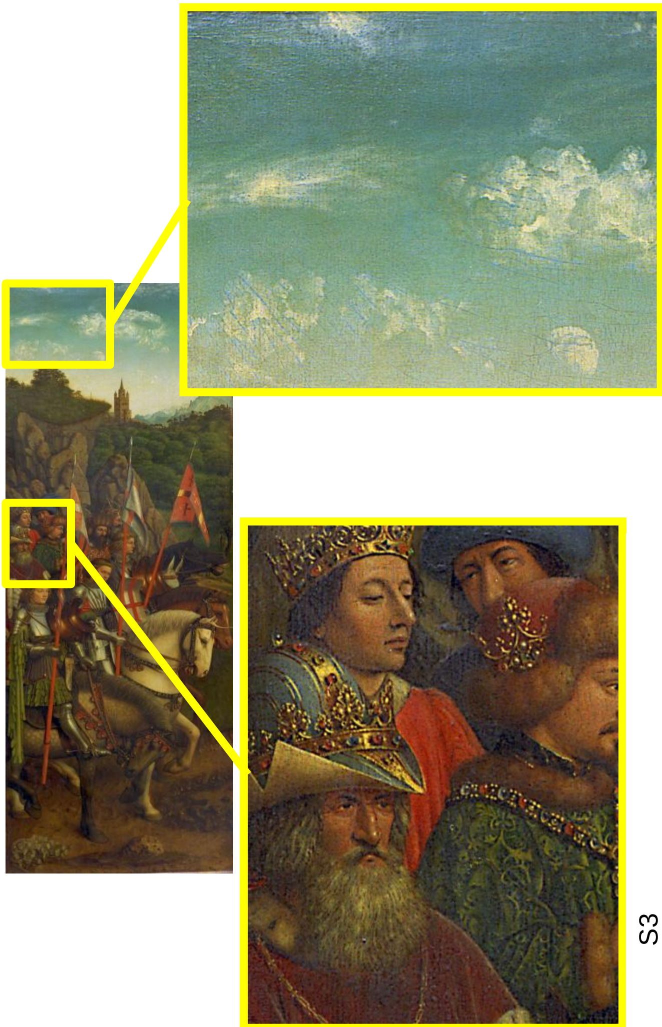
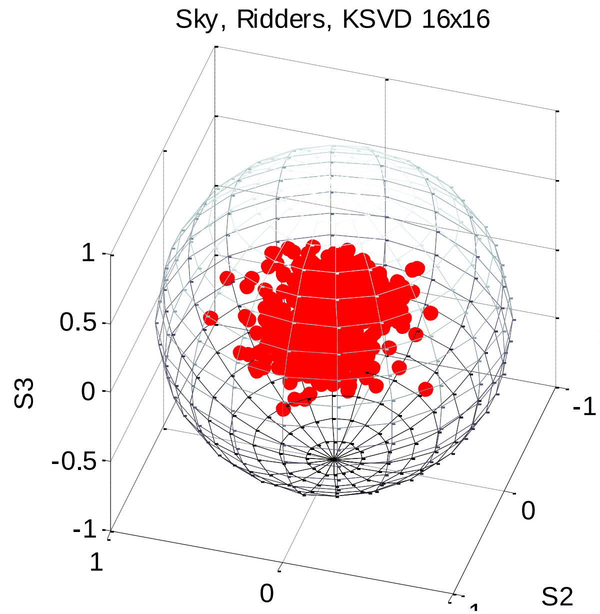
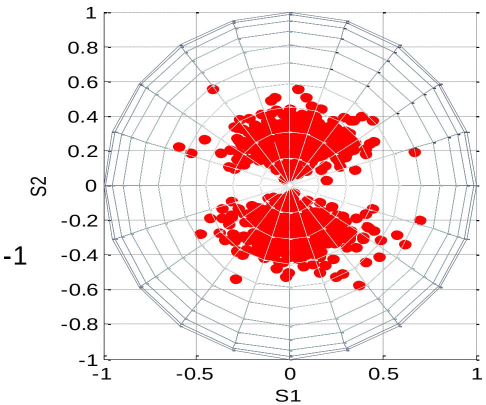


