Phase Distortion and Error Vector Magnitude for 8-PSK Systems

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Abstract: *The demand for higher data rates in mobile communications is leading to the use of higher order modulation schemes, such as 8-PSK. In such systems distortion is a consequence of magnitude and phase disturbances. In this work we investigate the performance of an 8-PSK with phase distortion due to Gaussian bandpass noise, through analytical and simulation studies. Results are presented relating EVM to SER and to Es/No.*

1. Introduction

The evolution of Global System for Mobile Communications (GSM) towards higher data rates is known as EDGE (Enhanced Data Rates for GSM Evolution). EDGE employs an 8 Phase Shift Keying (PSK) modulation scheme [1]. In such a system the figure of merit for the modulation accuracy is the Error Vector Magnitude (EVM), which represents the *distance* between the measured and the perfect modulated signals.

EVM is used instead of the typical figure of merit, bit-error-rate (BER), because BER suffers from some limiting factors, such as the requirement for dedicated equipment, long measurement intervals and a limited diagnostic value [2].

EVM encompasses the effects caused by magnitude and phase distortions. Simulation results for 8-PSK in additive white Gaussian noise (AWGN) show that phase distortion is more critical than magnitude distortion. Therefore, this work investigates the effects of phase distortion due to AWGN in 8-PSK systems, presenting analytical and simulation results for BER and EVM in noise and relating BER to EVM.

2. Error Vector Magnitude

Error vector represents the difference between the measured signal and a reference (a perfectly modulated signal) as illustrated by Figure 1. EVM is the magnitude of the error vector. Other quantities that are normally measured are the magnitude and phase errors, which are, respectively, the magnitude and phase difference between the measured and reference signals.

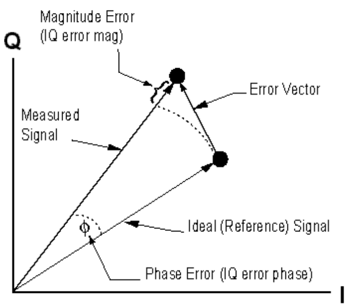


Figure 1: Error Vector Magnitude and related quantities [3]

The EVM, magnitude and phase errors can be expressed by:

$$Magnitude\ Error = \sqrt{I^2 + Q^2} - \sqrt{I_{ideal}^2 + Q_{ideal}^2} \quad [1]$$

$$Phase\ Error = \arctan\frac{Q}{I} - \arctan\frac{Q_{ideal}}{I_{ideal}} \quad [2]$$

$$EVM = \sqrt{(I - I_{ideal})^2 + (Q - Q_{ideal})^2} \quad [3]$$

where I and Q are the quadrature components of the measured signal, and I_{ideal} and Q_{ideal} are the quadrature components of the reference signal.

3. 8-PSK system performance with AWGN induced phase distortion

The performance of the basic 8-PSK system in the presence of AWGN was analysed through simulation, considering the separate and combined distortion effects on the magnitude and phase of the signal. The results are presented in Figure 2 in terms of symbol-error-rate (SER) versus EVM RMS (root-mean-square) values. It is clear from the figure that error degradation (for a given EVM value) due to phase distortion is more critical than that due to magnitude or complex (magnitude and phase) distortions. Therefore, the following sections investigate the probability of symbol error for such system with phase distortion, through analytical derivation, as well as, simulation.

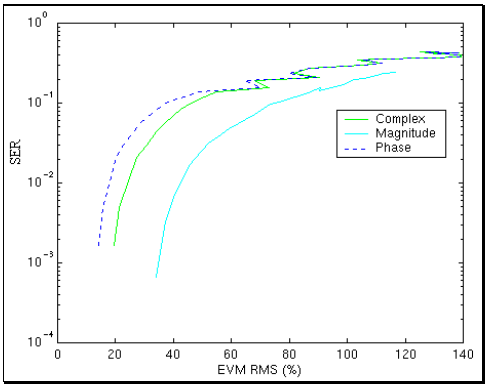


Figure 2: Symbol error rate versus EVM for 8-PSK system with complex (magnitude and phase), magnitude and phase distortion

3.1 Analytical results

Here we consider a sinusoidal signal such as $A_c cos(w_c t+\theta)$ distorted by bandpass Gaussian noise with zero mean and variance σ^2. Considering only the phase distortion, the resultant probability density function (PDF) of the signal phase is determined. Then using the relationship between phase error and EVM, the PDF of the EVM is determined as a function of the phase PDF of the signal with noise. Finally we compute the symbol error rate.

The phase PDF of a sinusoid with Gaussian bandpass noise can be expressed as [4]:

$$p_\phi(\phi) = \frac{1}{2\pi} e^{-\frac{A_c^2}{2\sigma^2}} + \frac{A_c \cos\phi}{\sqrt{2\pi\sigma^2}} e^{-\frac{A_c^2 \sin^2\phi}{2\sigma^2}} \left[1 - Q\left(\frac{A_c \cos\phi}{\sigma}\right)\right] \qquad [4]$$

for $-\pi<\phi<\pi$, where A_c is the magnitude of the sinusoid. Figure 3 depicts the transition of $p_\phi(\phi)$ from a uniform distribution when $A_c=0$ to a Gaussian distribution when A_c becomes large compared to σ.

