Error Probability Assessment of Radio over Fibre Based Wireless Networks Employing OFDM Signalling

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Abstract: *In this paper we propose a new analytic technique to assess the performance of radio over fibre based wireless networks employing OFDM signalling. With this technique, which is based on the Volterra series, the impact of optical system non-linearities on the error probability of OFDM signals can be evaluated.*

1. Introduction

Orthogonal frequency division multiplexing (OFDM) modulation schemes have recently been considered for high data rate transmission in wireless environments [1,2] and the extension of these modulation schemes to mm-wave frequencies allows very high capacity wireless networks to be deployed. Radio over fibre technology is appropriate to support the generation and remote delivery of these signals [3]. However, optical system non-linearities caused by the laser diode, the external modulator and the fibre infrastructure may significantly degrade the performance of these signals.

Optical system non-linearities can be divided in two broad classes, frequency dependent non-linearities (laser diodes belong to this class) and frequency independent non-linearities (external modulators belong to this class). Accordingly, a powerful analytic technique is required to assess the impact of these non-linearities on the performance of the mm-wave OFDM signals.

Elsewhere, we have described an analytic technique to determine the impact of optical system non-linearities on the statistics of OFDM signals [4]. Here, we describe an analytic technique to determine the impact of optical system non-linearities on the error probability of OFDM signals. These techniques are based on the Volterra series and can be used to evaluate the performance of radio over fibre based wireless networks employing OFDM signalling.

2. Volterra series representation of non-linearities

A non-linearity may, under certain general conditions, be represented by a Volterra series. The input-output relationship is given by [5]

$$y(t) = \sum_{n=1}^{\infty} y_n(t) \tag{1}$$

$$y_n(t) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \ldots, \tau_n) x(t-\tau_1) \ldots x(t-\tau_n) d\tau_1 \ldots d\tau_n \tag{2}$$

where $h_n(\tau_1,\ldots,\tau_n)$ is the nth-order kernel of the non-linearity. The n-dimensional Fourier transform of the nth-order kernel $h_n(\tau_1,\ldots,\tau_n)$ yields the nth-order transfer function $H_n(f_1,\ldots,f_n)$ and, conversely, the n-dimensional inverse Fourier transform of the nth-order transfer function yields the nth-order kernel.

A band-pass non-linearity may also be represented by a Volterra series. The input-output relationship is still given by eqn. 1 and eqn. 2 but in this case the even-order kernels are zero. For a band-pass non-linearity it is also possible to establish a relationship between the complex envelope of the input signal and the complex envelope of the output signal. This relationship is given by [5]

$$y_l(t)=\sum_{n=1}^{\infty} y_{(2n-1)l}(t) \tag{3}$$

$$y_{(2n-1)l}(t)=\int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty} h_{(2n-1)l}(\tau_1,\ldots,\tau_{2n-1})x_l(t-\tau_1)\ldots x_l(t-\tau_n)x_l^*(t-\tau_{n+1})\ldots x_l^*(t-\tau_{2n-1})d\tau_1\ldots d\tau_{2n-1} \tag{4}$$

where $h_{(2n-1)l}(\tau_1,\ldots,\tau_{2n-1})$ is the equivalent low-pass (2n-1)th-order kernel of the non-linearity. The (2n-1)-dimensional Fourier transform of the equivalent low-pass (2n-1)th-order kernel $h_{(2n-1)l}(\tau_1,\ldots,\tau_n)$ yields the (2n-1)th-order equivalent low-pass transfer function $H_{(2n-1)l}(f_1,\ldots,f_n)$ and, conversely, the (2n-1)-dimensional inverse Fourier transform of the (2n-1)th-order equivalent low-pass transfer function yields the (2n-1)th-order equivalent low-pass kernel. The equivalent low-pass kernels and the equivalent low-pass transfer functions can be obtained from the kernels and the transfer functions of the bandpass non-linearity [5].

This representation of a non-linearity will be used in the subsequent section to develop an analytic technique to determine the impact of non-linearities on the error probability of OFDM signals.

