The Effect of Oversampling on Aperture Jitter in Bandpass Sampling Receivers

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Abstract: Bandpass sampling allows for the digitisation of bandpass signals at RF or intermediate frequencies. This is desirable for reconfigurable radio systems, where, as many radio functions as possible should be defined in software to create a flexible transceiver. However, the effects of aperture jitter limit sampling at high carrier frequencies. This paper investigates the effects of jitter in bandpass sampling systems, and describes how oversampling the information signal has an impact on the effects of jitter.

1 Introduction.

Wireless communications receivers have generally adopted a superheterodyne architecture for its superior performance. However, the number of components required can be considerable, and with the concept of reconfigurable radio currently being developed, the need for more flexible transceiver architectures is necessary. Reconfigurable radios systems, or software radio essentially allows the characteristics of the radio to be changed, enabling the transceiver to adapt to different radio systems. This is possible by the use of programmable hardware on which software-defined radio functions are downloaded. The more radio functions that are defined in software, and hence encapsulated in the digital domain of the receiver imply that a more flexible radio transceiver design is possible. As a requirement for software radio, and wideband analogue font end is necessary to allow signals from different systems to be processed. Also it is desirable to digitise as close to the antenna as possible so that as many radio functions as possible can be defined in software, or using programmable hardware [1].

The alternatives to the superheterodyne receiver that may be suitable for software radios are the direct conversion architecture and bandpass sampling. Both methods reduce the number of analogue components, with bandpass sampling allowing digitisation further up the receive chain, and allowing downconversion of the signal to a low IF without the use of mixers.

However, the sampling process is never ideal, and many factors limit performance. One factor is aperture jitter of the sample-and-hold function of the ADC. This results in a variation of the sampling period from one sample to another, degrading performance. This effect becomes more serious as sampling occurs at higher carrier frequencies.

This paper describes bandpass sampling, and the effect of jitter on a bandpass signal. An expression is developed to describe the amplitude error due to aperture jitter, and its dependence on the jitter specification and carrier frequency. The amplitude error is also signal dependent, and will vary for different signal types. Also described is the effect of increasing the sampling rate and the result this has on the effect of jitter.

2. Bandpass Sampling

Conventional Receiver systems downconvert the RF signal to a low IF or to baseband using the superheterodyne or direct conversion receiver architectures. Digitisation of the received signal usually occurs at baseband, or at an intermediate frequency followed by a digital downcoverter. The downconverted signal is then sampled at the Nyquist rate, which is at least twice the maximum frequency of the signal. Oversampling is often used, where the sampling rate can be greater than twice the signal bandwidth. This has the effect of reducing the amount of ADC quantisation noise that falls in to the signal band, and hence improves the signal-to-noise ratio.

With a lot of research efforts directed at developing reconfigurable radio systems, it would be desirable to encapsulate as many radio functions in the digital domain, as possible. This means that digitisation and hence the position of the ADC should occur earlier in the receive chain, and hence minimise the number of front-end analogue components. Figure shows the architecture typical of a bandpass sampling receiver, which allows sampling at higher carrier frequencies, instead of sampling at baseband.

Sampling at higher frequencies using conventional Nyquist sampling would require rates of over twice the carrier frequency, which would result is a very large number of samples, and hence require a lot of computational power for further processing. The alternative is to intentionally alias the signal by sampling at rates which are significantly less than the carrier frequency. The original signal can be recovered if the bandpass signal is sampled at greater than twice its information bandwidth. A number of expressions are available for choosing the appropriate sampling frequency so that aliasing, or folding of the signal on top of itself is prevented [2][3]. Bandpass sampling has the advantage of downconversion without the use of mixers. This is due to the intentional aliasing of the signal band resulting in an inherent translation in frequency to a bw IF. Digital downconversion to baseband then allows for the phase information of the signal to be retained.

