A Simplified and Accurate Method to Analyse a Code Division Multiple-Access Performance

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Abstract - The performance of a random access CDMA packet network is analysed in the presence of multiple access interference (MAI). We use a new method based on the improved Gaussian approximation to model the MAI. We evaluate the accuracy of the new method on the bit error probability, packet success probability, throughput and delay.

1. Introduction

Multiple access interference (MAI) can have significant impact on the network performance [1]. However, it is not clear how exactly the MAI affects performance including throughput and delay. The overall performance also requires accurate calculation of bit error rate (BER). In the presence of MAI, such performance analysis is not easy to implement and can generally only be approximated with considerable computational effort.

To begin the analysis we first need to compute the BER of a DS/CDMA system, which is dependent on a set of parameters: the number of simultaneous users K, the packet length L and the coding gain N. The additional parameter such as error correcting capability t may also be included. However, the exact calculation of BER in the presence of MAI is difficult and emphasis has been given to bounds and approximations [1]. A simple approximation is to treat MAI as standard Gaussian and the bit errors are independent. The standard Gaussian approximation is simple but not very accurate, particularly for networks with small number of users. Another reason for lack of accuracy on this method is that for an asynchronous DS/CDMA network, each user has different time and phase delays after his transmission [2], thus the bit errors are dependent. The authors in [1] consider bit-to-bit error dependence caused by the phase shifts and time delays from all interfering users, using the improved Gaussian approximation. In this paper we use the improved Gaussian approximation proposed in [1] but instead of finding the probability distribution for the MAI required for calculation of BER, we use a simplified method [3] to obtain the BER. In this way we can obtain an accurate calculation while maintaining a low computational complexity.

2. MAI Representation

We consider the DS/CDMA system model used in [1] to represent the MAI. The k_{th} user's transmitted signal is given by

$$s_k(t-\tau_k) = \sqrt{2P}\, b_k(t-\tau_k)\, a_k(t-\tau_k)\cos(\omega_c t + \theta_k) \qquad (1)$$

where $b_k(t)$ and $a_k(t)$ are the data and chip signals, respectively, P is the received power, and ω_c is the carrier frequency. The time delays and phase shifts of the signal propagation are represented by τ_k and θ_k. We assume data signal $b(t)$ is a binary sequence of rectangular pulses with unit amplitude. The chip sequence $a(t)$ is also a binary sequence of rectangular pulses with unit amplitude but it is a much faster sequence. The multiplication of the data and the chip sequences produces data pulses. There are N chips per data pulse. The pulse and chip amplitudes are all independently and identically distributed random variables. The probability of being either positive or negative amplitude is 0.5. We use a receiver model below based on [1] to deal with the MAI. The received signal, $r(t)$ is given by:

$$r(t) = \sum_{k=1}^{K} s_k(t - \tau_k) + n(t) \tag{2}$$

where K is the total number of simultaneous users and $n(t)$ is the thermal noise, which we neglect in this paper. We assume that the MAI is the only source that contributes to the errors. During the demodulation process at the receiver, a synchronised replica of the original chip sequence multiplies all received signals. The two chip sequences are cancelled leaving only the data sequence at the output of the receiver. If the received signal $r(t)$ is the input to a correlation receiver 1 matched to $s_1(t)$, the output is:

$$\int_0^{T_b} r(t)\, a_1(t - \hat{\tau}_1) \cos(\omega_c t + \hat{\theta}_1) dt \tag{3}$$

where T_b is the data bit period, $\hat{\tau}_1$ and $\hat{\theta}_1$ are the estimates of time delay and phase shift of the desired signal 1. We assume that the data is modulated using binary phase shift keying (BPSK) format. The decision statistic of the desired receiver 1, normalised with respect to the chip duration, with all signals received power $P = 2$, and given K - 1 interfering transmitters, N chips per data bit, and rectangular chip pulses is given by [1]:

$$Z_1 = N + \sum_{k=2}^{K} W_k \cos \Phi_k \tag{4}$$

where N is the processing gain ($N = T_b/T_c$, T_c is the chip duration). The second term represents the MAI where W_k is related to the time and phase delays of the interfering transmitters. Φ_k is the carrier phase. If we let Ψ be the conditional MAI variance which depends on the time delays and phase shifts of the interfering users and of the desired sequence structure, then the BER will be the function of Ψ and is given by [1]:

$$p_e(\Psi) = Q\left(\frac{N}{\sqrt{\Psi}}\right) \tag{5}$$

where $Q(x)$ is related to the complementary error function and it is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt \tag{6}$$