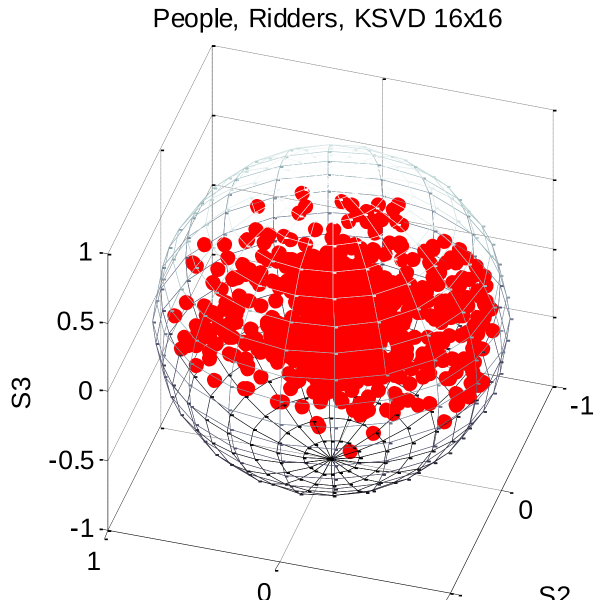


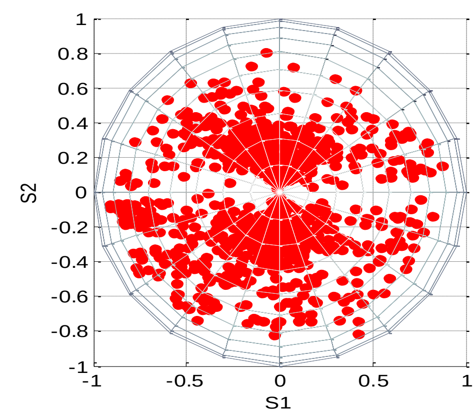
Applications: Encoding image atoms



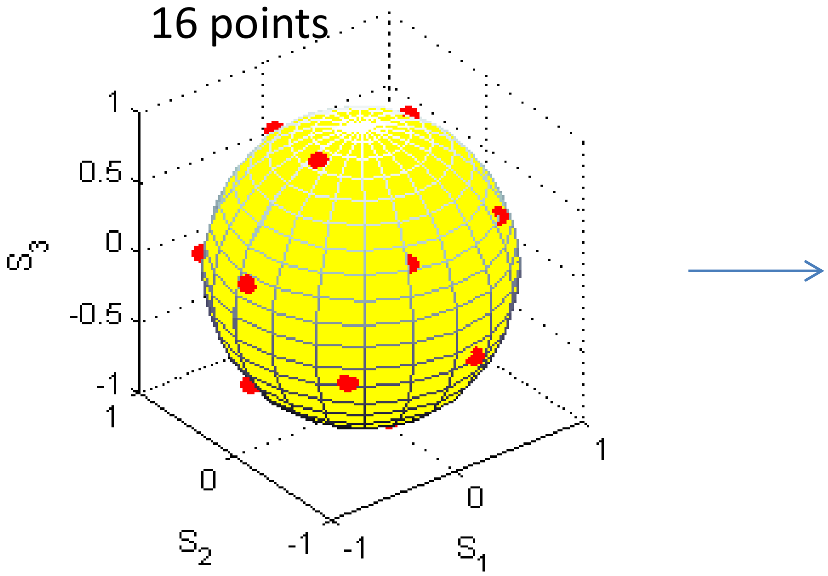


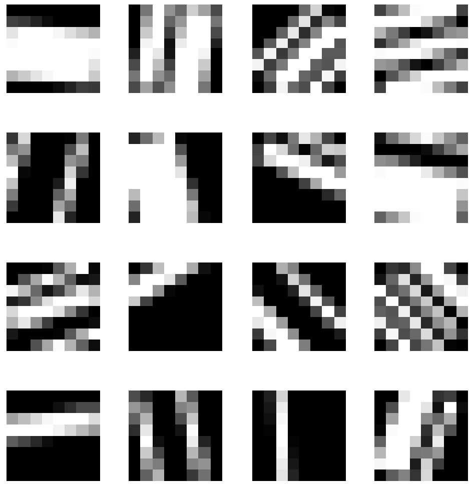






Generating dictionaries of image atoms

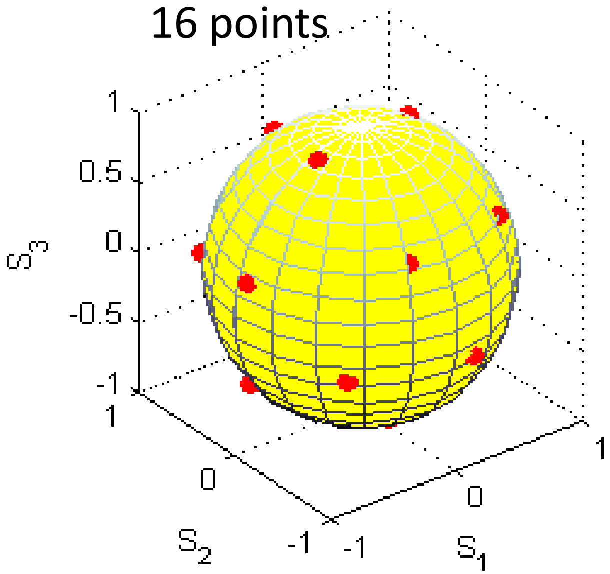


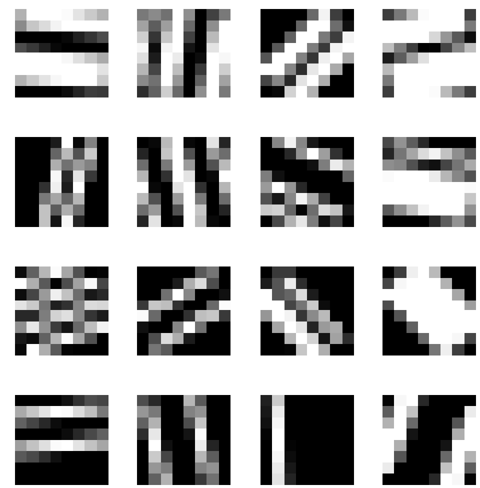


Examples with atom size 8x8

Extract from the three Stokes parameters the regularity, direction and elevation (mean grey tone) and generate randomly the corresponding patterns

Generating dictionaries of image atoms

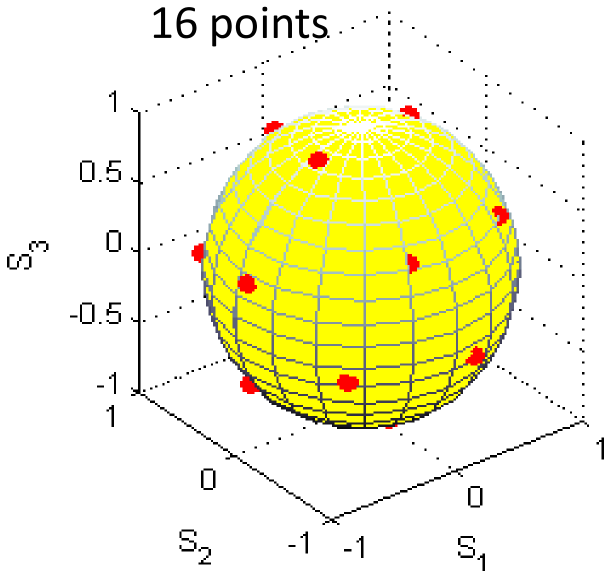


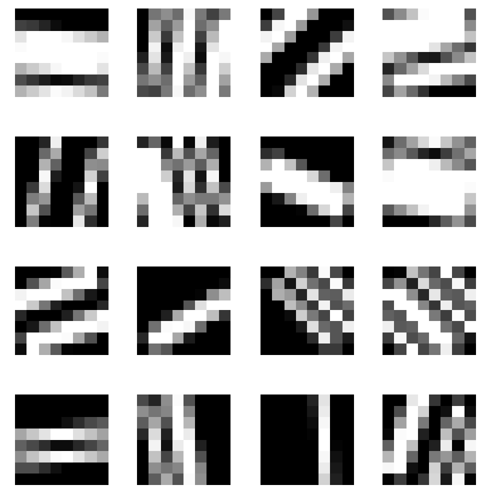


Examples with atom size 8x8

Extract from the three Stokes parameters the regularity, direction and elevation (mean grey tone) and generate randomly the corresponding patterns

Generating dictionaries of image atoms

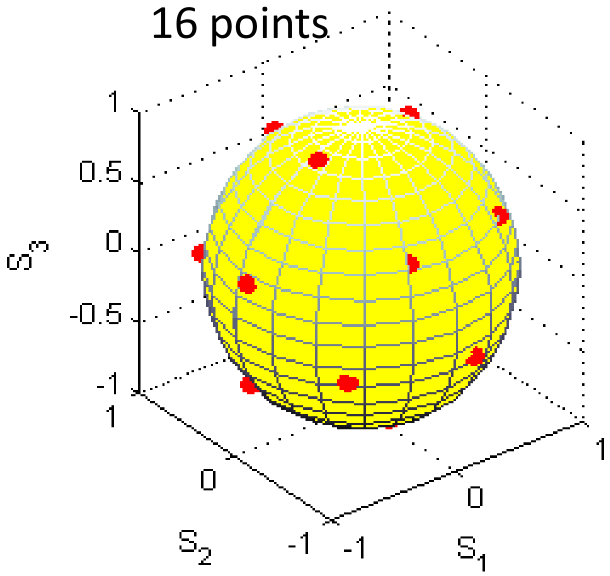


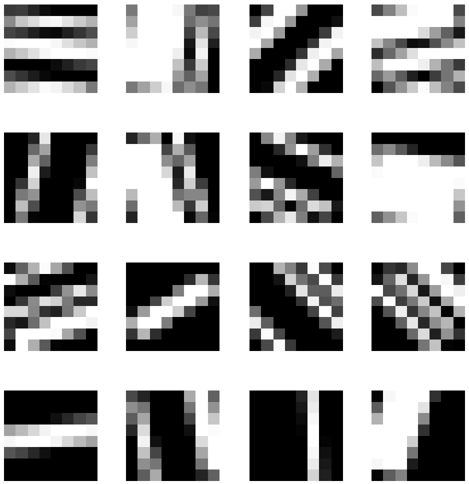


Examples with atom size 8x8

Extract from the three Stokes parameters the regularity, direction and elevation (mean grey tone) and generate randomly the corresponding patterns

