Estimation of average hop count using the grid pattern in multi-hop wireless ad-hoc network

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Abstract: Compared to a base station centralised wireless network, the multi-hop wireless ad-hoc network probably provides a flexible method of establishing a limitless, robust, inexpensive alternative in architecture. However, as far as concerned with its performance, throughputs of the multi-hop wireless ad-hoc networks are often believed to be lower than that in base station centralised networks. This is because the relay nodes in a multi-hop traffic traversal duplicate the whole traffic amount, which degrades the effective bandwidth usage in a propagation area. Hence the estimation of the average hop count in a multi-hop wireless ad-hoc network is important since it is beneficial to the design of network size and node connectivity in consideration of the throughput of the multi-hop wireless ad-hoc network. In this paper we try to give methods to estimate the average hop in a multi-hop wireless ad-hoc network with certain degree in connectivity.

1 Introduction.

Some opposed opinions have been existed upon the architecture of the multi-hop wireless ad-hoc network, like the poor throughput, the large latency and the time-sensitive routing performance due to the patterns in the dynamic mobility. The throughput is believed in the degradation because relay nodes in a multi-hop path duplicate the traffic when they are forwarding the packets. Hence a multi-hop ad-hoc network is easily to produce heavy redundant traffic. The average hop count anyhow indicates the severity of traffic duplication. The latency of a packet delivery is often larger in a multi-hop network because each node acts as a store-and-forward buffer and causes delay. Therefore the average hop count is related to the delay enlargement. The time-sensitive routing variance concerns with the mobility pattern directly, but it is actually has an indirect relationship with connectivity degree of the node. If a node has too few neighbours, it is easy to lose the routing path due to unexpected link disconnections. If a node has too many neighbours, the effective throughput with in the propagation cluster will be lower since more nodes share the same bandwidth. Thereafter a moderate value of node connectivity degree can stabilise the wireless routings while balancing the throughput. In a network with a certain size, the connectivity degree and average hop amount are contradicting things. So the average hop count is still the parameter under consideration for time-sensitive routings in a multi-hop ad-hoc network.

In the following sessions, we refers the objective network to the link connection-oriented one because it has clear point-to-point peer that helps us to know the result of the node connectivity degree vs. the average hop count. This paper is organised as follows. In section II, we introduce the approach to estimate the average hops. In section III, some simulations are presented in order to compare the estimations. Section IV leads the conclusion.

2. Estimation of average hop amount in ad-hoc networks

The difficulty in estimating the average hop count in a multi-hop ad-hoc network shows at *node distribution, propagation distance and dynamic patterns of mobility*. In the following discussion, we assume the nodes in a plane obey the uniform distribution. A large propagation radius implies that the node likely has a big connectivity degree to other nodes. As a result, the average hop count also decreases. Our estimation is based on a big plane with enough nodes. Thus we believe the estimation value still near the average hop count even in a plane with nodes in mobility.

In a grid pattern, we regard a 2-dimention rectangle plane $M \times N$ grids. Hopping direction will follow roughly vertical and horizontal. We assume a grid can reach the neighbour grid in south, east, north and west direction while in a circle. We make this assumption in order to know the effect of node connectivity degree and density.

For a node in grid (x, y), we iterate the possible grid (u, v) as receivers. Thus $hc(x,y) = \sum_{u=1}^{M}\sum_{v=1}^{N}(|x-u|+|y-v|)$.

Therefore, the total hop counts will be $all\_hc = \sum_{x=1}^{M}\sum_{y=1}^{N} hc(x,y)$ and the average hop count will be $avg\_hc(M, N) = \frac{1}{MN(MN-1)}\sum_{x=1}^{M}\sum_{y=1}^{N} hc(x,y)$.

2.1 Node density = 1 (the correspondent propagation radius $r = 1.26$)

We first put $M \times N$ nodes in a panel of $M \times N$ grids, which means each grid accommodate a node averagely. Thus the density of this network is 1 (node per grid). We denote $(M, N, density)$ the network. With a proper propagation radius in each node, we suppose each node can communicate with four nodes in neighbour grids. Considering the sum of hops between any node pair, i.e. $All\_hc(M, N, 1) = \sum_{x=1}^{M} \sum_{y=1}^{N} hc(x, y)$, we can deduce it in convenience by separating into six components.

Component 1: When the destination node (u, v) is in the northwest of the source node (x, y), the horizontal distance is $x - u$ and the vertical one is $y - v$. Without considering shortcuts in diagonal traversals, the hops between two nodes will be $x - u + y - v$.

$$Co1 = \sum_x \sum_y hc(component1) = \sum_{x=2}^{M} \sum_{y=2}^{N} \sum_{u=1}^{x-1} \sum_{v=1}^{y-1} (x - u + y - v) = \frac{MN(M-1)(N-1)(M+N-2)}{24}$$

Component 2: When (u, v) is in the southeast of (x, y), we have $Co2 = MN(M-1)(N-1)(M+N-2)/12$

Component 3: When (u, v) is in the southwest of (x, y), we have $Co3 = MN(M-1)(N-1)(M+N+2)/24$

