# Semi Analytic Fast Method to Dimension 3G downlink Air interface

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**Abstract:** There are two methods to dimension and plan downlink UMTS. The first method is based on an analytic formula, which is very fast, but inaccurate. The second method is based on Monte Carlo simulations, which is accurate, but time consuming. In this paper we will present a hybrid method based on statistical approximation, which combines the speed of the first method and the accuracy of the second method.

## 1. Introduction

DS-CDMA based systems, such as UMTS, are more difficult to dimension and plan compared to TDMA systems such as GSM. In GSM, the maximum achievable path loss in a cell is independent of the load in that cell. This makes coverage independent of achievable capacity. On the other hand, 3G CDMA based networks are highly affected by interference, which creates a complicated trade-off between capacity and coverage.

In 3G CDMA based networks all channels are reusing the same frequency. Consider the uplink where every additional user will contribute additional interference in the system. This forces the edge-of-cell mobile to increase its power to be able to achieve the required SIR at the base station. The cell radius is dictated by the maximum amount of power available to a mobile located at the edge of cell. Thus, the higher the interference in a cell the smaller the achievable coverage in that cell. This explains the very important characteristic of capacity-coverage trade-off in CDMA based networks. A similar phenomenon occurs on the downlink. These facts make dimensioning UMTS RF more complicated than conventional GSM networks. The dimensioning process should not only find the right cell area to carry a suitable amount of traffic, but it should also take the interference effect into account.

In GSM the number of time slots is known and constant, so the number of Erlangs that could be supported per transceiver is also known and constant (often calculated using the Erlang B model). In UMTS this is not the case because the number of simultaneous active links possible is variable depending on the instantaneous interference in the home cell and from neighbouring cells. So the way CDMA based networks cope with varying interference is another dimensioning complication. When dealing with mixed services this last complication becomes more pronounced.

To dimension and plan 3G networks the relationship between capacity and coverage has to be quantified. One way of doing this is to capture the relationship in a mathematical formula. Such a formula could be a link budget that specifies the achievable path loss given a certain loading in the network. It is possible to write a reasonably accurate mathematical equation that reflects the inter-dependency between capacity and coverage in the uplink direction. However, writing such a mathematical description -with a reasonable degree of accuracy - in the downlink is extremely difficult and complicated as we will see later (see [1] for an example of existing literature). Hence the attention has moved to Monte Carlo approach simulations. Undoubtedly Monte Carlo simulations can provide extremely accurate results, with the disadvantage of long run time. There are disadvantages associated with both methods. Since a reasonably accurate mathematical model is difficult to create, using the first method would yield inaccurate results. On the other hand, Monte Carlo simulations are relatively easy to perform, but at the same time are time-consuming. Updating any RF parameter will most certainly mean a new simulation run.

In this paper we propose a new hybrid method that relies on statistical approximation. This method offers the speed of the analytical approach while maintaining the accuracy of Monte Carlo approach. The paper is organised as follows. First we will investigate the difficulties of creating a meaningful downlink budget. Then we will explain the new proposed method. Finally we will compare the proposed method with the Monte Carlo method in terms of speed and accuracy.

## 2. Downlink Analytic Modeling Difficulties

In the uplink direction, the interference is coming from all other users in the system. Because of its immense randomness the uplink interference is often modelled as AWGN process. The story is slightly different in the downlink direction. There are two types of interference to the mobile in the downlink direction. The signal from the serving base station(s) to mobiles inside the serving cell(s) area is called "same-cell interference". The signal coming from other non-serving base stations is also seen as interference and is referred to as "other-cell interference". The other-cell interference can be assumed to be, more or less, random and arbitrary because it is coming from independent sources over independent paths. Consequently it could be modelled as an AWGN process (Additive White Gaussian Noise). On the other hand, the same-cell interference has a strong correlation with the useful mobile signal; it suffers the same fades that the useful signal goes through. For this reason same-cell interference has a different impact on the mobile compared to other-cell interference. This explains why the SIR requirement for a mobile depends on the relative strength of other-cell to same cell interference. In other words, the SIR requirement in the downlink direction, which is needed to guarantee a quality of service, depends on where the mobile is inside the serving cell. It stems from the above discussion that it is not possible to use one target SIR (or Eb/Io) as a representative of all possible cases in the downlink (e.g. location, handoff …etc).

