Analysis of External Optical Feedback Effects in the non-Linear Semiconductor Laser Response Considering Second Order Intermodulation products

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Abstract: *In SCM (sub-carrier multiplexing) communication systems, back-reflections caused by splices, connectors or the Fresnel effect at the end of the fibre, may contribute to signal degradation and increase intermodulation distortion. In analogue optical links this means a loss in SNR and linearity. In this paper, an adequate semiconductor laser non-linear model for the assessment of the effect of optical reflections on the second order intermodulation distortion is developed. A mathematical treatment along with simulation results is presented. It is shown that there is a periodicity in intermodulation distortion ripple with external cavity length and that for values of the latter in excess of a few meters this ripple becomes negligible. This work was done as part of a cooperation with CERN for the CMS-Tracker project.*

1. Introduction.

Important applications have been using analog transmission of microwave signals via optical fiber mostly because the associated electronics is inexpensive comparatively to its digital counterpart. One such example is TV transmission systems using SCM-WDM multiplexing techniques and phased array antennas where a stable signal is required to reach all elements [1,2]. In particular, the motivation for this work arose from the need to optimize the optical connection links between the detector front-end electronics and the remote data acquisition systems of the CMS tracker project at CERN. In this project 12 million detector strips are multiplexed and sent through 50000 channels that, in turn, are sent trough 100m fiber optic cables to the remote systems, each channel carrying a 40Msamples/s PAM signal. A future development that could be implemented to take advantage of the very high optic channel bandwidth is the use of SCM multiplexing techniques to send more than one channel per link. The advantages would be a reduction in the number of links and an improved system reliability resulting from sending the same channel through more than one link.

In low frequency modulation analog systems, the non-linearity of the semiconductor laser steady-state cavity is the main responsible factor for static non-linear distortion and loss of performance, whereas for higher modulation frequencies in the vicinity of resonance, non-linear intrinsic dynamic effects will be dominant. Up to moderate feedback levels, the intrinsic behavior of the laser diode under external feedback conditions can be determined using the dynamic rate equations.

A simple analytical approach starting from the rate equations and using the perturbation technique and Volterra Series analysis will be used to calculate the first-order small-signal transfer function and to extend those calculations to obtain the second order intermodulation transfer function. An important goal is that the final expressions be dependent on laser and external cavity physical parameters only. For the laser these can be extracted by doing laboratory measurements of the first order transfer function and then by fitting it to a simulated one [3].

2. Theory.

The laser rate equations with feedback terms are

$$\frac{dP_{(t)}}{dt} = \Gamma g_0 (N_{(t)} - N_{og})(1 - \varepsilon P_{(t)})P_{(t)} - \frac{P_{(t)}}{\tau_{ph}} + \frac{\beta \Gamma N_{(t)}}{\tau_s} + 2K_{ext}\sqrt{P_{(t)}P_{(t-\tau ext)}} \cos\left(\omega_0 \tau_{ext} + \phi_{(t)} - \phi_{(t-\tau ext)}\right) + F_{p(t)} \qquad (1)$$

$$\frac{dN_{(t)}}{dt} = \frac{I_{A(t)}}{qV_{act}} - \frac{N_{(t)}}{\tau_s} - g_0 (N_{(t)} - N_{og})(1 - \varepsilon P_{(t)})P_{(t)} + F_{N(t)} \qquad (2)$$

$$\frac{d\phi_{(t)}}{dt} = \frac{1}{2}\alpha \Gamma g_0 (N_{(t)} - N_{og}) - K_{ext}\sqrt{\frac{P_{(t-\tau ext)}}{P_{(t)}}} \sin\left(\omega_0 \tau_{ext} + \phi_{(t)} - \phi_{(t-\tau ext)}\right) + F_{\phi(t)} \qquad (3)$$

in which $P_{(t)}$ is the photon density, $N_{(t)}$ is the carrier density and $\phi_{(t)}$ is the change in phase. Other terms are K_{ext} the feedback rate, τ_{ext} the external cavity round-trip time; $F_{P\,(t)}$, $F_{N\,(t)}$ and $F_{\phi\,(t)}$ are the Langevin noise terms. Neglecting the Langevin noise terms for the sake of simplicity, normalizing these equations and solving for the first order photon transfer function $H_1(\omega)$ [4], we get

$$H_1(\omega) = \frac{D(i\omega + M) + EN}{(i\omega - C)(i\omega + M)(i\omega + A) + DB(i\omega + M) + BEN - EQ(i\omega + A)} \qquad (4)$$

$$G_1(\omega) = \frac{1 - BH_1(\omega)}{(i\omega + A)} \qquad (5) \qquad\qquad F_1(\omega) = \frac{NG_1(\omega) + QH_1(\omega)}{(i\omega + M)} \qquad (6)$$