Component 4: When (u, v) is in the northeast of (x, y), we have $Co4 = MN(M-1)(N-1)(M+N+2)/12$

Component 5: When the destination node (u, v) is in the same latitude of the source node (x, y), then $y = v$. The hops between two nodes will be $x - u$ if $x > u$, or $u - x$ if $x < u$.

$$Co5 = \sum_x \sum_y hc(component5) = \sum_{x=1}^{M} \sum_{y=1}^{N} [\sum_{u=1}^{x} (x-u) + \sum_{u=x}^{M} (u-x)] = \frac{MN(M-1)(M+1)}{3}$$

Component 6: When (u, v) is in the same longitude of node (x, y), we have $Co6 = MN(N-1)(N+1)/3$

Combining all the six components, we get the whole hops count in this $M \times N$ community with node density 1.

$$All\_hc(M, N, 1) = \sum_{x=1}^{M} \sum_{y=1}^{N} hc(x, y) = M^2 N^2 [\frac{M+N}{4} + \frac{M/N + N/M}{3}]$$

For simplicity, we consider a square plane with side length a. And let $M = N = a$ and $M \pm 1 \approx M,\ N \pm 1 \approx N$ while the plane is large enough. Therefore the average hop count will be the follows.

$$avg\_hc(a, a, 1) = \frac{1}{a^2} All\_hc(a, a, 1) = \frac{a}{2} + \frac{2}{3} \qquad (1)$$

Until now we have made an assumption that the propagation radius of a node just covers four adjacent nodes. It is hard to get the proper value of the radius since the nodes are randomly placed in the plane. Here we give an approximation in the value of the radius. We put a number of N^2 nodes in a grid plane of $N \times N$. Considering the possible four adjacent nodes in four directions, we hope the each node should have a radius to cover the space of 5 nodes, including that node and its four neighbours. Therefore $\pi r^2 = 5$, and we get $r = 1.26$.

2.2 Node density > 1

When node density is bigger than 1, it implicitly means that each grid accommodates more than one node. This can be achieved by increasing the propagation radius of a node. It is expected that the average hop count will decrease, however, the effective throughput per node will also decrease because more nodes compete for the limit bandwidth in the same propagation coverage. Meanwhile, it requires more energy consumption in nodes to obtain a longer propagation distance. As the propagation radius increases, we rescale the side length of the plane. We still keep the plane to accommodate a number of N^2 nodes, i.e. $a = N$. We denote $r_0 = 1.26$, and use r to represent the actual propagate radius. Then we convert the new side length as a normalise one, which equals to r / r_0. Sequentially we get the new side length of the square plane to be $a' = a/(r/r_0)$. Finally we obtain the average hop count in a N^2-node-plane with node density of r / r_0 as the following.

$$avg\_hc(N, N, r/r_0) = \frac{1.26}{r} \frac{a}{2} + \frac{2}{3} \qquad (2)$$

2.3 Node density < 1

We use the concept of the node density to describe how many nodes in a grid. However, in the last discussion about node density bigger than 1, we already realise that the grid actually is decided by the node propagation radius. According to the value of propagation radius, we can covert the plane to the new one with effective grids for further estimation in equation (2). In the simulation, we will know that average hop count will decrease dramatically when propagation radius r takes a small value, e.g. $r=1$, then many nodes will lose the connectivity to adjacent nodes. We denote $Den(N,a)=N/a^2$ the density for N nodes in a square plane with side length a. Thus each grid owns a space of a^2/N, which the side length $a/\sqrt{N}$. If we still consider the typical value 1.26, then the propagation radius per node should satisfy $r>1.26a/\sqrt{N}$.

3. Comparison in simulation

Our simulation firstly sets up a rectangle plane with the side length of M and N. We predetermine the propagation radius of the node. Next, we iterate each node to judge the neighbour nodes in propagation distance, which means the existence of links. We use Dijkstra algorithm [5] to deduce the hop count.

3.1 Node density = 1

Under the node density as 1, we compare the result of equation (1) and simulation. The simulation was done in a square plane with side length a. For each node, the propagation radius ($r=1.26$) implicitly covers four other nodes. We range the side length of the simulation plane from 5 to 30 to see the results (see Figure 1). We can see the simulation result is near the estimation value while it is always smaller.

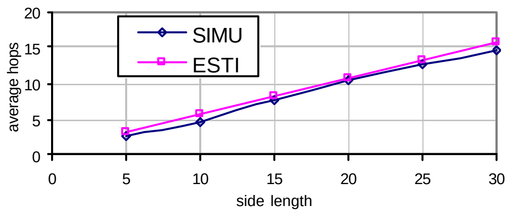

Figure 1 Comparison of avg_hc(a,a,1) and simulation results

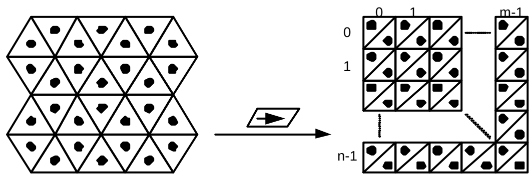

Figure 2 A sample layout for the diagonal traversal

Reason 1: During the simulation, the hopping traversal does not strictly follow the horizontal or vertical paths. A typical sample shows in Figure 2. The left plane is the original layout. Each node has a connectivity degree of three. We convert it to the right plane in order to show the rectangle plane clearly. We observe the traversal sometimes goes in a diagonal shortcut path. As a result, the average hop count will be decreased.