However, there are many problems associated with sampling at high carrier frequencies. An important limitation would be that currently available ADCs cannot operate at RF and high intermediate frequencies since their input analogue bandwidth is limited by the gate length of the active devices used in the sample and hold within ADCs [4]. Currently it is not feasible to sample at RF frequencies, particularly for applications such as cellular radio. One possible method could be however to sample the signal at RF using a fast sample-and-hold device. This undersampling results in an inherent frequency translation to a lower intermediate frequency, where a more conventional high performance ADC can be used to digitise the signal.

The front end of a bandpass sampling receiver also requires a very high Q filter, which may be difficult to achieve. This is necessary to limit the bandwidth of the signal, and prevent distortion due to aliasing.

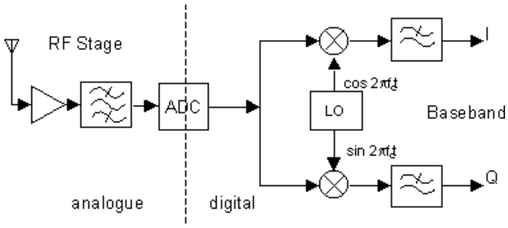

Figure 1: bandpass sampling receiver architecture

3. Aperture Jitter in Bandpass Sampling systems

Aperture jitter is caused by the uncertainty in aperture time in the sample-and-hold amplifier. It results in a sampling period that varies from sample to sample. Jitter is the result of noise, which is superimposed on the hold command, which affects it's timing [5].

As the frequency of the input signal increases, the effect of jitter becomes more severe. In fact the aperture jitter sets an upper limit on the maximum frequency sinusoidal signal that can be accurately sampled by a sample-and-hold. In order not to lose accuracy, the rule of thumb is that the signal must not change by more than ±1/2 LSB during the aperture jitter time. Using a full-scale sinusoidal signal V = Asin(2πft), we have [5]

$$\frac{dV}{dt} = 2\pi f A \cos(2\pi f t) < \frac{\pm \frac{1}{2} LSB}{t_{aj}} \tag{1}$$

where A is half the analogue input voltage range and t_{aj} is the rms aperture jitter. This results in a frequency limitation set by the aperture jitter, and the resolution of the converter as

$$f < \frac{1}{2^{n+1}\pi t_{aj}} \tag{2}$$

n is the resolution of the ADC specified as a number of bits. The signal-to-noise ratio due to jitter can be specified by [6]

$$SNR = \frac{1}{(2\pi f_{IF} \sigma_{\tau})^2} \tag{3}$$

where f_{IF} is the carrier frequency at which the signal is centred, and σ_{τ} is the rms aperture jitter. As the jitter specification becomes worse, or if the signal is sampled at a higher carrier frequency, there is degradation in the SNR due to jitter.

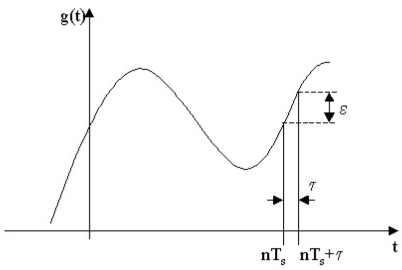

Figure 2: amplitude error due to aperture jitter

Consider a signal g(t) that is to be sampled, as shown in Figure 2. This signal would ideally be sampled at points that are multiples of T_s, the sampling period. However, due to jitter, sampling does not occur at exactly multiples of T_s, and hence the sampling period varies between samples. This in turn results in an amplitude error ε. Therefore the uncertainty in sampling position results in an amplitude error of the sampled signal. This amplitude error and hence the signal-to-noise ratio due to jitter become worse, if the signal to be sampled varies fast i.e. such as a high frequency carrier, or if the deviation of sampling points from the ideal position is large, as deduced from equations above.

