A Novel Technique To Estimate the Voltage-Dependent Capacitance of a Weakly Nonlinear Device At Microwave Frequencies

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Abstract: *This paper formulates a novel matrix-based solution to estimate the voltage-dependent capacitance of a weakly nonlinear device at microwave and millimeter wave frequencies. Unlike other approaches, which rely on direct capacitance measurement at different bias voltages, the proposed technique generates a large signal model for the device capacitance directly from the frequency domain measurement and the known I-V characteristic. The resulting capacitance versus voltage characteristic can be incorporated as a nonlinear simulation model for harmonic balance simulation. Measurement of the capacitance using this approach agrees with the simulation result for microwave frequency band. In this paper, the analytical treatment that gives the voltage-dependent-capacitance model is first presented. Experimental characterization of a highly abrupt device is then given to substantiate the presentation.*

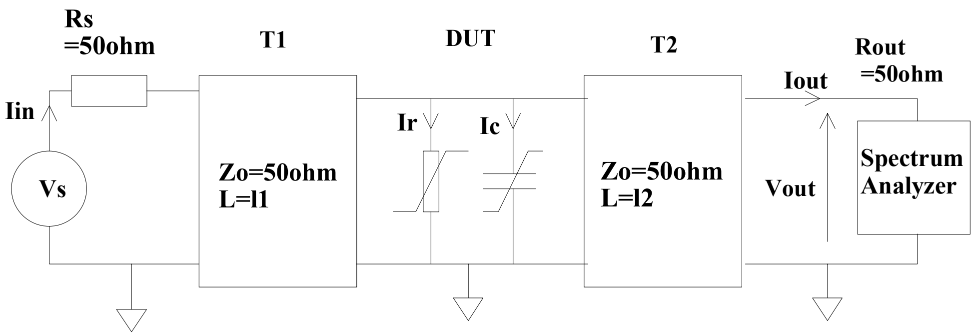
1. Introduction.

In microwave or millimeter-wave system design, nonlinear simulation and modeling forms a critically important part of the whole design process. In some systems, such as power amplifier and millimeter wave receiver, the elements that contribute to the system nonlinearities are mainly the nonlinear conductance's and capacitance's. Nonlinear conductance of a device can be known simply from the measurement of its current-versus-voltage relationship from a curve tracer. At low operating frequencies, whereby the frequency dependent effects are not dominant, the nonlinear capacitance of a device in a circuit can be neglected during the circuit analysis. But at higher operating frequencies, the nonlinear capacitance starts to shunt the imaginary current flowing through the device, with lesser magnitude of real current available to the nonlinear conductance. In some applications, such as millimeter-wave mixer design, the nonlinear capacitance affects the mixing performance significantly. At millimeter wave frequencies, the current-versus-voltage dependency in the dc domain will not be adequate to yield any accurate analysis without the capacitance information. In this respect, it is of paramount importance to incorporate the nonlinear capacitance of the device into the simulation model.

The frequency dependent behavior of the device is normally characterized as a time-variant of nonlinear capacitance or nonlinear inductance connected in parallel with the nonlinear conductance. Rough measurement of capacitance versus voltage characteristic can be done with a commercial C-V meter at a frequency below 1 Ghz. In most situation, however, the nonlinear capacitance information obtained from the C-V meter is not reliable for circuit analysis at millimeter-wave frequencies. In this paper, a method is derived to measure the nonlinear capacitance directly from the frequency domain taken from a spectrum analyzer or a power meter. The background theories discussed in this paper are similar to that in [1], except for the fact that complexity of the algorithm proposed in this paper is greatly improved.

2. Analytical Treatment

Figure. 1 illustrates the schematic of an experimental setup for measurement of a nonlinear device. The nonlinear device is excited by a single tone signal V_s through the source impedance R_s and the input transmission line T_1. Because of the nonlinear nature of the device, multiple harmonic is expected to be measurable by the spectrum analyzer through the output transmission line T_2. Measurement of the device nonlinearity involves two steps: a) resolving the spectral balances of the device by de-embedding the measurements taken from the spectrum analyzer; b) resolving the nonlinearity of the device with the given harmonic voltages and currents. What follows is an analytical description of these two steps.



***2.1. Resolving the spectral balances of the devic**e:* Referring to the diagram shown in figure.1, the kth harmonic of the device current for l_{in} and l_{out} can be expressed as follows:

$$i_k = \frac{(Z_s I_{out_k} + V_{out_k})}{\left(Z_s \cos(\beta l_{in}) + jZ_o \sin(\beta l_{out})\right)}\left[Z_s I_{out_k} + V_{out_k} + V_{sk}\right] \tag{1}$$

where V_{sk} is zero for $k \neq 1$. The kth harmonic of the voltage across the device V_k can be expressed as:

$$v_k = \cos(\beta l_{out}) V_{out_k} - jZ_o I_{out_k} \sin(\beta l_{out}) \tag{2}$$

The expression of (2) and (1) will be used to determine the nonlinear capacitance in the section that follows.

