

# Derivation of an upper limit on the performance of indoor Direct-Sequence ranging systems

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**Abstract:** This paper derives an upper limit on the accuracy with which a given range can be measured by a Direct-Sequence ranging system, and the maximum range which can be measured to a specified accuracy. This upper limit is achieved by an ideal receiver which is able to remove the effects of fast-fading and is able to resolve the line-of-sight component. Examples are given for a system operating in the 2.4GHz license-free band.

## 1 Introduction.

There are many applications in which the ability to locate people or objects indoors would be useful, such as a mother tracking her child whilst in a shopping mall, or a hospital tracking its expensive equipment to make sure that it can be found quickly and utilised in an emergency [1]. Looking further into the future, measurement of position indoors will be a foundational technology for ‘Ambient Intelligence’ or ‘Pervasive Computing’ in which electronic appliances are aware of their surroundings and adapt to them in a pro-active manner, with the aim of helping the user [2].

Proposals to increase the coverage of Global Navigation Satellite Systems (GNSS), such as the Global Positioning System (GPS), to enable indoor operation have been widely publicised [3]. The major obstacle is that the signals which are received indoors are very weak, and it is difficult to improve receiver sensitivity without some sort of assistance, or prior information. This obstacle is a direct result of having to use satellites in order to achieve global coverage. For many of the applications of interest, however, only a local solution is required, i.e. one which works in the confines of a house, a hospital, or a shopping mall. In these circumstances it is sensible to investigate non-satellite based, self-contained solutions for performing positioning indoors. Several companies have already proposed solutions using Direct-Sequence Spread-Spectrum (DSSS) [1][4].

DSSS is common in the fields of ranging and radar, since it achieves pulse compression, enabling high multipath-resolution along with long operating ranges. Current DSSS ranging systems on sale use several milliwatts of transmission power, chipping rates of a few tens of Mchips/s, and commonly operate in license-free bands such as the one at 2.4GHz [1]. However, DSSS systems have also been demonstrated which use chipping rates of several hundred Mchips/s, and several GHz of spectrum [5]. These systems would be regulated in future as ‘Ultra Wideband’ systems, and are likely to be subject to lower power limits [6].

In designing an indoor ranging system, it is important to quantify the upper limit on performance, in terms of the accuracy with which range can be measured, and the maximum range which can be measured to a specified accuracy. It is particularly important when considering DSSS ranging systems using chipping rates of only a few tens of Mchips/s that the upper limit on performance exceeds the specification by a wide margin, since these systems will only be able to partially resolve the multipath in the indoor environment. This means that they will tend to operate well below the upper limit on performance, unless additional signal processing is applied.

This paper derives the upper limit on performance, then applies it to a system operating in the 2.4GHz band.

## 2 Derivation of a the upper limit on performance

### 2.1 Definition of the ideal receiver

This paper considers ranging systems which measure range from the Time-of-Arrival (TOA) of the transmitted signal at the receiver. The transmitter generates a pseudo-random sequence of chips, and applies a pulse-shaping filter before transmission. The receiver is assumed to know the sequence of chips, and generates a local replica

of the sequence. The basic task of the receiver is to correlate the incoming signal with a local replica until synchronisation with the line-of-sight is achieved.

The ‘ideal receiver’ is defined here as one which performs correlations, but in addition is able to remove the effects of fast-fading, and to resolve the line-of-sight signal from amongst the multipath components. This ideal receiver will achieve the best ranging performance, but is not necessarily realisable. This paper will not aim to assess how close a real receiver comes to this ideal, but a few comments are in order.

Fast-fading effects occur when the receiver moves by a few wavelengths of the carrier around a particular location. The relative phases of the multipath components will change rapidly even in this small region, resulting in points where the multipath power adds constructively, and points where it adds destructively, causing a fading effect. One example of a method which aims to remove the effects of fast-fading is to make repeated measurements, and to build up a statistics of the multipath arrivals, so that the principle multipath components can be distinguished [7]. Even if destructive fading causes a particular multipath component to be very weak during a single measurement, over several measurements its importance will be reflected in the statistical distribution. This relies on the fact that a stationary user is unlikely to be perfectly stationary, but in fact, when considering carrier frequencies of a few GHz, will be moving the device by a few wavelengths during operation.

The task of resolving the line-of-sight signal is easier for systems using chipping rates of several hundred Mchips/s than systems using chipping rates of only tens of Mchips/s, since the resolution is determined by the chip width. In addition, we assume here that the line-of-sight can be measured in the presence of multipath, which will require a receiver with sufficient dynamic range to process both the line-of-sight and also potentially much stronger multipath components. The ability to resolve the line-of-sight can be greatly aided by the knowledge that it is the first signal to arrive at the receiver.