Generating dictionaries of image atoms

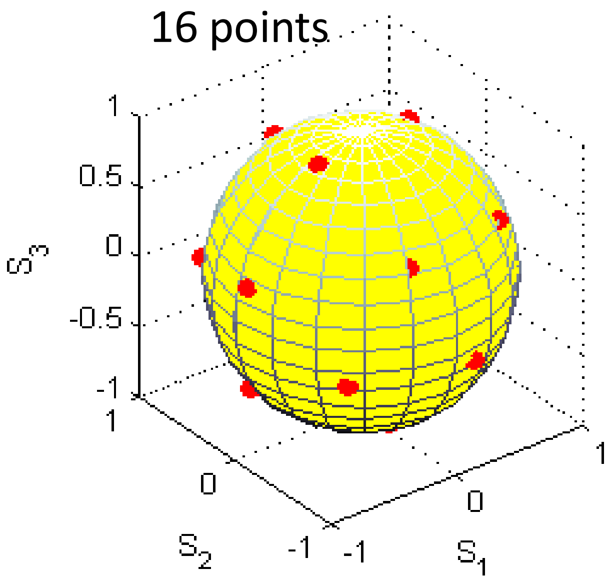


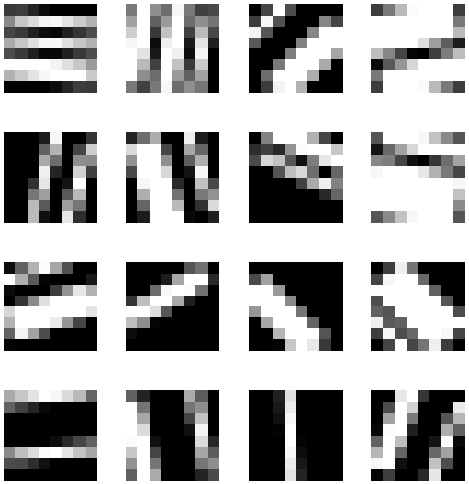


Examples with atom size 8x8

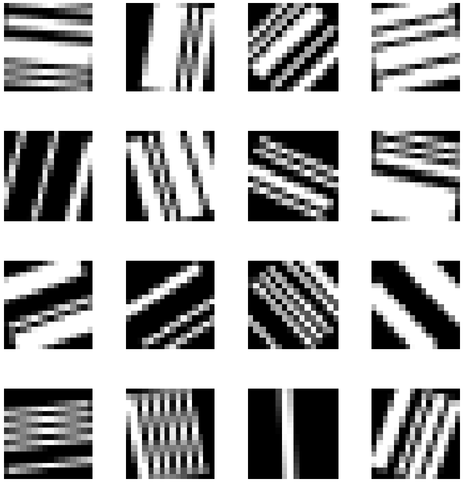
Extract from the three Stokes parameters the regularity, direction and elevation (mean grey tone) and generate randomly the corresponding patterns

Generating dictionaries of image atoms





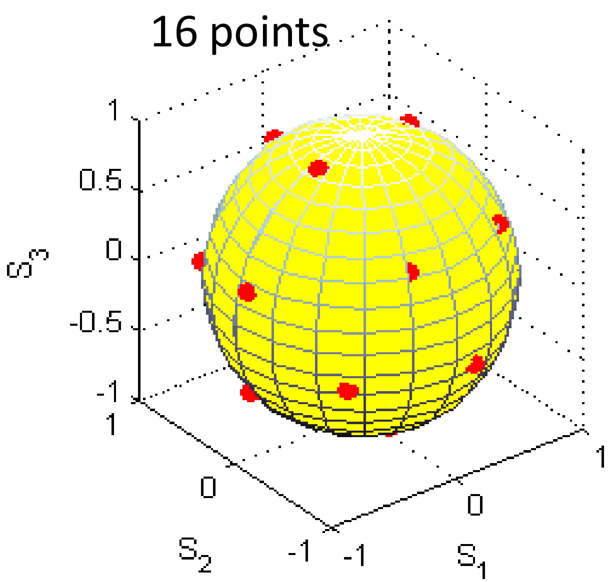
Examples with atom size 8x8

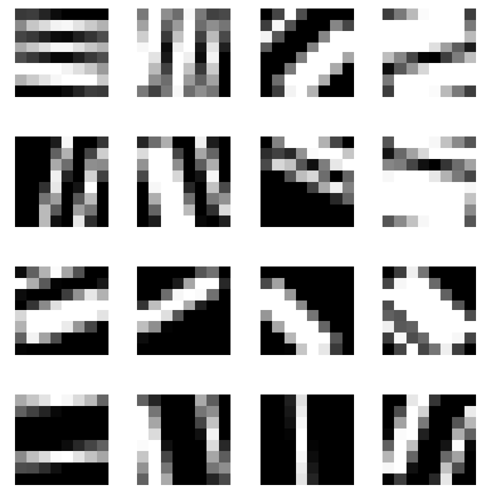


Examples with atom size 16x16

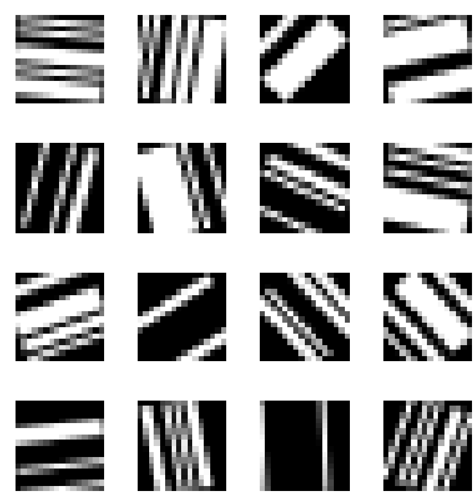
Extract from the three Stokes parameters the regularity, direction and elevation (mean grey tone) and generate randomly the corresponding patterns

Generating dictionaries of image atoms





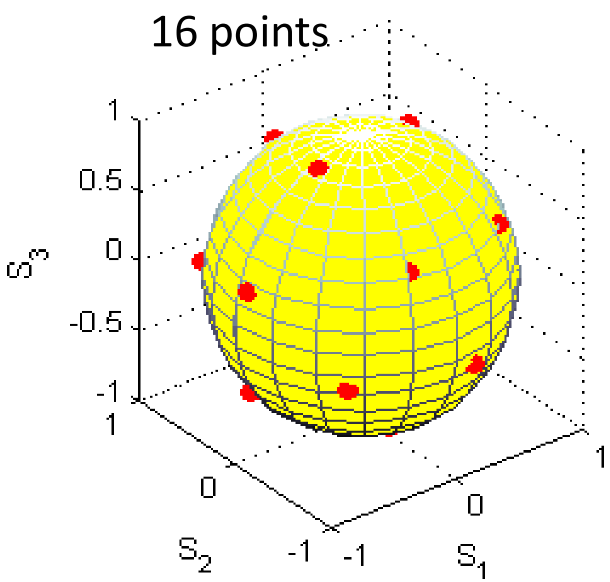
Examples with atom size 8x8

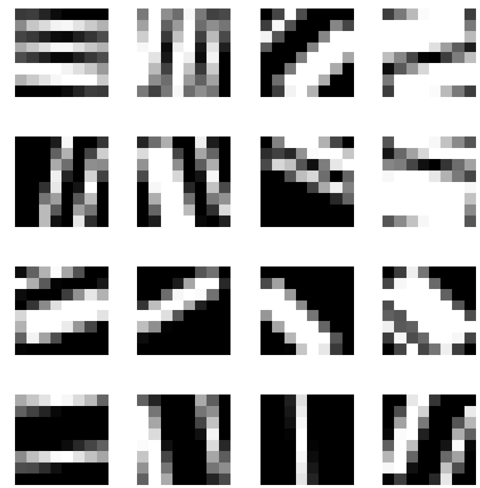


Examples with atom size 16x16

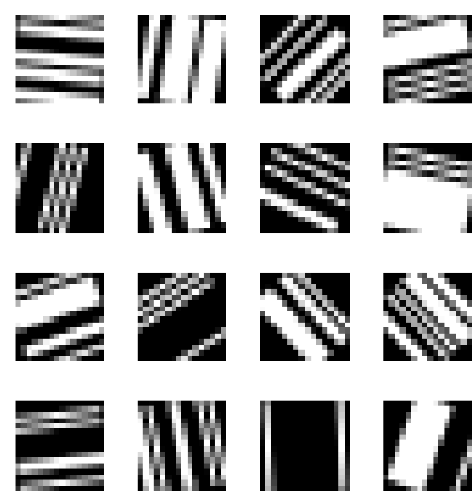
Extract from the three Stokes parameters the regularity, direction and elevation (mean grey tone) and generate randomly the corresponding patterns