A link budget looks at one point that is assumed to be located at the edge-of-cell. The maximum allowable path loss in the downlink direction depends on the amount of power the base station allows to this hypothetical edge-of-cell mobile. The power that a base station allocates to a mobile located at the edge-of cell depends on the amount of available power after some power has been reserved for other active mobiles in the cell, which in turn depends on the location of these other mobiles.

The users share the same total available traffic power. The traffic power allocated per user is dependent on user’s location, velocity, multipath scenario, fading and interference from other base stations. Also some traffic power has to be allocated to support mobiles in soft handoff from neighbouring cells, and the number of these soft handoff mobiles is a random variable.

All of this makes it difficult to create a meaningful link budget formula in the downlink direction. Any attempt to construct a downlink budget will only represent an example or a snapshot of the system and will not be valid for other possible scenarios.

## 3. Downlink Monte Carlo Simulations

Running Monte Carlo simulations may provide a good alternative to analytic models. Also they can provide answers to very specific cases which can not be reflected in an analytic formula. In this approach, cell sites are chosen and then the traffic is thrown according to the desired distribution. The performance of the system is assessed and recorded. This process is repeated a large number of times until the totality of results reflect a good approximation of the system behaviour. (We will not discuss details of such simulation in this paper). The problem with this method is that it is time consuming. It may take minutes, or hours to run a reasonably accurate Monte Carlo simulation. Changing few RF parameters, like antenna height, antenna orientation, or site location may necessitate the repeat of the whole simulation process.

## 4. Proposed Statistical Approximation Method

### A. Initial Calculation

This method relies on the knowledge of power requirement statistics in a sector. So first we will present a description on how to calculate these statistics. We explained earlier that one Eb/Io target is not enough to describe the link level performance in the downlink, because the required target depends on the mobile's location. The ratio of other-cell interference to same-cell interference influences the performance in the downlink and determines the amount of base station power that has to be allocated to a mobile (we will call this ratio: the interference ratio from now on). Hence, link level performance is described as a relationship between average power requirements and interference ratio. Figure 1 shows the power requirement vs. interference ratio for a 12.2 kbps voice service in a Pedestrian environment to achieve 1% BLER.

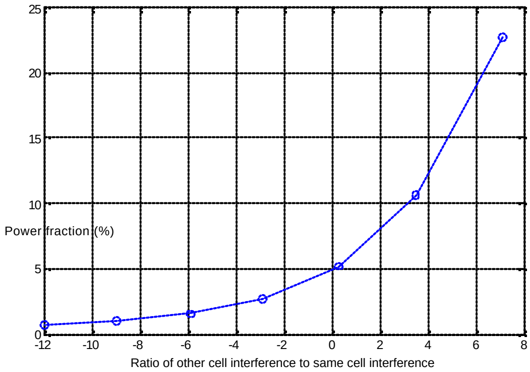


**Figure 1 Power Requirements vs. Interference ratio for 12.2k voice link in a Pedestrian environment**

The average power requirements are shown as a percentage of the total transmitted power. The figure shows that when other-cell interference increases in comparison to same-cell interference, then more power needs to be allocated to serve the mobile with an adequate quality of service.

Based on the above discussion, we need to calculate the interference ratio in order to decide the power requirements to a mobile location. So we begin by locating sites: a cell is surrounded by a number of cell rings with the required inter-site distance. Each cell has the required number of sectors and is assumed to be transmitting at a certain wanted level. Then we look at every pixel (bin) served by each sector in the sample cell. For each of these pixels the interference ratio is calculated. We can assume that all sites are transmitting maximum power. This is a reasonable assumption if the system is to operate at its maximum load. Log-normal shadow fading samples and directional antenna gain should be included at this stage.

To achieve a desired coverage confidence, e.g. 95% overall cell coverage, only the 95% best pixels are looked at. The better pixels are those with a small interference ratio. It is possible to determine the power fraction to be given to each of these pixels using the link level performance curves (e.g. like the curve in figure 1). For example, if a mobile falls in a pixel (bin) that experiences 0 dB interference ratio, then around 5% of the base station power has to be allocated to that mobile.

### B. Using Central Limit Theorem

The central limit theorem (CLT) states that if there are N identically distributed random variables which have a mean (μ) and variance (σ 2), then the sum of these random variables can be approximated as a normally distributed random variable with a mean (N.μ) and a variance (N.σ 2).