## 2.2 Ranging accuracy of the ideal receiver

Given these basic assumptions, the range error generated by the ideal receiver will be determined by the ability of the receiver to locate the time-of-arrival of the line-of-sight component in the presence of thermal noise. Classical radar theory states that the accuracy with which the receiver can do this is determined by the received line-of-sight energy, the thermal noise floor, and the mean-square bandwidth of the signal [8]. The mean-square bandwidth is equal to the second derivative of the correlation function at its peak, i.e. the ‘sharpness’ of the peak.

As long as the bandwidth of the pulse-shaping filter is much greater than the chipping rate, the mean-square bandwidth of a DSSS signal is approximately  $b^2 \approx 2Bf_c$ , where  $B$  is the bandwidth of the pulse-shaping filter and  $f_c$  is the chipping rate. The root-mean-square range error in our ideal receiver will be [9]:

$$dR_R = \sqrt{\frac{B}{f_c} \frac{c}{2B\sqrt{E_T/N_0}}} \quad \text{Equation 1}$$

where  $c$  is the speed of propagation in free space,  $E_T$  is the total received signal energy (which might be collected from several separate measurements), and  $N_0$  is the thermal noise floor.

Equation 1 is used in this work in one of two ways: firstly, it can be used to define a minimum energy-to-noise ratio  $(E_T/N_0)_{\min}$  which is required to guarantee range errors smaller than a specified maximum error  $(dR_R)_{\max}$ . This then leads to the concept of a Maximum Acceptable Path Loss (MAPL), which is the largest power loss which the signal can experience in travelling from transmitter to receiver if the range error is to be confined to  $(dR_R)_{\max}$ . Alternatively, given a certain energy-to-noise ratio arriving at the receiver, Equation 1 can be used to predict the accuracy of the range measurement.

## 2.3 Propagation considerations

In order to convert the Maximum Acceptable Path Loss (MAPL) into a maximum range at which a given range accuracy can be achieved, or to determine the power received at the receiver given a certain transmission power, knowledge of the propagation environment is required.

In this work, the results of published propagation studies have been used, which measured the received power at many locations throughout a building, and at several points within a few wavelengths of each location. The relation between the average power  $\overline{P_{Rx}}(d)$  received in the vicinity of a certain location and the distance  $d$  of that location from the transmitter is often modelled by a straight-line fit to the measurement results [10]:

$$\overline{P_{Rx}}(d) = P_{Rx}(d_0) + 10n \log_{10}(d/d_0) + X_s \quad [\text{dB}] \quad \text{Equation 2}$$

where  $P_{Rx}(d_0)$  is the received power at a reference distance  $d_0$  very close to the transmitter,  $n$  is known as the ‘path loss exponent’ and  $X_s$  is a Gaussian random variable when expressed in dB, with a standard deviation of  $s$  dB. This equation predicts the average power which our ideal receiver will retrieve once it has removed the fast-fading effects, but includes both the power in the line-of-sight component and the power in the multipath.

Typical indoor environments such as the one measured in [11] exhibit Rician fading, which is indicative of receiving a strong line-of-sight signal and several multipath components. Once our ideal receiver has resolved the line-of-sight component from the multipath components, the power in the line-of-sight component can be determined from the Rician K-factor of the environment, which is defined as the ratio of the power in the line-of-sight components to the power in the multipath components.

Table 1 below shows a typical example of the calculation of the upper limit. A handheld device is assumed to be transmitting a DSSS ranging signal at a power of 1mW to another handheld device in the 2.4GHz frequency band. Both have very compact and largely omnidirectional antennas, giving only 1dBi of antenna gain. The path loss measurements from [11] have been used, and the random variable  $X_s = 2s$ . Since  $X_s$  is a log-normal random variable, our ideal receiver will achieve the upper performance limit with 97.5% reliability. Finally, a receiver noise figure typical of currently available wireless-networking chip-sets has been used, and the receiver bandwidth is assumed to be much larger than the chipping rate.

