# Peak-to-Average Power Ratio in Higher-Order MC-CDMA 

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#### Abstract

The combination of code division multiple access (CDMA) with multi-carrier modulation (MCM) is currently being investigated by a number of authors as a possible multiple access scheme for future wireless systems. In this paper we explore the implications for peak-toaverage power ratio (PAPR) of such systems using higher order mapping schemes. Simulation results for PAPR for various PSK/QAM schemes have been presented and a comparison is made between the performance of the different schemes.


## 1. Introduction

Future wireless/mobile radio systems will need to be able to transport multimedia traffic at very high bit rates and the communications research community is already working on the concept of such $4^{\text {th }}$ generation ( 4 G ) wireless/mobile systems. The services proposed for 4G type systems will require data rates of up to 155 Mbps . This extends considerably beyond the capabilities planned for $3^{\mathrm{d}}$ generation mobile systems and presents numerous challenges for the physical layer scheme to be adopted for use over the time varying frequency selective radio channel.

CDMA has been proposed as a candidate for 4G systems. Another possible candidate is a MCM scheme such as OFDM. The observation that there are to some extent complementary benefits for CDMA and MCM has led to consideration of the merits of combining CDMA with MCM to realize so-called multi-carrier CDMA (MCCDMA); such a combination offers the prospect of ameliorating many of the limitations of adopting just one or other scheme.

One of the main disadvantages of a multi-carrier based scheme such as multi-carrier CDMA is the PAPR of the transmitted signal. There has been a significant amount of work reported into the investigation of PAPR reduction techniques and derivation of a distribution for PAPR in a multi-carrier system, however this has been predominantly based on single user multi-carrier schemes such $\propto$ OFDM and discrete multitone transmission (DMT). Here we consider PAPR in multi-user Multi-Carrier scheme.

The intention of this paper is to describe the framework used to investigate the PAPR performance of multi carrier CDMA with higher order modulation schemes and present simulation results for PAPR for various PSK/QAM schemes.

The paper is organised as follows: Following this introduction, section 2 describes multi-carrier CDMA. It provides background on MC-CDMA and describes the key advantages and dis advantages of this scheme. Section 3 describes the system model used for the investigation. Section 4 gives details of the PAPR calculations. Section 5 presents the results and discusses the difference between the various mapping schemes for different users. Finally, section 6 provides concluding remarks and indications of future work.

## 2. Multi-Carrier CDMA

Multi-Carrier CDMA schemes were first proposed in 1993. Their development has been aimed at improving the performance of broadband signals over the multipath link. In DS-CDMA, the coding provides a spreading operation that both separates users from one another and makes the signal more robust to multipath. In contrast, in MC-CDMA the coding separates individual users while the spreading operation effected by the OFDM signalling format also combats multipath.

The combined scheme benefits from the advantages of both systems. The advantages of CDMA are: high immunity against multipath distortion (as the signal is spread over a wide bandwidth) and easier variable rate transmission. The advantages of MCM are robustness to frequency selective fading, capability of narrowband interference rejection, higher spectral efficiency (because of overlapping subcarriers), reduced signal processing complexity (as no equalisation is required in the time domain) and easier implementation (as the system can be implemented using FFTs).

The main advantages of the combined system are: higher flexibility (adding new users is the same as adding more noise to the system), robustness to frequency selective fading, higher spectral efficiency, simpler transmitter and receiver structure compared to DS-CDMA and narrowband interference rejection capability.

The combined system also has a number of drawbacks. The main ones being high PAPR and sensitivity to frequency offset and phase noise.

In this paper, we focus on the PAPR. High PAPR occurs in a multi-carrier system when several subchannels add in phase at the output of the transmitter. In the case of linear amplification, this high PAPR imposes a substantial reduction in power efficiency which is undesirable in mobile transmitter where energy is a limited resource. As a result, nonlinear amplification has to be used. Nonlinear amplification can result in distortion of signal waveform which causes in-band and out-of-band interference and hence should only be permitted to a certain degree.

## 3. System model

A number of schemes have been proposed for combining CDMA with MCM [1]. In this study, we have further developed the scheme described in [2] to allow for higher order modulation formats.

We are specifically addressing the synchronous downlink case for which the transmitter and receiver are syncronised.

Complex envelope representation of the signal has been used in order to simplify the analytic formulation and also reduce the simulation time and complexity. User separation is achieved using walsh hadamard (WH) codes.

### 3.1 Transmitter

In MC-CDMA, each user data is first spread using a user specific spreading code and then modulated using multi-carrier modulation. The baseband equivalent representation of the transmitter model used in this work is given in Fig. 1a. The user data is first mapped onto a constellation point using a PSK/QAM mapper. The output complex value is then copied into N parallel data streams (where N is equal to the length of the spreading code). Each stream is then multiplied by a single chip of the spreading code, $\mathrm{c}_{\mathrm{n}}{ }^{\mathrm{u}}$ and then modulated onto a set of subcarriers, $\mathrm{g}(\mathrm{t}-\mathrm{mT})$. The modulated output is then summed and transmitted. In most practical systems a guard period is introduced before transmission.

The complex envelope signal at the output of the transmitter is given as:

$$
\begin{equation*}
S_{m c-c d m a}(t)=\sum_{u=1}^{N_{u}} \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} D_{m}^{u} c_{n}^{u} g_{n}(t-m T) \tag{1}
\end{equation*}
$$

Where $D_{m}{ }^{u}$ is the output of the PSK/QAM mapper in time slot $m$ for user $u, c_{n}{ }^{u}$ is the $n^{\text {th }}$ chip from the user specific spreading code and $g_{n}(t-m T)$ represents the $\mathrm{n}^{\text {th }}$ subcarrier defined as:
$g_{n}(t)= \begin{cases}e^{\frac{{ }^{j 2 \pi n\left(t-T_{c p}\right)}}{T-T_{c p}}} & t \in[0, T] \\ 0 & t \notin[0, T]\end{cases}$

Where T is the duration of the MC-CDMA symbol and $\mathrm{T}_{\mathrm{cp}}$ is the duration of the guard period.