Generating dictionaries of image atoms





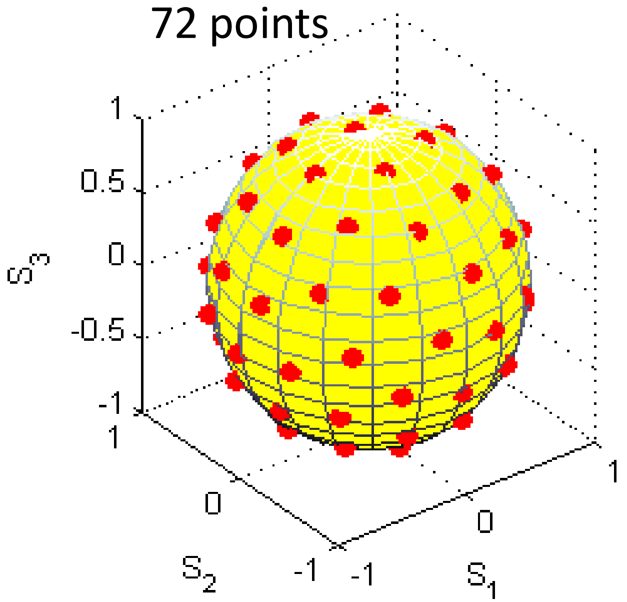
Examples with atom size 8x8

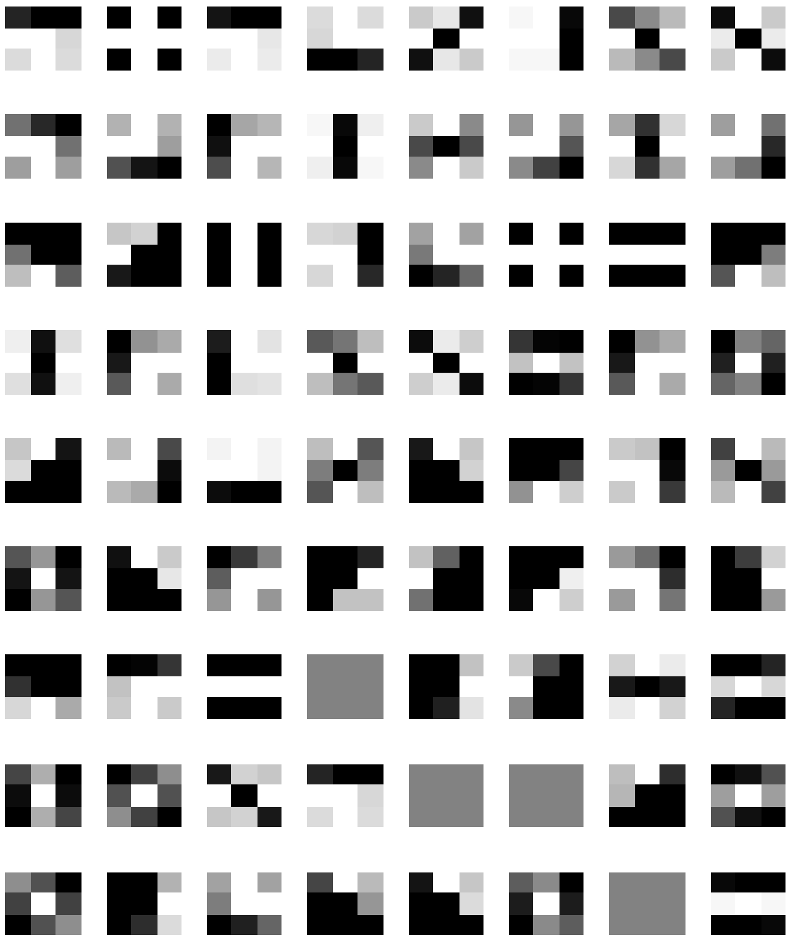


Examples with atom size 16x16

Extract from the three Stokes parameters the regularity, direction and elevation (mean grey tone) and generate randomly the corresponding patterns