2.2. Resolving the nonlinearity of the device: If the device under test is not over-discontinuous and there exists no analytic regions in the device characteristic, the conductive current flowing through the device can be characterized as:

$$I_r = \sum_{k=0}^{\infty} a_k V^k \tag{3}$$

The nonlinear reactance of the device can be obtained from the nonlinear capacitance, which can be characterized as:

$$C = \sum_{k=0}^{\infty} C_k V^k \tag{4}$$

Now the voltage V across the device is a time-domain super-imposition of all the harmonic components across the device, as given in exponential form as follows:

$$V = \sum_{m=-N}^{N} V_m \exp(jm\omega t)$$

(5)

with $V_m = v_m \exp(j\psi_m)/2$. Following some algebraic manipulations, the current in the device can be expressed as:

$$I = \sum_{k=0}^{\infty}\left(a_k + (k+1)C_k \sum_{m=-N}^{N} jm\omega V_m \exp(jm\omega t)\right)\sum_{n=-kN}^{kN} A_{k,n} \exp(jnwt)$$

(6)

where $A_{k,n}$ is a complex quantity that can be numerically resolved by the following routine:-

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for  (r=1; r<=N; r++)
    for (i=-(r-1)*M; i<=(r-1)*M; i++)
        for (j=-M;j<=M; j++)
            A_{r,i+j} = A_{r,i+j} + A_{r-1,i} * V_j
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Although the phase between each harmonic voltage and current pair is not clear at this stage, we can still use the magnitude of each vector variable to resolve equation (6). Equating the real part and the imaginary part of equation (6), the vector of coefficients for the nonlinear capacitance becomes:

$$\sum_{m=0}^{M-1} \left|B_{k,m}\right| C_m = \frac{1}{\omega}\sqrt{\left|I_k\right|^2 - \left(\sum_{n=0}^{N} \left|A_{k,n}\right| a_n\right)^2}$$

(7)

C_m of equation (7), which represents the coefficient vector of the voltage-dependent capacitance, can be resolved by gaussian elimination. It is assumed that the I-V characteristic of the device, a_n, and the spectral information are known.

3. Measurement

To validate the frequency domain solution of voltage-dependent capacitance, a series of characterizations based on the theory previously described have been conducted on a two-terminal resonate tunneling device. The measurement setup is shown in figure 1. What follows is an example set of data:-

Frequency (Ghz)	Power (dbm)
2.054	2.13
4.160	-46.17
6.220	-32.00
8.240	-49.33
10.29	-37.83
12.34	-47.67
14.40	-42.50

Table 1. Frequency domain measurement taken from the spectrum analyzer. (Power Input from the source, V_s, = 5.77 dbm)

The resulting capacitance model as obtained from the data set of table 1 is graphically illustrated in figure 2:

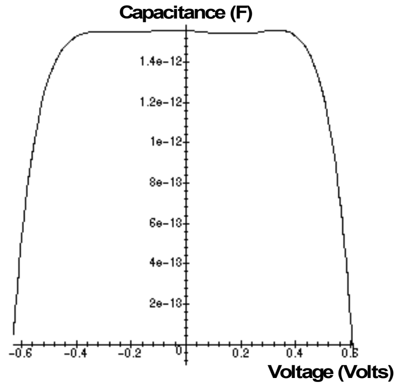

Figure 2. Capacitance versus voltage of the resonant tunneling device (Device Diameter = 10 microns)

The zero-biased capacitance according to figure 2 is around 1.5 pF in this measurement, while the value obtained from an independent VNA measurement was around 1.3 pF. The tunneling device has the negative differential resistance regions right at +/- 0.5 volts, corresponding to the thresholds, where the nonlinear capacitance decreases with voltage in figure 2. The capacitance model has been tested in harmonic balance simulation and was found to be accurate enough to predict the conversion performance of a second subharmonic mixing system operating at microwave frequencies. Further improvement can be made if we measure the harmonic with a transition analyzer, instead of spectrum analyzer which is limited by its noise floor.

5. Conclusions

This paper has presented a novel and reliable technique of extracting the nonlinear capacitance of a device from the frequency domain information. The extracted model of the device under test has been validated by experiment and by harmonic balance simulation at microwave frequencies.

References.

[1] Chao-Ren Chang, Michael B. Steer, "Frequency-Domain Nonlinear Microwave Circuit Simulation Using the Arithmetic Operator Method" *IEEE Transactions On Microwave Theory and Techniques*, Vol.38, No.8, August 1990.