The following analysis investigates the effect of sampling rate on the amplitude error that occurs due to aperture jitter, particularly in a bandpass sampling system. The amplitude error which occurs due to sampling the signal g(t) at positions that deviate from the ideal sampling points defined as multiples of T_s can be approximated by

$$\varepsilon \approx \tau . g'(t) \tag{4}$$

Consider a simple sinusoid as the input signal g(t). This could be a high frequency carrier of the form $A\cos(2\pi f_c t+\Theta)$, where Θ is a random variable, uniformly distributed over the interval $(-\pi, \pi)$.

$$g(t) = A\cos(2\pi f_c t + \Theta) \tag{5}$$

$$x(t) = g'(t) = \frac{d}{dt}\left[A\cos(2\pi f_c t + \Theta)\right] = -2\pi f_c A \sin(2\pi f_c t + \Theta) \tag{6}$$

The variance, or mean square value of the amplitude error as a result of sampling is given as

$$\sigma_\varepsilon^2 = \sigma_\tau^2 E\left[x^2(t)\right] \tag{7}$$

$$E\left[x^2(t)\right] = \int_{-\infty}^{\infty} S_x(f)\,df \tag{8}$$

where σ_τ^2 is the mean square jitter value. g(t) can be considered as a power signal. $E[x^2(t)]$ can be expressed in terms of the power spectral density $S_x(f)$. Since the power spectral density can be obtained by taking the Fourier transform of the autocorrelation function

$$R_x(\tau) = E\left[x(t+\tau)x(t)\right] = 2A^2\pi^2 f_c^2 \cos(2\pi f_c \tau) \tag{9}$$

and hence $S_x(f) = A^2\pi^2 f_c^2\left[\delta(f - f_c) + \delta(f + f_c)\right]$ **(10)**

This results in an expression, equation 11, for the variance of the amplitude error that occurs due to jitter, with the signal described in the frequency domain as power spectral density, leading to the conclusion that as the input frequency of the signal increases, the amplitude error that results, also increases. As the amount of jitter increases, again the amplitude error increases. This can also be seen in equation 3.

$$\sigma_{\varepsilon_1}^2 = \sigma_\tau^2 A^2 \pi^2 f_c^2 \int_{-\infty}^{\infty}\left[\delta(f - f_c) + \delta(f + f_c)\right]df \tag{11}$$

The expression for $\sigma_{\varepsilon 1}^2$ considers jitter expressed as a fixed amount, i.e. a particular mean square value. The expression implies that there is no change in amplitude error if the sampling rate is changed, since the amplitude error affects each sample in the same way, regardless of the number of samples. Therefore increasing the sampling rate does not reduce amplitude error due to jitter according to equation 11.

A sampling clock can be implemented using several methods, which may treat jitter in different ways. The jitter superimposed on the clock signal may be due to a variety of noise effects within the clock circuitry. The jitter may be treated such that a fixed value of jitter occurs for a range of sampling rates within the operating range of the clock.

However, another clock implementation may scale the amount of jitter as the sample rate is varied. This results in a fixed percentage jitter defined as $\sigma_j^2=\sigma_\tau^2/T_s^2$, where T_s is the sampling period. This modifies the expression for amplitude error due to aperture jitter, resulting in equation 12.

$$\sigma_{\varepsilon_2}^2 = T_s^2 \sigma_j^2 A^2 \pi^2 f_c^2 \int_{-\infty}^{\infty}\left[\delta(f - f_c) + \delta(f + f_c)\right]df \tag{12}$$

From equation 12 it can clearly be seen that using a sampling clock with a fixed percentage jitter reduces the amplitude error due to jitter as the sampling frequency is increased. This means that an increase in sampling frequency in a bandpass sampling system, i.e. increasing the oversampling ratio (ratio of sampling rate to 2 × signal BW) results in an improved SNR due to jitter.