3. Impact of non-linearities on the error probability of OFDM

In figure 1 we show the model used to determine the impact of non-linearities on the error probability of OFDM signals. In figure 1, $s_l(t)$ is the complex envelope of the transmitted OFDM signal, $r_l(t)$ is the complex envelope of the received OFDM signal, $n_l(t)$ is the additive white complex Gaussian noise (with power spectral density N_0) and the band-pass non-linearity is represented by its equivalent low-pass kernels $h_{(2n-1)l}(\tau_1,\ldots,\tau_{2n-1})$ or by its equivalent low-pass transfer functions $H_{(2n-1)l}(f_1,\ldots,f_{2n-1})$. S_{mn} is the complex transmitted symbol in time slot m and sub-channel n, R_{mn} is the complex received symbol in time slot m and sub-channel n, $g_n(t-mT)$ denotes the complex waveform transmitted in time slot m and sub-channel n and $g^*_n(t-mT)$ denotes its complex conjugate, T is the OFDM symbol duration and T_{CP} is the OFDM cyclic prefix duration.

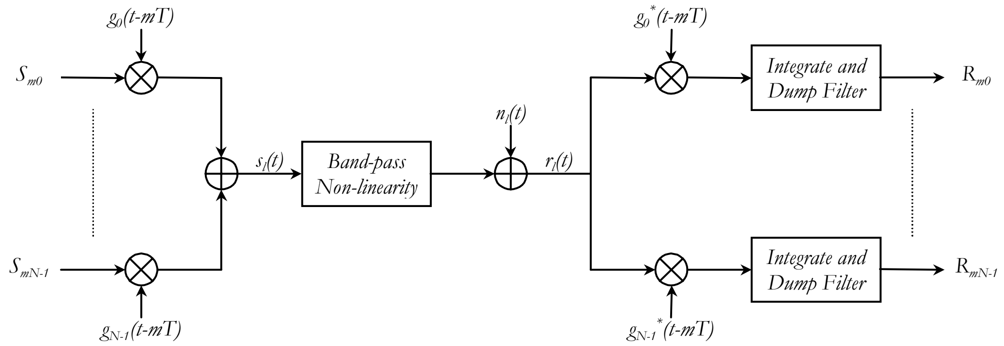

Figure 1 – Model used to determine the impact of non-linearities on the error probability of OFDM signals.

The complex envelope of the transmitted OFDM signal is

$$s_l(t) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} S_{mn} g_n(t - mT) \tag{5}$$

$$g_n(t) = \begin{cases} \frac{1}{\sqrt{T - T_{CP}}} e^{j\frac{2\pi n(t - T_{CP})}{T - T_{CP}}} & t \in [0, T] \\ 0 & t \notin [0, T] \end{cases} \tag{6}$$

and the complex envelope of the received OFDM signal is

$$\begin{aligned} r_l(t) = &\int_{-\infty}^{\infty} h_{1l}(\tau_1) s_l(t - \tau_1) d\tau_1 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_{3l}(\tau_1, \tau_2, \tau_3) s_l(t - \tau_1) s_l(t - \tau_2) s_l^*(t - \tau_3) d\tau_1 d\tau_2 d\tau_3 + \cdots \\ &+ n_l(t) \end{aligned} \tag{7}$$

The complex received symbol is given by

$$R_{mn} = \int_{mT + T_{CP}}^{(m+1)T} r_l(t) g_n^*(t - mT) dt \tag{8}$$

and assuming that the first-order low-pass equivalent kernel is restricted to $\tau_1 \in [0,T_{CP}]$; the third-order low-pass equivalent kernel is restricted to $\tau_1,\tau_2,\tau_3 \in [0,T_{CP}]$;…(note that these conditions are often met for optical system non-linearities) then eqn. 8 reduces to

$$\begin{aligned} R_{mn} = &\sum_{n_1=0}^{N-1} S_{mn_1} \frac{1}{(T - T_{CP})^0} H_{1l}\left(\frac{n_1}{T - T_{CP}}\right) \delta(n_1 - n) \\ &+ \sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1}\sum_{n_3=0}^{N-1} S_{mn_1} S_{mn_2} S_{mn_3}^* \frac{1}{(T - T_{CP})^1} H_{3l}\left(\frac{n_1}{T - T_{CP}}, \frac{n_2}{T - T_{CP}}, -\frac{n_3}{T - T_{CP}}\right) \delta(n_1 + n_2 - n_3 - n) + \cdots \\ &+ N_{mn} \end{aligned} \tag{9}$$

where

$$N_{mn} = \int_{mT + T_{CP}}^{(m+1)T} n_l(t) g_n^*(t - mT) dt \tag{10}$$

N_{mn} is a complex Gaussian random variable with mean $\mu=0$ and variance $\sigma^2=N_0$.