$G_1(\omega)$ and $F_1(\omega)$ are the carrier and phase first order transfer functions, respectively. *A, B, C, D, E, M, N* and *Q* are expressions described in the appendix. Using Volterra Series Analysis, and starting from the second order rate equations, we get the second order photon transfer function $H_2(\omega_i + \omega_j)$ as a function of (4) and (5)

$$H_2(\omega_i + \omega_j) = \frac{1}{J}\left\{\begin{matrix} LH_1(\omega_i)H_1(\omega_j) + M\left[H_1(\omega_i)G_1(\omega_j) + H_1(\omega_j)G_1(\omega_i)\right] + \\ + \beta_j H_1(\omega_i) + \beta_i H_1(\omega_j) + \mathrm{Z} \end{matrix}\right\} \qquad (7)$$

with

$$J, L, M = f(n_0, p_0, \omega_i + \omega_j, Kext) \qquad (8)$$

$$\beta j = 2K_{ext}\, e^{i(\varphi_j + \omega_j \tau_{ext})}(1 + e^{-i\omega_i \tau_{ext}})J_0{}^2(n_i)J_0(n_j)J_1(n_j) \qquad (9)$$

$$\beta_i = 2K_{ext}\, e^{i(\varphi_i + \omega_i \tau_{ext})}(1 + e^{-i\omega_j \tau_{ext}})J_0{}^2(n_j)J_0(n_i)J_1(n_i) \qquad (10)$$

$$Z = -8K_{ext} p_0\, e^{-i(\omega_i + \omega_j)\tau_{ext}}\, e^{i(\varphi_i + \varphi_j)}\, J_0(n_i)J_0(n_j)J_1(n_i)J_1(n_j) \qquad (11)$$

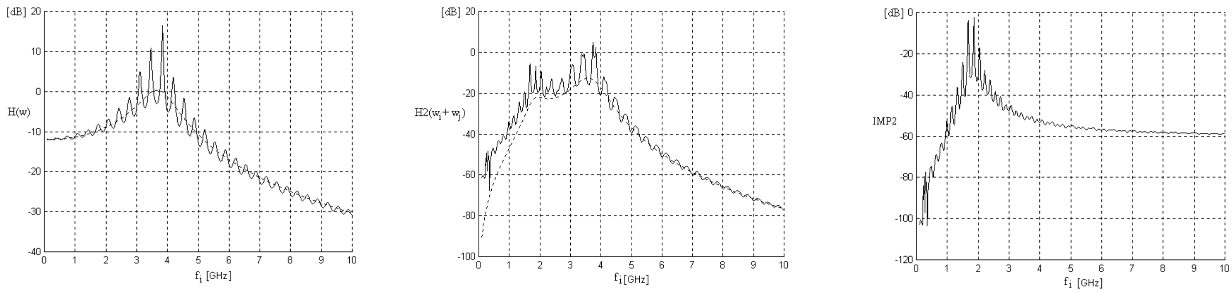
$$K_{ext} = \frac{(1 - R_m)}{\tau_L}\left(\frac{f_{ext}}{R_m}\right)^{1/2} \text{ is the external cavity feedback rate.} \qquad (12)$$

In this analysis, the laser is described in the frequency domain by the use of transfer functions. Second and superior order transfer functions determine the non-linearity of the laser device. The second order intermodulation product is given by

$$IMP_2(\omega_i + \omega_j) = 20\log\left(m\ (\omega_j) p_0 \frac{\left|H_2(\omega_i + \omega_j)\right|}{\left|H_1(\omega_i)H_1(\omega_j)\right|} \right) \tag{13}$$

For an SCM-FM system the signal-to-noise ratio (SNR) will be related to the carrier-to-noise ratio (CNR) by the expression $SNR = \frac{3 * \Delta f^2}{B^3} CNR$. In an FM system, to reach a high quality video signal a value of 56 dB is necessary. This limits the maximum value that the intermodulation distortion is allowed to have and, therefore, its calculation as a function of measured parameters in a physical implementation is of importance.