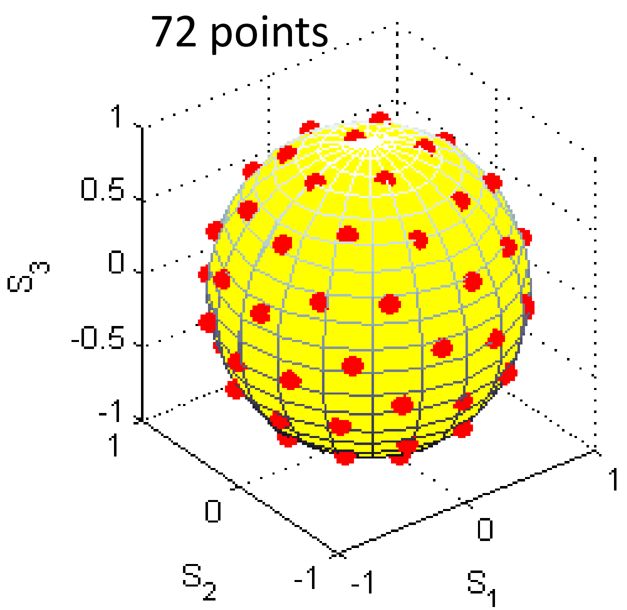
Generating dictionaries of image atoms

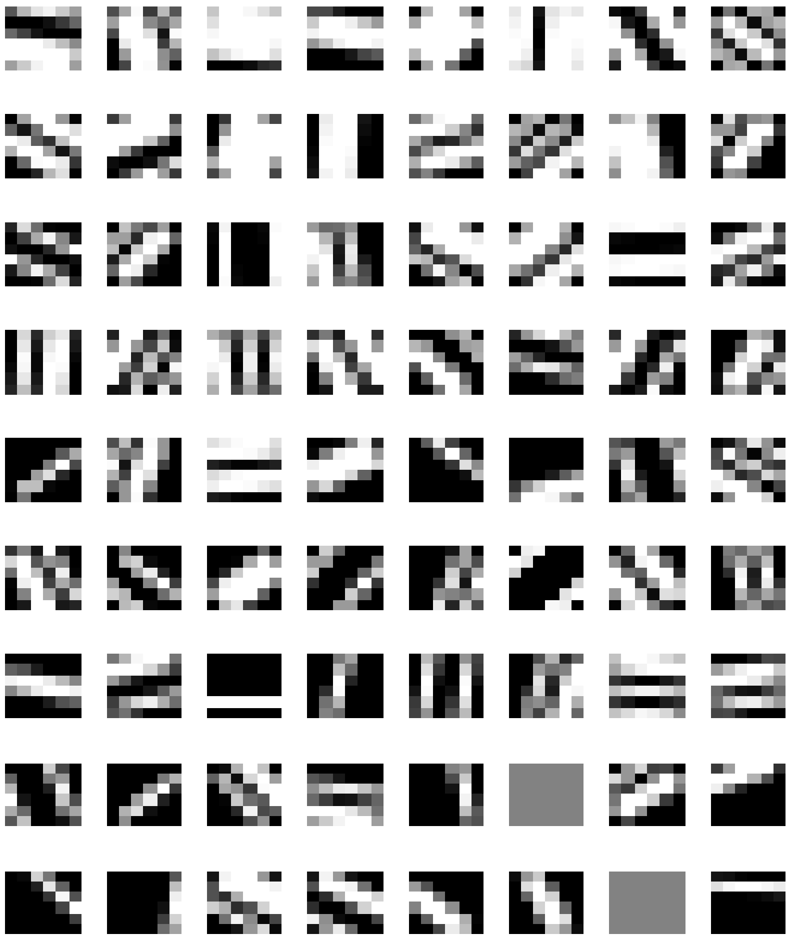




3x3

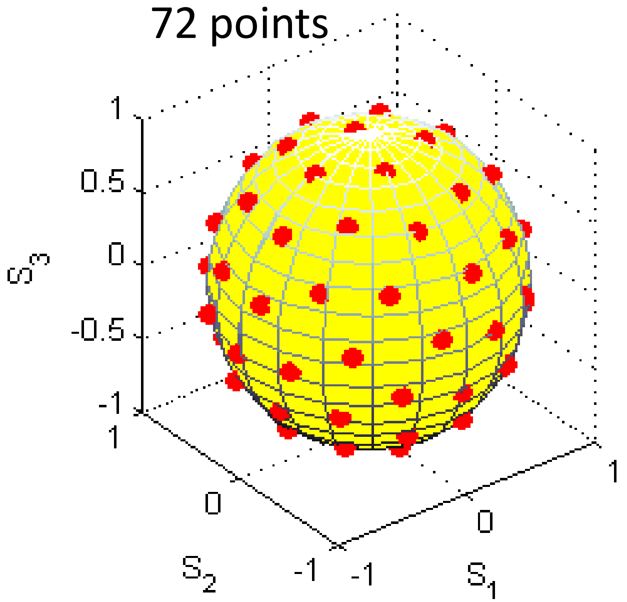
Generating dictionaries of image atoms

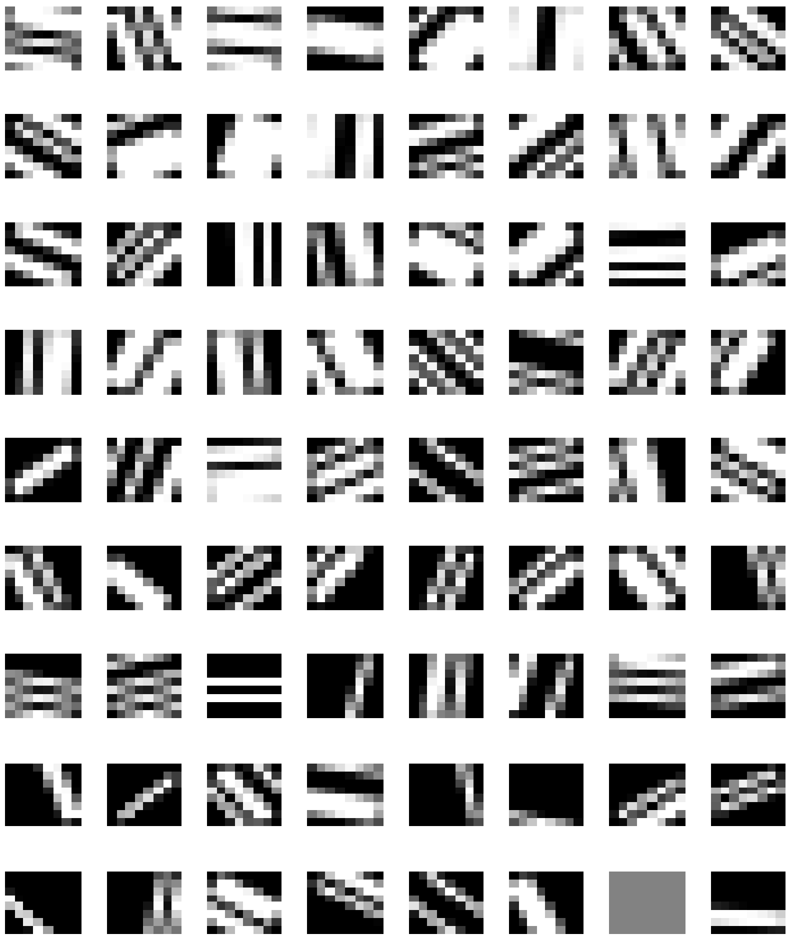




6x6

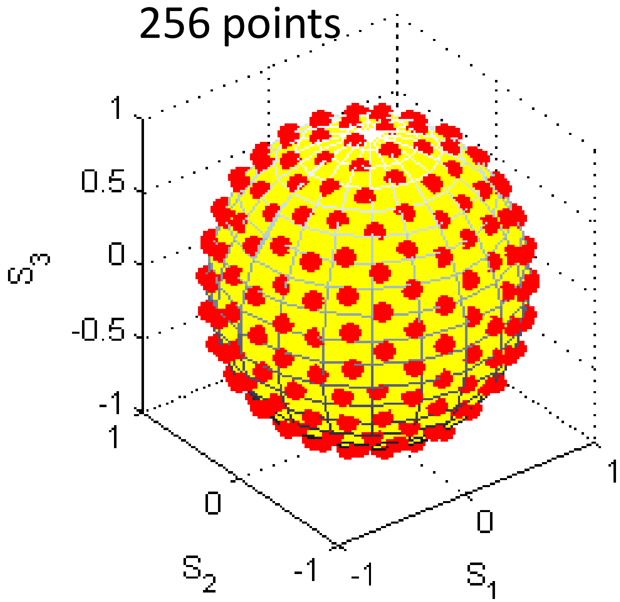
Generating dictionaries of image atoms

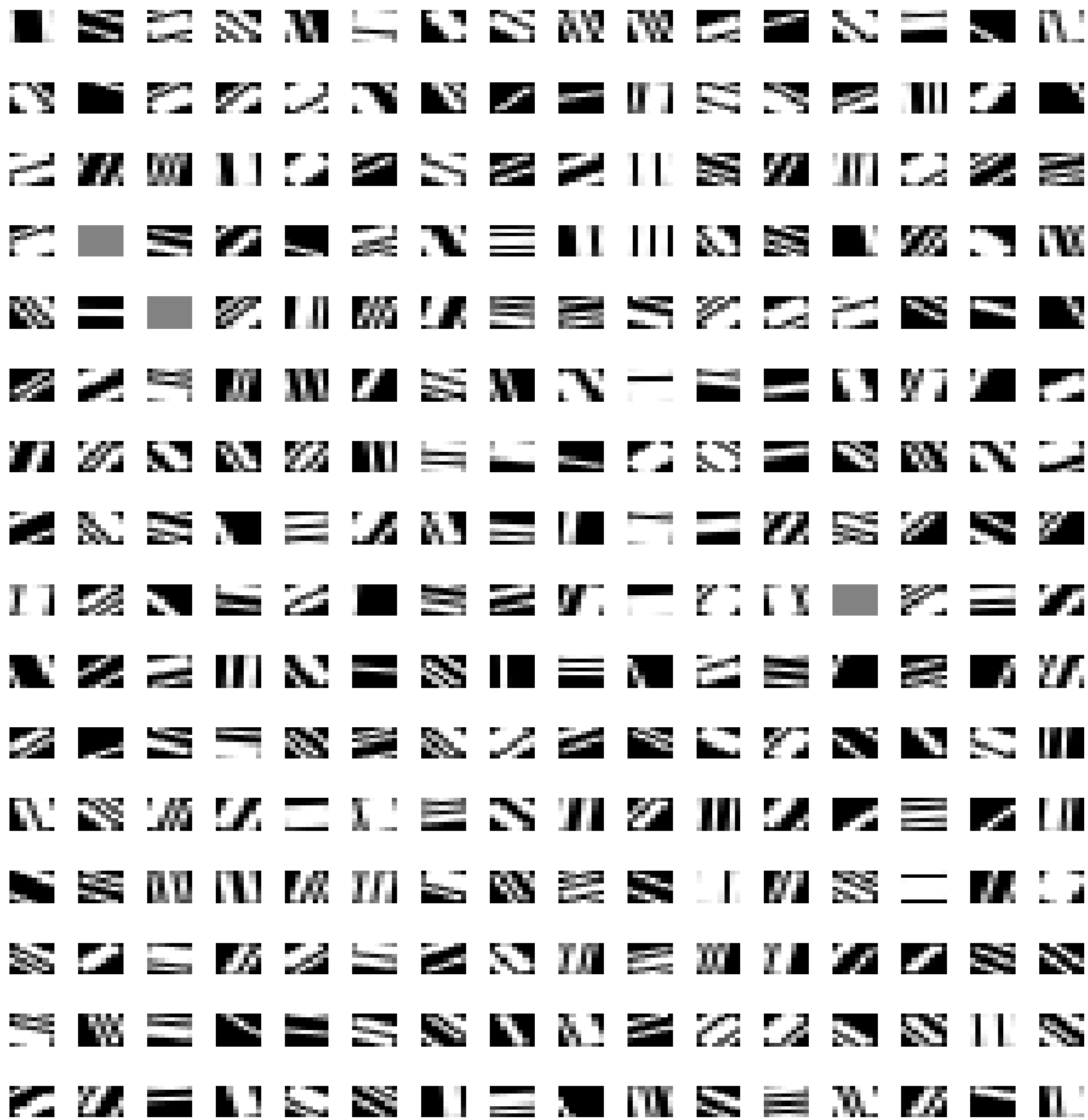




8x8

Generating dictionaries of image atoms

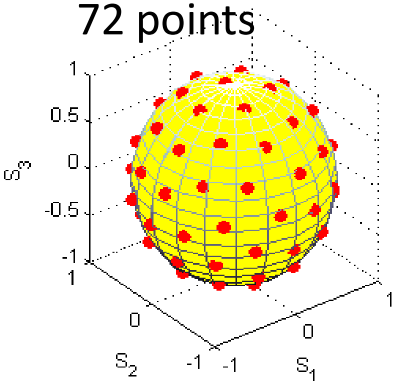




PD_{256} (8x8)