The BER using the standard Gaussian approximation is given by [1]

$$p_e = Q\left(\sqrt{\frac{3N}{K-1}}\right) \tag{7}$$

It is know that BER based on the standard Gaussian is not very accurate. The improved Gaussian approximation given in [1] is based on the observation that MAI is only approximately Gaussian, but conditioned on the delays and phases of all the interfering transmitters. An accurate approximation of the BER is given by

$$\hat{p}_e = E\left\{Q\left[\frac{N}{\sqrt{\Psi}}\right]\right\} = \int_0^{\infty} Q\left[\frac{N}{\sqrt{\psi}}\right] f_\Psi(\psi) d\psi \tag{8}$$

where $f_\Psi(\psi)$ is the density of Ψ and Ψ is the variance of MAI stated earlier. Each outcome $\Psi = \psi$ is produced by specific outcomes of the time delays and phase shifts parameters defined in [1]. As N becomes large, $Q[N/\psi^{0.5}]$ gives an accurate approximation of error probability for a particular ψ. Because Ψ is random so the approximate BER is taken as mathematical expectation of the function $Q[N/\sqrt{\Psi}]$ defined for all Ψ. It is shown in [1] that a significant improvement on accuracy over the standard Gaussian approximation can be obtained. However, the computation of $f_\Psi(\psi)$ is complex. In the next section, we use a simple method found in [3] to calculate the BER that does not use $f_\Psi(\psi)$ in equation (8).

3. A simplified method

We notice that equation (8), the average BER, is the expectation of the function, $Q[N/\sqrt{\Psi}]$ of the random variable Ψ. Let p be a real function of Ψ, a random variable with mean u_Ψ and variance σ_Ψ^2, assuming the existence of derivatives of p, the computation of BER can be started by expanding $p_e(\Psi)$ using the Taylor series

$$p_e(\Psi) = p_e(u_\Psi) + (\Psi - u_\Psi) p_e'(u_\Psi) + \frac{1}{2!}(\Psi - u_\Psi)^2 p_e''(u_\Psi) + \ldots \quad (9)$$

Taking expectations of $P_e(\Psi)$, the first and the second derivatives and substitution, one obtains

$$E[p_e(\Psi)] = (1 - \frac{1}{\gamma^2}) p_e(u_\Psi) + \frac{1}{2\gamma^2} p_e(u_\Psi + \sigma_\Psi) + \frac{1}{2\gamma^2} p_e(u_\Psi - \sigma_\Psi) \quad (10)$$

where γ is a constant and the value is approximately 1.7. The mean of Ψ is given by [1]:

$$u_\Psi = \frac{(K-1)N}{3} \quad (11)$$

and the variance of Ψ is given by:

$$\sigma_\Psi^2 = \frac{(K-1)}{360}\left[(2 - 10K)(1 - N) + 23N^2\right] \quad (12)$$

Substitute (11) and (12) into (10) one obtains a new approximation for the BER:

$$\begin{aligned} \hat{p}_e \approx & \left(1 - \frac{1}{\gamma^2}\right) Q\left[\sqrt{\frac{3N}{(K-1)}}\right] + \left(\frac{1}{2\gamma^2}\right) Q\left[\frac{N}{((K-1)N/3 + \gamma\sigma_\Psi)^{0.5}}\right] \\ & + \left(\frac{1}{2\gamma^2}\right) Q\left[\frac{N}{((K-1)N/3 - \gamma\sigma_\Psi)^{0.5}}\right] \end{aligned} \quad (13)$$

4. Performance analysis

In this section we calculate the network performance including probability of bit error, probability of packet success, throughput and delay using both the standard and the improved Gaussian approximations.

4.1 Probability of bit error

We first computed the probability of bit error for the DS/CDMA system using the standard Gaussian and the improved Gaussian methods. Fig. 1 is the plot of probability of bit error showing the comparison between the two methods.

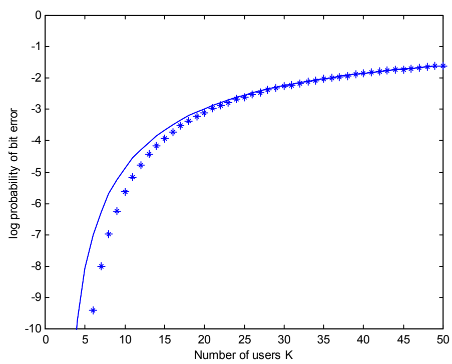

Fig. 1 BER for DS/CDMA system. The starred and solid lines represent the standard and improved Gaussian approximations, respectively. $N = 63$.

The results show that the BER using the standard Gaussian approximation gives optimistic evaluation. We can compare these results with that in reference [1], which used $f_\Psi(\psi)$. It is seen that the BER obtained in this work gives accurate evaluation for error rates greater than 10^{-6}. This performance applies to many applications for packet transmission. It is also seen that for a small number of users, the standard Gaussian approximation method gives very optimistic result.

4.2 Probability of packet success

Next we evaluated the packet success probability for various t, the error correcting capability, using both the standard Gaussian approximation and improved Gaussian approximations. With the use of linear (n, k) block code, we can write the probability of packet success as:

$$Q_E = \sum_{i=0}^{t} \binom{L}{i} p_e^i (1-p_e)^{(L-i)} \tag{14}$$

where L is the packet length, t is the error correcting capability and p_e is the BER. Fig. 2 shows that we have optimistic result for a small number of users and pessimistic result for a large number of users using the standard Gaussian approximation.