In the following sub-section we deal with error probability evaluation of OFDM/BPSK signals. It is straightforward to extend the methods presented to OFDM/M-PSK signals and OFDM/M-QAM signals.

1.1 OFDM/BPSK signals

We will present two methods to evaluate the error probability of OFDM/BPSK signals: the exhaustive method (a rather inefficient method) and the moment based method (a more efficient method). We assume that the complex transmitted symbols S_{mn} are independent and take any value belonging to the set $\{Ae^{j\pi/2}, Ae^{j3\pi/2}\}$ with equal probability.

1.1.1 Exhaustive method

The overall error probability is given by

$$P(error) = \frac{1}{N} P(error\ in\ sub\text{-}channel\ 0) + \cdots + \frac{1}{N} P(error\ in\ sub\text{-}channel\ N\text{-}1) \tag{11}$$

and we exhaustively compute the error probability in sub-channels n as follows

$$\begin{aligned} P(error\ in\ sub\text{-}channel\ n) = &\frac{1}{2^N} P\left(error\ in\ sub\text{-}channel\ n \,\middle|\, S_{m0} = Ae^{j\frac{\pi}{2}}, \ldots, S_{mN-1} = Ae^{j\frac{\pi}{2}}\right) + \cdots \\ &+ \frac{1}{2^N} P\left(error\ in\ sub\text{-}channel\ n \,\middle|\, S_{m0} = Ae^{j\frac{3\pi}{2}}, \ldots, S_{mN-1} = Ae^{j\frac{3\pi}{2}}\right) \end{aligned} \tag{12}$$

In order to calculate the conditional error probabilities appearing in eqn. 12, we write eqn. 9 as

$$R_{mn} = S'_{mn} + N_{mn} \tag{13}$$

where

$$\begin{aligned} S'_{mn} &= \sum_{n_1=0}^{N-1} S_{mn_1} \frac{1}{(T-T_{CP})^0} H_{1l}\left(\frac{n_1}{T-T_{CP}}\right)\delta(n_1-n) \\ &+ \sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1}\sum_{n_3=0}^{N-1} S_{mn_1} S_{mn_2} S^*_{mn_3} \frac{1}{(T-T_{CP})^1} H_{3l}\left(\frac{n_1}{T-T_{CP}}, \frac{n_2}{T-T_{CP}}, -\frac{n_3}{T-T_{CP}}\right)\delta(n_1+n_2-n_3-n)+\cdots \end{aligned} \tag{14}$$

Note that S'_{mn} is a quantity that depends not only on the complex transmitted symbol conveyed in sub-channel n but also on the complex transmitted symbols conveyed in other sub-channels.

Assuming maximum likelihood detection we have that

$$P\left(error\ in\ sub\text{-}channel\ n \,\middle|\, \ldots, S_{mn} = Ae^{j\frac{\pi}{2}}, \ldots\right) = \frac{1}{2} erfc\left(\frac{\operatorname{Im}\{S'_{mn}\}}{\sqrt{2N_0}}\right) \tag{15}$$

$$P\left(error\ in\ sub\text{-}channel\ n \,\middle|\, \ldots, S_{mn} = Ae^{j\frac{3\pi}{2}}, \ldots\right) = \frac{1}{2} erfc\left(-\frac{\operatorname{Im}\{S'_{mn}\}}{\sqrt{2N_0}}\right) \tag{16}$$