Image reconstruction examples

72 points



PSNR= 31.62 dB

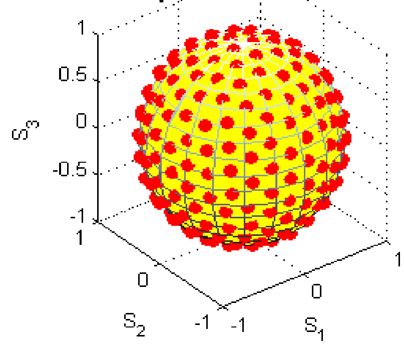
In all reconstructions:

atom size: 8x8; sparsity: 5;
reconstruction method: OMP

Image reconstruction examples

256 points

PSNR= 33.63 dB



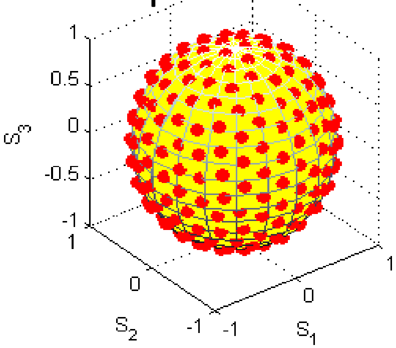
In all reconstructions:

atom size: 8x8; sparsity: 5;
reconstruction method: OMP

Image reconstruction examples

256 points

PSNR= 34.33 dB

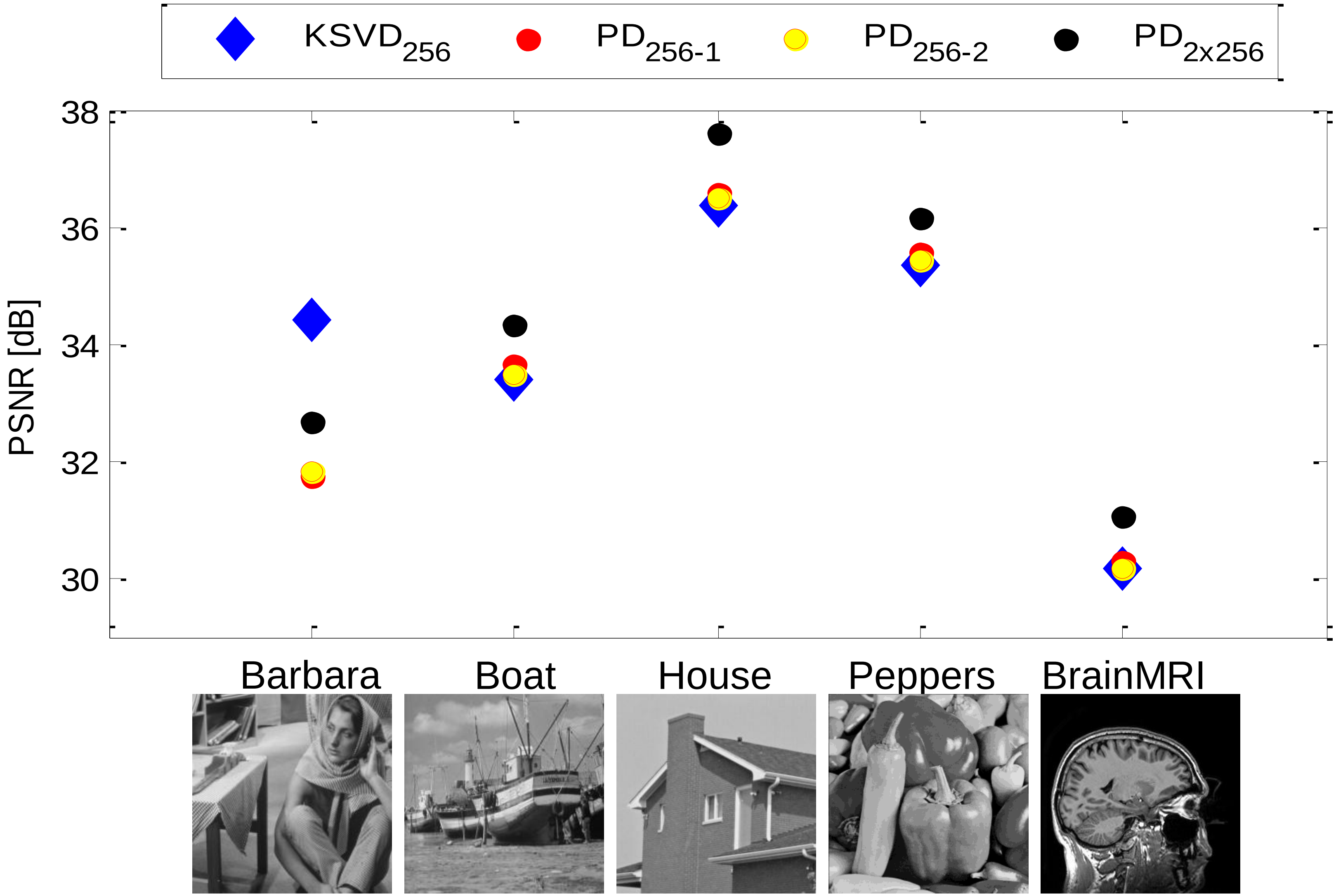


2 random dictionaries (2x256)

In all reconstructions:

atom size: 8x8; sparsity: 5;
reconstruction method: OMP

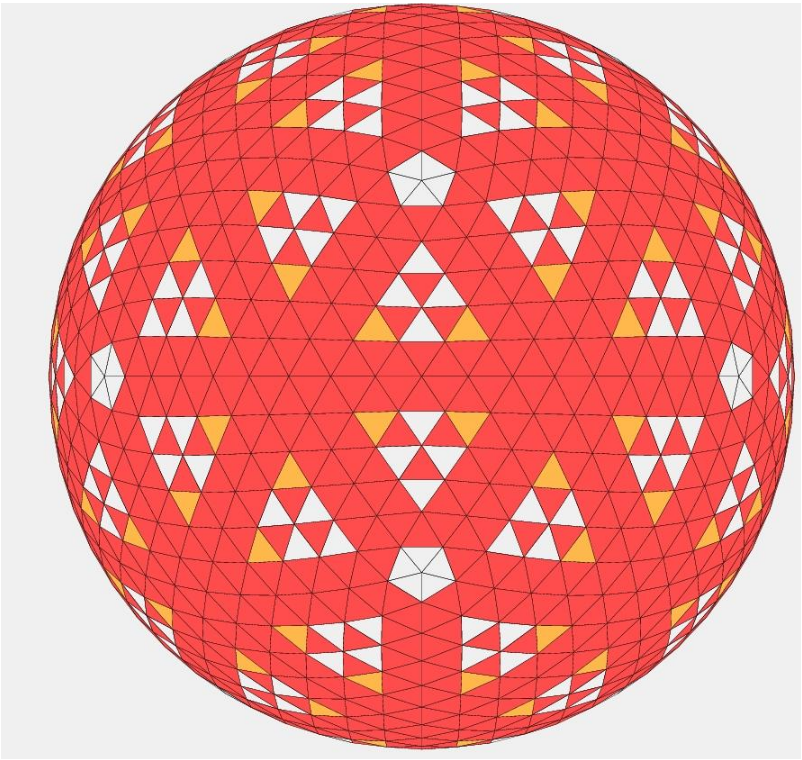
Reconstruction performance

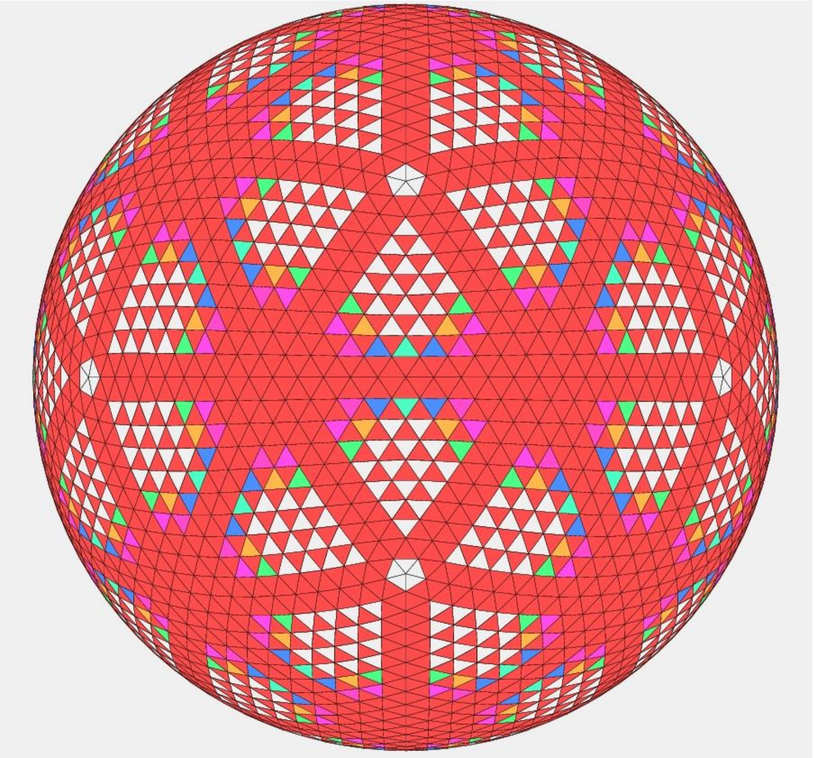


atom size: 8x8; sparsity: 5; reconstruction method: OMP

Sphere packings

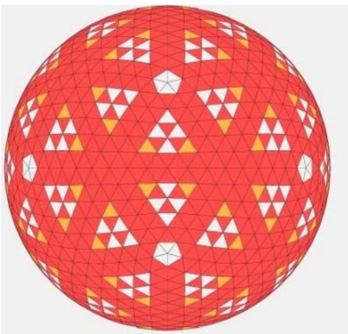
N.A.J. Sloane http://neilsloane.com/icosahedral.codes/index.html



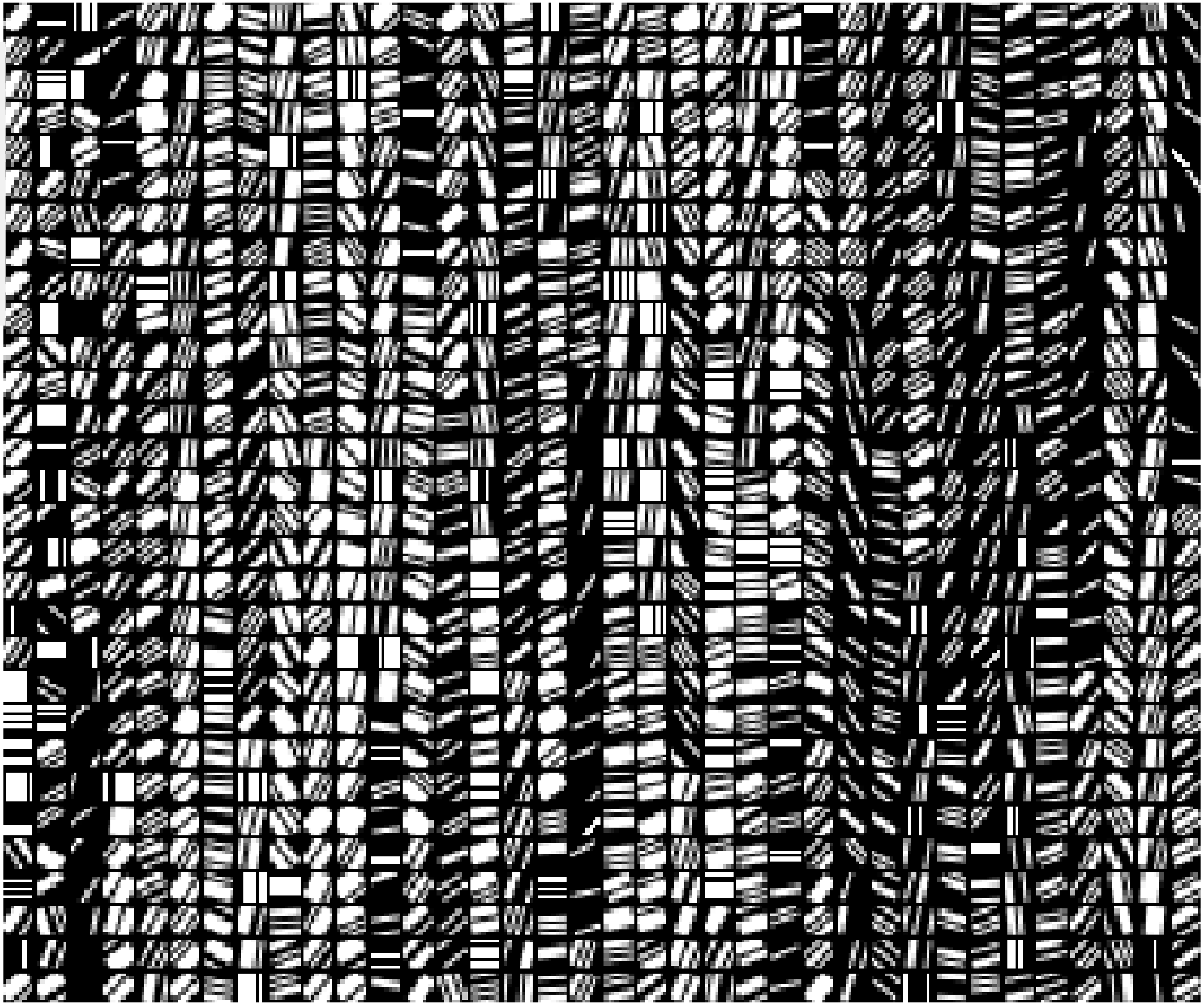


Tables of Spherical Codes with Icosahedral Symmetry
R. H. Hardin, N. J. A. Sloane and W. D. Smith