Description	Equation	Value	Units	Legend
Transmission power		0	dBm	<b>A</b>
Transmitter antenna gain		1	dBi	<b>B</b>
Close-in path loss	$-20 \log_{10}(1/4\pi d_0^2)$	42.0	dB	<b>C</b>
Path loss	$10n \log_{10}(d/d_0)$	$30.86 \log_{10}(d/1.25)$	dB	<b>D</b>
Worst-case shadowing	$2s$	11.7	dB	<b>E</b>
Discard power in multipath	$-10 \log_{10}[K/(K+1)]$	1.8	dB	<b>F</b>
Receiver antenna gain		1	dBi	<b>G</b>
Receiver noise figure		6.8	dB	<b>H</b>
Thermal noise floor ( $N_0$ )	$10 \log_{10}(kT) + \mathbf{H}$	-167.2	dBm/Hz	<b>I</b>
Receiver bandwidth		10	MHz	<b>J</b>
Chipping rate		1	Mchips/s	<b>K</b>
Measurement time		0.1	s	<b>L</b>
Required signal power to achieve 1m rms range accuracy	$20 \log_{10}\left(\sqrt{\frac{J}{K}} \frac{c}{2J}\right) + \mathbf{I} - 10 \log_{10}(\mathbf{L})$	-123.7	dBm	<b>M</b>
Maximum allowable path loss	$[\mathbf{A} + (\mathbf{B} + \mathbf{G})] - \mathbf{M}$	125.7	dB	<b>N</b>
Maximum range	$d_0 10^{\frac{N-(C+E+F)}{10n}}$	235	M	

Table 1: Calculation of the upper limit using  $d_0 = 1.25\text{m}$ ,  $n = 3.086$ ,  $\sigma = 5.84$ , and  $K=2$  from [11]

### 3. Examples of the use of the upper limit

Figure 1 shows an example of using Equation 1 to predict a MAPL, and from this an upper limit on the range which can be measured to a given accuracy. This calculation used the path loss models from [11], and where not

stated, the parameter values were the same as in Table 1. Figure 1 shows that when the line-of-sight from the transmitter to the receiver is clear, longer ranges can be measured than when it is cluttered. Although the same model for path loss is recommended in [11] for both clear and cluttered line-of-sight cases, a smaller proportion of the received power is contained in the line-of-sight in the cluttered case. Figure 1 shows that the upper limit on the range that can be measured can be greatly extended by increasing the transmission power.

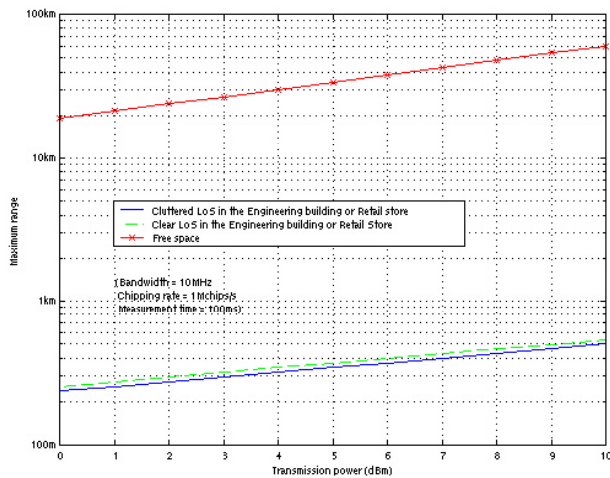


Figure 1: The maximum range which can be measured to 1m rms accuracy as a function of transmission power

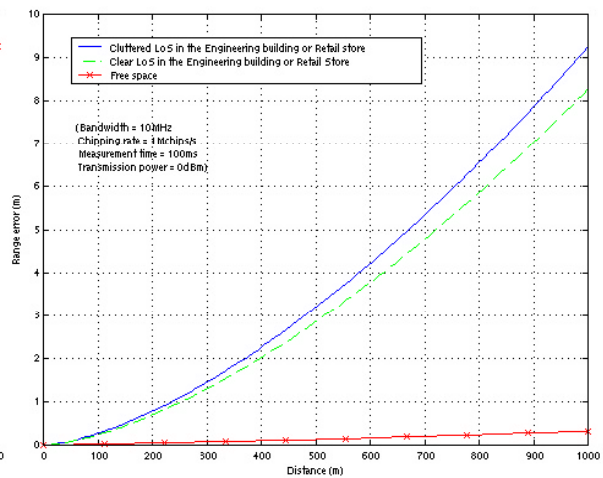


Figure 2: The rms accuracy with which a given range can be measured

Figure 2 shows an example of using Equation 1 to predict the accuracy with which a given range can be measured. Figure 2 shows that the ideal receiver delivers a noticeable difference in ranging accuracy when close to the transmitter and when far away from it. For a real system, this curve is likely to be flatter: when measuring short ranges, the accuracy is likely to be multipath-limited (i.e. limited by the ability to resolve the line-of-sight component and to remove the effects of fast-fading), and only when measuring long ranges will the accuracy be noise-limited, revealing a relation between range and range accuracy.

## Conclusions

This paper has derived an upper limit on the performance of an indoor DSSS ranging system. The upper limit can be used both to check the feasibility of using DSSS for a particular indoor ranging application and for benchmarking the performance of real receivers.

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