1.1.2 Moment based method

The overall error probability is also given by

$$P(error) = \frac{1}{N} P(error\ in\ sub\text{-}channel\ 0) + \cdots + \frac{1}{N} P(error\ in\ sub\text{-}channel\ N\text{-}1) \tag{17}$$

and, in this case, we compute the error probability in each sub-channel as follows

$$\begin{aligned} P(error\ in\ sub\text{-}channel\ n) &= \frac{1}{2} P\left(error\ in\ sub\text{-}channel\ n \,\middle|\, S_{mn} = Ae^{j\frac{\pi}{2}}\right) + \\ &+ \frac{1}{2} P\left(error\ in\ sub\text{-}channel\ n \,\middle|\, S_{mn} = Ae^{j\frac{3\pi}{2}}\right) \end{aligned} \tag{18}$$

In order to calculate the conditional error probabilities appearing in eqn. 18, we now write eqn. 9 as

$$R_{mn} = S'_{mn} + S''_{mn} + N_{mn} \tag{19}$$

where

$$\begin{aligned} S'_{mn} &= S_{mn} \frac{1}{(T-T_{CP})^0} H_{1l}\left(\frac{n}{T-T_{CP}}\right)\delta(n-n) \\ &+ S_{mn} S_{mn} S^*_{mn} \frac{1}{(T-T_{CP})^1} H_{3l}\left(\frac{n}{T-T_{CP}}, \frac{n}{T-T_{CP}}, -\frac{n}{T-T_{CP}}\right)\delta(n+n-n-n)+\cdots \end{aligned} \tag{20}$$

$$\begin{aligned} S''_{mn} &= \sum_{\substack{n_1=0 \\ n_1 \neq n}}^{N-1} S_{mn_1} \frac{1}{(T-T_{CP})^0} H_{1l}\left(\frac{n_1}{T-T_{CP}}\right)\delta(n_1-n) \\ &+ \sum_{\substack{n_1=0 \\ n_1,n_2,n_3 \neq n}}^{N-1} \sum_{n_2=0}^{N-1} \sum_{\substack{n_3=0 \\ ,n,n}}^{N-1} S_{mn_1} S_{mn_2} S^*_{mn_3} \frac{1}{(T-T_{CP})^1} H_{3l}\left(\frac{n_1}{T-T_{CP}}, \frac{n_2}{T-T_{CP}}, -\frac{n_3}{T-T_{CP}}\right)\delta(n_1+n_2-n_3-n)+\cdots \end{aligned} \tag{21}$$

Note that S'_{mn} is a quantity that depends only on the complex transmitted symbol conveyed in sub-channel n and S''_{mn} is a random quantity that depends also on the complex transmitted symbols conveyed in other sub-channels.

Assuming maximum likelihood detection then

$$P\left(error\ in\ sub\text{-}channel\ n \;\middle|\; S_{mn} = e^{j\frac{\pi}{2}} \right) = E_{\mathrm{Im}\{S''_{mn}\}}\left[\frac{1}{2} erfc\left(\frac{\mathrm{Im}\{S'_{mn}\} + \mathrm{Im}\{S''_{mn}\}}{\sqrt{2N_0}} \right) \middle| S_{mn} = Ae^{j\frac{\pi}{2}} \right] \tag{22}$$

$$P\left(error\ in\ sub\text{-}channel\ n \;\middle|\; S_{mn} = e^{j\frac{3\pi}{2}} \right) = E_{\mathrm{Im}\{S''_{mn}\}}\left[\frac{1}{2} erfc\left(-\frac{\mathrm{Im}\{S'_{mn}\} + \mathrm{Im}\{S''_{mn}\}}{\sqrt{2N_0}} \right) \middle| S_{mn} = Ae^{j\frac{3\pi}{2}} \right] \tag{23}$$

The statistical averages appearing in eqn. 22 and eqn. 23 can be evaluated by either of the following techniques:

1. By expanding the complementary error function in a Taylor series and using a sufficient number of moments of $Im\{S''_{mn}\}$ to approximate either eqn. 22 or eqn. 23 [6].
2. By using a sufficient number of moments of $Im\{S''_{mn}\}$ to compute a Gaussian quadrature rule that approximates either eqn. 22 or eqn. 23 [7].

A method to compute a sufficient number of moments of $Im\{S''_{mn}\}$ is described in [5].

4. Conclusions

In this paper we have proposed a new analytic technique to assess the performance of radio over fibre based wireless networks employing OFDM signalling. With this technique, which is based on the Volterra series, the impact of optical system non-linearities on the error probability of OFDM signals can be evaluated.

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