Example dictionary from a spherical code

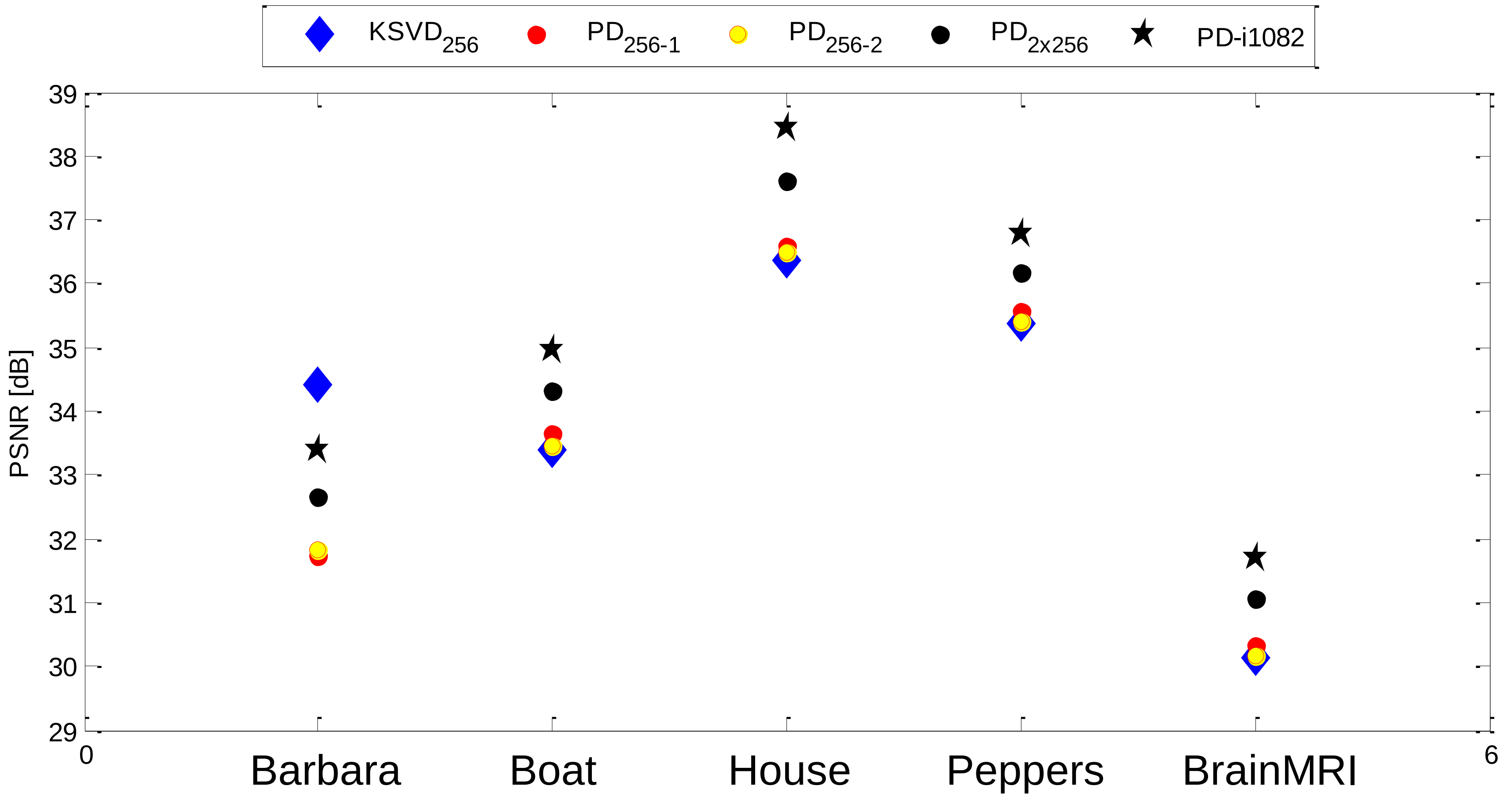


icover 1082



PD-i1082
(8x8)

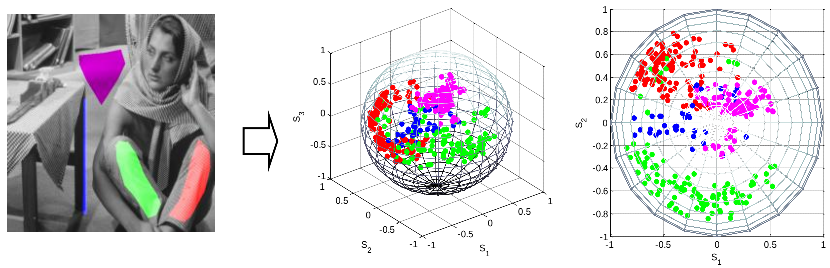
Reconstruction performance



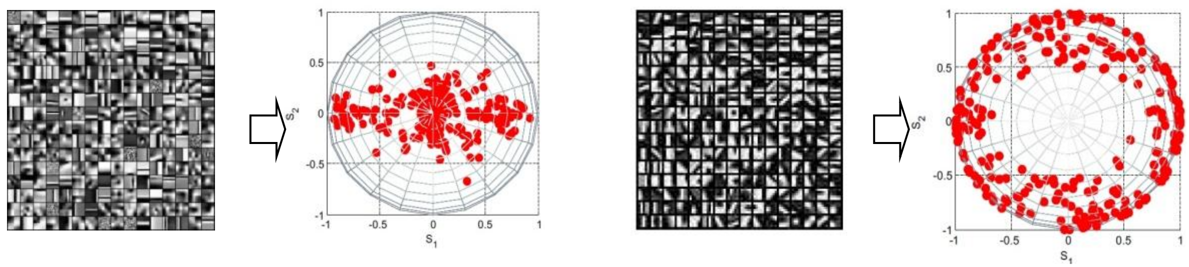
atom size: 8x8; sparsity: 5; reconstruction method: OMP

Summary

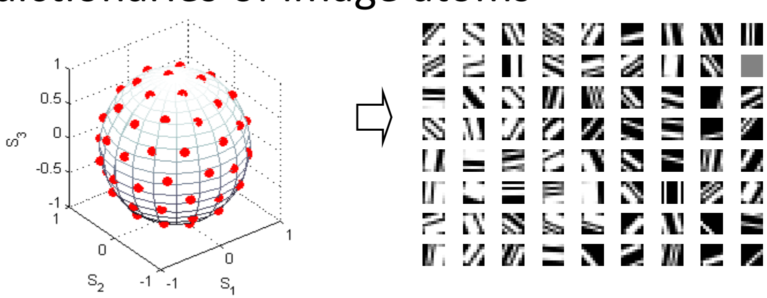
- A graphical tool was presented for encoding visual patterns
- Possible applications include
  - Patch clustering



  - Visualizing properties of learned dictionaries of image atoms



  - Generating dictionaries of image atoms



References

Material from this presentation:

A. Pizurica. **Pattern encoding on the Poincaré sphere**, arXiv:1410.0243 [cs.CV], 2014.

Spherical codes, packings, lattice coding

N. J. A. Sloane. **Tables of sphere packings and spherical codes**, IEEE Trans. Inf. Theory, 27:327-338, 1981.

J. H. Conway, R. H. Hardin and N. J. A. Sloane. **Packing Lines, Planes, etc.: Packings in Grassmannian Spaces,** *Experimental Mathematics,* 5(2):139-159, 1996.

R. Calderbank, R. H. Hardin, E. M. Rains, P. W. Shor and N. J. A. Sloane. **A Group-Theoretic Framework for the Construction of Packings in Grassmannian Spaces**, *J. Algebraic Combinatorics*, 9:129-140, 1999.

POLSK systems with spherical codes

S. Benedetto and P. Poggiolini. **Theory of polarization shift keying modulation**, IEEE Trans. Commun., 40(4):708-721, 1992.

A. Pizurica, V. Senk and V. Pizurica. **An Application of Spherical Codes to Polarization Shift Keying Modulation**, Facta Universitatis, 11(2):207-221, 1998.

Learning dictionaries of image atoms

J. Mairal, G. Sapiro and M. Elad. **Learning multiscale sparse representations for image and video restoration**, Multiscale Model. Simul, 7(1):214-241, 2008.