Depending on the clock implementation, the dependence of amplitude jitter on the sampling frequency varies. In some cases the jitter may be a fixed amount, or vary very slightly with sampling rate. With other methods, the error due to jitter may significantly depend on the sampling rate used, as in the fixed percentage jitter case. A model can be developed to illustrate that the effect of sampling frequency on the amplitude error due to aperture jitter varies with clock implementation. An expression, equation 13 can be used to simply model the behaviour of aperture jitter in various sampling clocks.

$$\alpha\sigma_{\varepsilon 1}^2 + (1-\alpha)\sigma_{\varepsilon 2}^2 \tag{13}$$

where α is a value with range 0≤α≤1, and is a parameter related to clock implementation. As α is varied we obtain a number of clock implementation cases whose treatment of aperture jitter vary between the case of a fixed value jitter (α = 1), and a fixed percentage jitter (α = 0). This is illustrated in Figure 3, which is a graph of amplitude error due to jitter versus sampling rate. The two solid lines, one diagonal, and one horizontal, represent the cases for fixed percentage jitter and for a fixed amount of jitter respectively. The dashed lines in between these two extremes represent models of clock implementations with 0<α<1, whose treatment of jitter ranges between a result of fixed amount of jitter, and jitter as a fixed percentage of the sampling period.

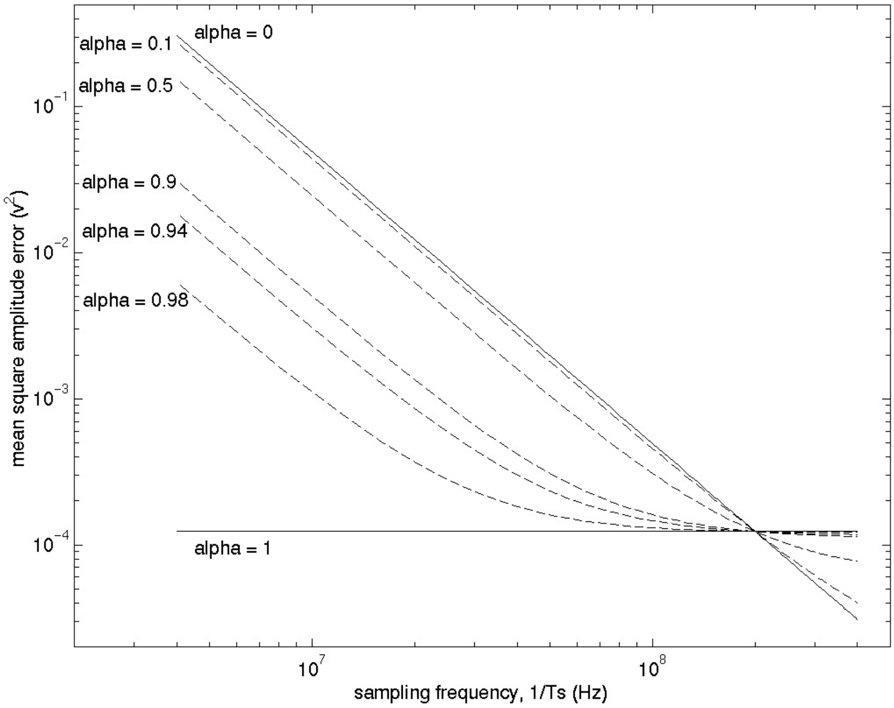

Figure 3: graph of sampling frequency versus mean square amplitude error due to jitter

4. Conclusions

Bandpass sampling has been briefly described with an emphasis on use for reconfigurable radios. The effect of aperture jitter on bandpass signals has been mentioned and an expression for the amplitude error obtained due to jitter has been developed. This expression implies that the amplitude error depends on the amount of jitter, the frequency of the signal, and the signal shape. If the jitter is expressed as a percentage of the sampling period, an improvement in the amplitude error occurs as the sampling frequency is increased. Therefore oversampling with respect to the signal, but undersampling with respect to the carrier frequency, can reduce the impact of jitter on the signal amplitude, depending on the implementation of the sampling clock. A simple model is developed to show that the effects of jitter on signal amplitude vary as sampling rate is changed and depend on how the sampling clock is implemented.

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