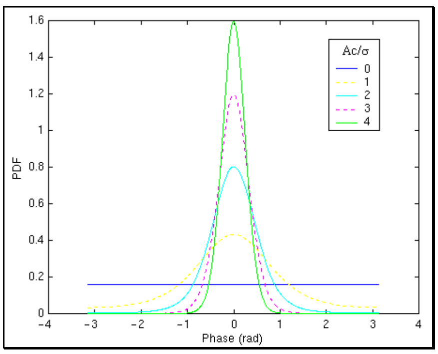


Figure 3: Phase PDFs for the PSK signal with bandpass Gaussian noise for A_c/σ ratios of 0,1,..,4

In a phase distortion system, the EVM is related to the phase distortion ø by the following expression:

$$EVM_{Pha} = \sqrt{2 - 2\cos\phi} \qquad [5]$$

The resultant PDF of the EVM can be derived by applying a transformation of random variables [4] to the phase PDF. This gives:

$$p_{EVM\ PHA}(EVM) = \frac{4}{\sqrt{4 - EVM^2}}\, p_\phi\left[\arccos\left(\frac{2 - EVM^2}{2}\right)\right],\ 0 \le EVM \le 2 \qquad [6]$$

In an 8-PSK system an error occurs when the phase is greater than π/8. The EVM value correspondent to such a phase value, using equation [5], is 0.3902. Therefore the symbol error rate can be expressed as:

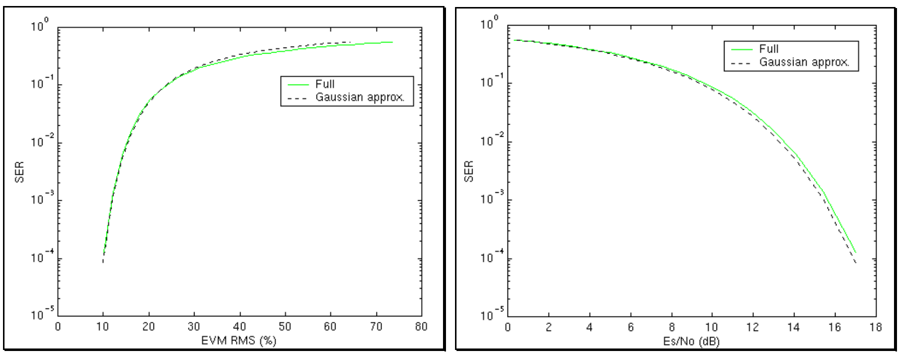
$$Pe_{SER} = P(EVM > 0.3902) = \int_{0.3902}^{2} p_{EVM\ PHA}(EVM)\, d(EVM) \qquad [7]$$

According to Figure 3 the PDF of the phase for large values of the ratio A_c/σ becomes Gaussian. Therefore, we use the Gaussian approximation for the phase PDF and then compare the results with those computed with the full phase PDF given by equation [4]. The phase PDF approximated to Gaussian distribution is expressed as:

$$p_{\phi G}(\phi) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\phi^2}{2\sigma^2}} \qquad [8]$$

The probability of symbol error can be calculated from equation [7], using the EVM PDF computed from equation [6] from the Gaussian PDF defined by [8].

The computed results for symbol error rate in phase distortion when the phase PDF is given by equation [4] (called *Full*) and for Gaussian phase approximation, are shown in Figure 4, for SER versus EVM and versus energy per symbol (Es) over single-sided noise power spectral density (No). The curves are very similar indicating that the Gaussian approximation for the phase PDF is valid, for the range of Es/No used.

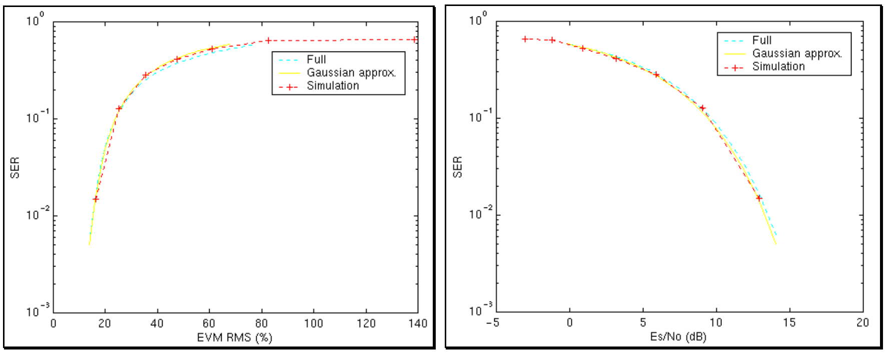


(a) (b)

Figure 4: Symbol error rate results for 8-PSK with phase distortion when the phase has *Full* and Gaussian approximation PDFs versus: (a) EVM RMS; (b) Es/No,

3.2 Simulation results

Simulations were carried out on an 8-PSK system with a Gaussian random variable of zero mean and variance σ^2 added to the signal phase. The symbol error rate versus Es/No and EVM are presented in Figure 5. The simulation results closely match those obtained analytically confirming that the Gaussian approximation for the phase PDF is accurate. Another important conclusion is that the SER value of 10^{-2} (typically expected in mobile communication systems) corresponds to an EVM value of approximately 15 % and to an Es/No value of approximately 13 dB.



(a) (b)

Figure 5: Symbol error rate results for 8-PSK with phase distortion when using analytical expressions (phase PDF employing *Full* and Gaussian approximation expressions) and computer simulations versus: (a) EVM RMS; (b) Es/No

4. Conclusions

This work has investigated the performance of the 8-PSK system with phase distortion due to Gaussian bandpass noise. Analytical and simulation results show that the PDF of the phase signal with noise can be approximated to a Gaussian. EVM values were computed and simulated in order to relate them to SER and Es/No figures. Further studies of the magnitude and complex distortion effects are in progress.

Acknowledgements

This work is supported by Nokia Networks. João Lima Pinto acknowledges the support of the Foundation for Science and Technology of the Portuguese Government through the research program Praxis XXI.

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