Assuming that the number of users in a sector is known, then it is possible to predict the statistics of the total power consumption using CLT. It is essential that the total power requirement for all mobiles in the sector does not exceed the total available power. If that occurs then a new user coming to the network will experience blocking due to lack of resources. So the problem reduces to finding the number of users N that would make the total power consumption exceed the available power for a limited and controlled percentage of time.

According to Central Limit Theorem, the summation of power fractions of mobiles in one cell will follow a normal distribution [2]. The probability that a new user will be blocked is the probability that the summation of power exceeds the available total power. Since the summation follows a normal distribution it is very easy to calculate that probability as follows:

The summation of power fractions is denoted χ and the number of users is N. Keeping in mind that a certain fraction of the total power is reserved as overhead (OH) for common channels:

$$\Pr[c > (1-OH)] = \int_{1-OH}^{\infty} \frac{1}{s\sqrt{2Np}} \exp\left(-\frac{(c-Nm)^2}{2Ns^2}\right) \cdot dc \qquad (1)$$

This formula can be described by using the classical Q function as follows:

$$\Pr[c > (1-OH)] = Q\left(\frac{(1-OH)-Nm}{s\sqrt{N}}\right) \qquad (2)$$

The Q (.) function is defined as follows:

$$Q(t) = \frac{1}{2} - \frac{1}{2} erf\left(\frac{t}{\sqrt{2}}\right) \tag{3}$$

It is possible then to calculate the number of users N that will ensure a specific blocking probability is (bp), as follows:

$$bp = Q\left(\frac{(1-OH) - Nm}{s\sqrt{N}}\right) \tag{4}$$

$$Q^{-1}(bp) = \frac{(1-OH) - Nm}{s\sqrt{N}} \tag{5}$$

$$m.N + Q^{-1}(bp).s.\sqrt{N} - (1-OH) = 0 \tag{6}$$

This is a 2nd order equation in terms of $\sqrt{N}$ that could be easily solved. The positive solution is then picked.

**C. Improving the central limit theorem**

The central limit theorem (CLT) works very well for a summation of a large number of random variables. As the number of random variables get smaller, the approximate CLT value deviates from the true statistics of the summation [2]. This is a problem when we have fewer mobiles per cell, as for the 64kbps service.

A closer investigation reveals that the probability density function of the power fraction random variable has an exponential distribution. It can be proven that the summation of exponentially distributed random variables has a gamma distribution. [3]. In other words, assuming that $x_1, x_2, ..., x_N$ are samples from the exponential distribution $f(x) = l \cdot e^{-lx}$ with x>0, then the sum of $x_1, x_2, ..., x_N$ has a gamma distribution with the density function:

$$\sum_N x_N = g(y) = \frac{l^n \cdot y^{N-1} \cdot e^{-ly}}{(N-1)!} \quad \text{with y>0.} \tag{7}$$

Consequently, the blocking probability can be calculated as follows:

$$\Pr[c > (1-OH)] = \int_{1-OH}^{\infty} \frac{l^n \cdot x^{N-1} \cdot e^{-lx}}{(N-1)!} dx = \frac{l^N}{(N-1)!} \int_{1-OH}^{\infty} x^{N-1} \cdot e^{-lx} d \tag{8}$$

To solve the above integral we note that:

$$W(x,a,b) \equiv \int x^a \cdot e^{-bx} dx = \frac{-e^{bx}}{b^{a+1}} \cdot \sum_{k=0}^{k=a} \frac{a!}{(a-k)!} (bx)^{a-k} + C \tag{9}$$

This implies that:

$$\Pr[c > (1-OH)] = \frac{l^N}{(N-1)!} [W(\infty, N-1, l) - W(1-OH, N-1, l)] \tag{10}$$

It is possible to solve the formula numerically to find the capacity N that would satisfy a given blocking probability.

**D. Multiple Services and accurate dimensioning**

3G systems offer a wide range of applications with different bit rates. Thus any dimensioning process should consider the impact of multiple services. The process described above could easily be extended for multiple services. It is very easy to extend the CLT analysis to include multiple services.

A cell might have mobiles belonging to different classes of service. We calculate the captured offered load per site contributed by different services. Let us assume that the number of mobiles in the cell (offered load) is $N_1, N_2, ..., N_n$ which belong to services $1, 2, ...., n$. At each pixel, the power requirement is calculated for each potential service (application). Then, the mean and standard deviation of power consumption of mobiles belonging to each of the services types is calculated and we denote them $m_1, m_2, ..., m_n$ and $s_1, s_2, ..., s_n$ respectively.