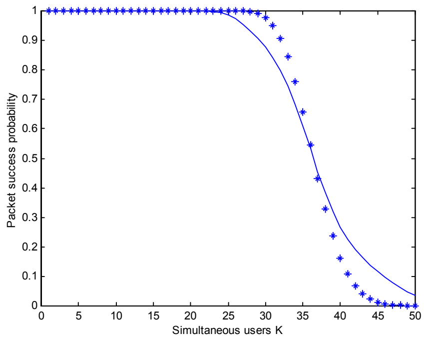

Fig. 2 Probability of packet success for DS/CDMA system (N = 63, L =1023, t = 10). Starred lines: standard Gaussian. Solid lines: improved Gaussian.

4.3 Throughput

The throughput is defined as the expected value of the number of successful packets transmitted in a slot. We consider the slotted Aloha as random access method for the DS/CDMA network. The channel contains both newly generated packets at rate S packets per slot as well as retransmitted packets at rate R packets per slot. We assume that the new packets and retransmitted packets are Poisson distributed, the offered load is thus Poisson distributed with rate $G = S + R$. When calculating the throughput we assume the channel is at equilibrium state and is stable, the throughput rate is also S and all newly generated packets will be successfully transmitted within a finite time period. Using the result of (14), the probability of a successful packet transmission given K simultaneous users, the throughput of the DS/SSMA slotted Aloha packet network is given by

$$S = Ge^{-G} + Ge^{-G} \sum_{k=1}^{\infty} \frac{G^k}{k!} Q_E(k+1) \tag{15}$$

where G is the arrival rate. The first term represents the throughput for a narrow-band slotted Aloha. The second term represents the additional throughput resulting from the use of the spread spectrum CDMA.

Fig. 3 is the comparison of the throughput using both the standard Gaussian and the improved Gaussian approximations. The effect of using different error correction codes is also shown in the figure. The standard Gaussian method gives optimistic evaluation compared to the improved Gaussian approximation. For packets with length $L = 1023$ and coding gain $N = 63$, there is an over estimate of about 10 % of the total throughput by using the standard Gaussian method compared with the improved Gaussian method. At the peak throughput the corresponding offered load is defined as G^* which will be used for the following section.

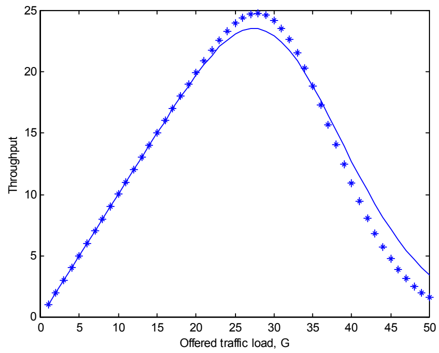

Fig. 3 Throughput comparison. The starred lines represent the standard Gaussian method. N = 63, L = 1023, t =10.

4.4 Delay

The network delay is defined as the number of transmissions required to successfully transmitting a packet. The propagation and other delays are assumed to be zero. Consider the one-persistent transmission protocol where a blocked packet is retransmitted in successive time slots until it is successfully transmitted. We can use the result of probability of packet success to determine the delay against the offered load of the DS/CDMA packet network. The expression for the average network delay is found as

$$D = \sum_{k=0}^{\infty} [Q_E(k)]^{-1} \frac{e^{-G} G^k}{k!} \tag{16}$$

where G is the offered load, k is the number of packets in one slot. Fig. 4 is the average delay for the DS/CDMA system.

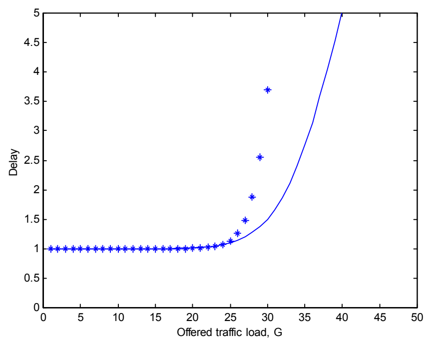

Fig. 4 Network delay (N = 63, L = 1023, t = 10). The starred lines are for the standard Gaussian method.

Fig. 4 shows that for offered load below G^*, the delay is relatively constant. This property can be compared with the throughput curves shown in Fig. 3 where for offered load $G < G^*$, the throughput increases as the offered load. Recall the probability of packet success, the constant delay corresponds to the high probability of packet success. We also observe that delays obtained using the standard Gaussian method have a much sharper change effect. Similar effect may also be observed for the packet success probability. Using the throughput and delay, we can roughly examine the stability of the network: for offered load $G > G^*$, the network tends to be unstable.

5. Conclusions

We have analysed the performance of a random access CDMA packet network in the presence of MAI. The standard Gaussian approximation overestimates the BER, particularly for a small number of users. The overestimation or underestimation also exhibits in the analysis of the packet success probability, throughput and delay. The effect of the MAI on the network performance is evaluated using the improved Gaussian approximation that takes into account the bit-to-bit error dependence. It has been shown that the simplified BER calculation method used in the paper is sufficiently accurate with reduced computational complexity. This also reduces the difficulty for the subsequent analysis including throughput and delay that depend on BER performance.

References

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