### 3.2 Receiver

Fig. 1 b shows the baseband equivalent representation of the MC-CDMA receiver for user, $u$. At the receiver, the complex envelope of the received signal, $\mathrm{r}(\mathrm{t})$ is copied N times. Each copy is multiplied by the complex conjugate of the set of subcarriers, $\mathrm{g}(\mathrm{t}-\mathrm{mT})$ and the individual chips of the spreading code, $\mathrm{c}_{\mathrm{n}}{ }^{\mathrm{u}}$ and summed together. The output is then integrated over the symbol period in order to recover the user data (The integration period does not include the guard interval).

The complex output, $\mathrm{R}_{\mathrm{m}}{ }^{\mathrm{u}}$ in timeslot m for user $u$ is:

$$
\begin{equation*}
R_{m}^{u}=\frac{1}{N\left(T-T_{c p}\right)} \int_{m T+T_{c p}}^{(m+1) T} \sum_{n=0}^{N-1} r(t) c_{n}^{u} g_{n}^{*}(t-m T) \tag{3}
\end{equation*}
$$

In this expression, $r(t)$ is the complex envelope received signal, $\mathrm{c}_{\mathrm{n}}{ }^{\mathrm{u}}$ are the individual chips of the spreading codes and $\mathrm{g}_{\mathrm{n}}{ }^{*}(\mathrm{t}-\mathrm{mT})$ is the complex conjugate of the $\mathrm{n}^{\text {th }}$ subcarrier.


Fig.1a: Transmitter Model


Fig. 1b: Receiver model

## 4. PAPR Calculations

The PAPR is defined as:
$P A P R_{d B}=10 \log \left(\frac{P_{\text {peak }}}{P_{\text {average }}}\right)$
where $P_{\text {peak }}$ is the peak power and $P_{\text {average }}$ is the average power of the MC-CDMA signal.
In this work, the value of the peak power is obtained by calculating the maximum amplitude of the transmitted signal, $\mathrm{S}_{\mathrm{mc} \text {-cdma }}(\mathrm{t})$.

$$
\begin{align*}
P_{\text {peak }} & =A_{\text {peak }}^{2} \\
A_{\text {peak }} & =\operatorname{Max} .\left\{\left|S_{\text {mc-cdma }}(t)\right|\right\} \tag{5}
\end{align*}
$$

The average power of the complex envelope of the MC-CDMA signal is given by:
$P_{\text {average }}=\left.\lim _{a \rightarrow \infty} \frac{1}{2 a}\left(\frac{1}{2}\right)_{-a}^{a} \int_{-a} S_{m-c d m a}(t)\right|^{2} d t$

## 5. Results

A Multi-carrier CDMA system with 24 subcarriers has been simulated. The maximum number of users in the system is 24 . An m-sequence pseudo random signal is used for the input user data and the symbol duration is set to be $1 \mathrm{e}-6$. The simulations are run for duration of 10000 symbols (separate tests have established that this is sufficiently long duration).

Fig. $2 \mathrm{a}, 2 \mathrm{~b}$ and 2 c show the average power, peak power and the $\mathrm{PAPR} R_{d B}$ for different numbers of users for various modulation schemes.


Fig.2a: Average Power


Fig. 2b: Peak Power


Fig. 2c: PAPR

From Fig. 2a it can be seen that, for all modulation schemes, the average power increases linearly with the number of users. As expected, the graph for all PSK schemes is the same (because of the fact that it is a constant envelope signal). For the QAM schemes, the gradient of the graphs increases with the modulation index.

In the case of peak power (Fig. 2b), the gradient of all the curves increases linearly with number of users up to (maximum users) $/ 2$. After this point, increasing the number of users is seen to have no significant effect on the gradient of the curves. The WH coding may reduce the peak value of the transmitted signal.

It can be seen from the figure that the peak value for the different modulation schemes increases with the modulation index for both PSK and QAM schemes.

Fig. 2c shows the PAPR values for different users. From this figure, it can be concluded that the PAPR for a large number of users is lower then that for a small number of users. This may be attributed the spreading operation of the WH codes. Further simulation work is required to confirm this. It is interesting to note that the curves peak at (maximum users) $/ 2$. This is currently being investigated.

Overall, the PSK schemes have a lower PAPR value then the QAM schemes. In both cases, as the modulation index increases the PAPR values also increase.

## 6. Concluding Remarks

This paper presents results of a study of the PAPR performance of MC-CDMA system incorporating higher order modulation schemes. Simulation results have been presented for a number of different PSK and QAM schemes and it has been confirmed that PSK schemes give lower PAPR as compared with QAM schemes for a number of users. As the modulation index increases, the PAPR increases in both cases. Significantly, results show that the PAPR of a fully loaded system is lower then that for a single user casewith potential implications for system design.

Future work will involve the study of the MC-CDMA system with various mapping schemes in the presence of a nonlinear amplifier.

## References.

[1] R. Prasad and S. Hara, "Overview of multi-carrier CDMA," IEEE Communications Magazine, pp. 126-133, December 1997.
[2] N. Yee, J.P. Linnartz and G. Fettweis, "Multi-carrier CDMA in indoor wireless radio networks," Proceedings of IEEE PIMRC'93, pp. 109-113, September 1993.