The total power consumption of mobiles belonging to one type of service is approximately normally distributed according to CLT. For example the power consumption of mobiles belonging to service 1 has a normal distribution characterised by the mean: $N_1 \times m_1$ and standard deviation $\sqrt{N_1} \times s_1$. If the power consumption of each mobile group is normal, then the total power consumption is a sum of normal processes, which is also normal. In other words, the total power consumption of all N users will have a normal distribution with mean:

$$m_T = N_1 \times m_1 + N_2 \times m_2 + ... + N_n \times m_n \tag{11}$$

and variance:

$$s_T^2 = N_1 \times s_1^2 + N_2 \times s_2^2 + \ldots + N_n \times s_n^2 \tag{12}$$

The blocking probability is then calculated as follows:

$$Blocking\_probability = Q\left(\frac{(1-OH)-m_T}{s_T}\right) \quad (13)$$

The blocking probability is an indication of the quality of service in the cell. As the inter-site distance is changed, the captured traffic per cell will change, and consequently the blocking probability will change. The inter-site distance can be varied until the desired blocking probability is achieved.

## 5. Comparison and Results

Figure 2 presents comparison results between Monte Carlo simulation and the two semi-analytic methods presented in this paper. The results are applicable to a system with 1.2 km inter-site distance and 2° down-tilted antenna beams.

The figures shows how the Gamma sum approximation method has a better performance than the Central Limit Theorem (CLT) approximation.

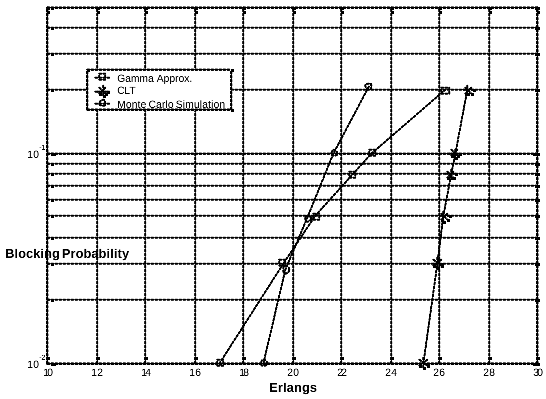


**Figure 2 Capacity Comparison between Monte Carlo and the semi-analytic methods**

In terms of running time, a Matlab program was used to assess the speed of each of the two methods. It took around 40 minutes to generate the Monte Carlo curve presented in the figure above. The semi-analytic method took only 8 minutes to run.

## 6. Applications of the semi-analytic method

As mentioned before, to perform a UMTS dimensioning exercise, the relationship between coverage and capacity has to be created. To create the capacity-coverage relationship using a Monte Carlo method, the capacity has to be evaluated for a range of possible inter-site distances.

Using the alternative method described above, the capacity N is calculated based on the power requirement distribution for a given inter-site distance. By noting that the interference ratio is independent of the inter-site distance, it is possible to incorporate the change in performance due to different inter-site distances by including the effect of thermal noise. Instead of repeating the interference ratio calculation for another inter-site distance, it is possible to correct the interference ratio by including the thermal noise floor in the calculation. At the end a capacity-coverage trade-off curve can be created.

Cell planning tools that rely on Monte Carlo simulations can improve their speed with a slight degradation in accuracy by using the semi-analytic method. In contrast to the dimensioning process, cell-planning tools perform the calculation based on the knowledge of the site locations and the intensity of traffic in a given scenario. Hence a slightly more complicated process has to be adopted in order to apply the semi-analytic method in cell-planning tools.

Briefly, the statistics of power requirements in each pixel inside the coverage of each sector has to be calculated first, then based on the offered traffic the blocking probability could be calculated.

## 7. Conclusion

In this paper we presented a semi-analytic method which would enhance the speed of downlink simulations while maintaining a reasonably accurate result, and which can be used for dimensioning, planning and optimisation of DS-CDMA based networks, such as UMTS.

Two different variations of the semi-analytic methods were presented. We showed that the Gamma approximation has a better fit to simulation results than the CLT approximation. The theory was presented in this paper, and both accuracy and speed were compared with Monte Carlo simulations.

## 8. References


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[3] Craig, 1970, "Introduction to Mathematical Statistics" Collier Macmillan