The expressions (4) and (7) were simulated as a function of modulation frequency, using MATLAB™ software. The parameters used were a wavelength of 1300nm, an external cavity length of 0.3m and a modulation index of 0.1. The results are presented in figures 1a) and 1b), which also represent, using a dashed line, the transfer function for the same situation without external feedback.



a) b) c)

Figure 1 – a) First order transfer function $H_1(\omega)$, b) Second order transfer function $H_2(\omega_i + \omega_j)$, c) Second order intermodulation distortion function $IMP_2(\omega_i + \omega_j)$, as a function of modulation frequency (f_i-f_j = 200MHz).

These results show that the ripple resulting from the feedback effect of the external cavity is more notorious in the vicinity of the resonance frequency peaks. This suggests that biasing the laser for higher output power levels would result in a flattening of the frequency response and, therefore, would also decrease the dependence on back reflection.

In figure 1c) it can be seen that maximum sensitivity to back-reflected light occurs at approximately half the laser's resonance frequency, where the second order transfer function $H_2(\omega_i + \omega_j)$ has the first resonant peak, figure 1b). This is due to the canceling effect of the first order transfer function resonance peak [eq.(13) and figure 1a)]. Several simulations for different external cavity lengths were carried out. The results showed that the ripple of both first and second order transfer functions varies periodically with the length, with a period of 1.3μm for the used parameters, and that for changes of about 0.1μm, there is a significant change in the shape of this ripple. Periodicity should be the result of trigonometric expressions in the photon density and phase rate equations, (14) and (15) respectively.

$$\cos(\omega_0 \tau_{ext} + \phi(t) - \phi(t - \tau_{ext})) \tag{14}$$

$$\sin(\omega_0 \tau_{ext} + \phi(t) - \phi(t - \tau_{ext})) \tag{15}$$

According to simulation results, for external cavities with a length in excess of about a few meters, the external feedback effects on ripple become very small and the transfer functions are similar to the situation without feedback. Clearly, this phenomenon will depend significantly on the lasers coherence length being more significant for longer coherence length lasers.

3. Summary.

The results show that there is a significant increase in intermodulation distortion ripple with optical feedback and that it varies periodically with external cavity length. Also this effect becomes negligible for cavity lengths in excess of a few meters. However, it can be seen that the second order intermodulation distortion plot has a pronounced maximum peak at half the value of the laser's resonance frequency. This technique is relevant for the performance assessment of dynamic distortion in semiconductor systems.

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Appendix.

$$A = 1 + p_0(1-\varepsilon p_0) \qquad B = (n_0 - n_{og})(1-2\varepsilon p_0) \qquad C = \gamma\Gamma(n_0 - n_{og})(1-2\varepsilon p_0) - \gamma + K_{ext}P_1(1+e^{-i\omega_0\tau_{ext}})$$

$$D = \gamma\Gamma[\beta + p_0(1-\varepsilon p_0)] \qquad E = 2K_{ext}p_0\alpha P_2(1-e^{-i\omega_0\tau_{ext}}) \qquad M = K_{ext}P_2(1-e^{-i\omega_0\tau_{ext}})$$

$$N = \alpha\gamma\Gamma/2 \qquad Q = K_{ext}\frac{\alpha P_1}{2p_0}(e^{-i\omega_0\tau_{ext}} - 1)$$

$$P_2 = \frac{J_0(n') + J_2(n')}{\sqrt{1+\alpha^2}} \qquad P_1 = \frac{J_0(n')}{\sqrt{1+\alpha^2}} \qquad n' = 2 \cdot n \cdot sen(\omega\frac{\tau_{ext}}{2})$$

n_0 and p_0 are the steady-state values of the carrier and photon density. P_1 and P_2 result from the expansion of the rate equations using Bessel functions representing the external cavity feedback effect [5].