Reason 2: In the simulation, we define the propagation radius as 1.26, which equals to the total space occupied by five nodes. But the selection of proper radius is complex and $r=1.26$ was deduced in a risky approximation. In fact, $r=1.26$ is more than expected for a node to cover four adjacent nodes in propagation. We show this in simulation (see Figure 3). It is noted that the actual node connectivity is really more than the expected value of 4. But we also notice an exception happens when side length is small, e.g. $a=5$. It is due to the edge effect in lower connectivity.

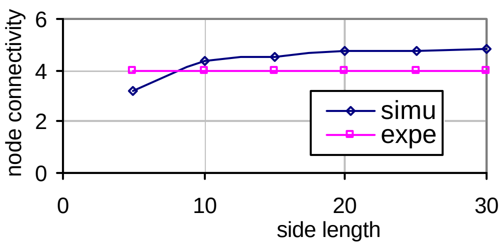

Figure 3 average node connectivity when r=1.26

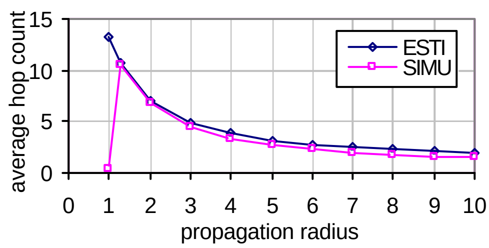

Figure 4 average hc vs. propagation radius

3.2 Node density > 1

In the simulation for node density bigger than 1, we put 400 nodes (i.e. $N=20$) in a square plane. We do not explicit the grid numbers and the side length of the plane, as we will dynamically decide the value of side length of the plane according to the propagation radius. With this dynamical decision we try to keep the propagation of a node still covering 5 new grids in terms of new coordinate system. In our simulation, we range the propagation radius from 1 to 10 units in old coordinate system.

It is noted that an abnormal value appears when propagation radius $r = 1$. The average hop count in simulation drops down to below one hop. This is because the some nodes are hardly to build any links to the neighbours under such a short propagation distance. We can imagine that the plane will show a pattern with many lonely nodes like islands (See Figure 5). When we take a typical value $r = 1.26$, ad-hoc nodes increase the inter-connectivity and eliminate the "lonely island" effectively. (See Figure 6)

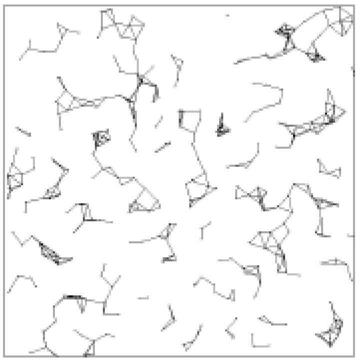

Figure 5 Sample ad-hoc (20,20,1)

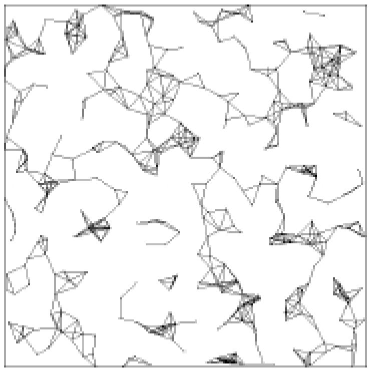

Figure 6 Sample ad-hoc (20,20,1.26)

3.3 Node density < 1

We have pointed out that the nodes still obtain enough connectivity to construct a proper ad-hoc network even though node density is less than 1. It is because the node density is a concept of node number vs. network layout space, so it is not an absolute description for the capacity of nodes in connectivity. Instead, a better understanding should base on the propagation radius per node, i.e. r. With the parameter r, we can re-scale the grid pattern and estimate the average hop count, and that is similar to the conditions above. Therefore no more simulations are presented here.

4. Conclusions.

In this paper, we present a method in the estimation of the average hop count in an ad-hoc network using grid pattern. Considering a N node ad-hoc network in a square plane, the average hop count will follow $O(\sqrt{N})$. This estimation can help us to evaluate other network parameters such as the throughput since the intermediate nodes in hop paths duplicate the traffic.

Furthermore, we also discuss the effort of node density and propagation radius against the average hop count. Increasing the density and propagation radius can escalate the connectivity degree per node. It means the average hop count will be decreased as a result when the total amount of nodes is certain. We use the grid space formulation to obtain the new grid system so that we can get the average hop count quantitatively with different node density and propagation radius.

The ratio of the average node space over the whole plane size determines the accuracy of our estimation. If the ratio is too big that plane accommodates only a few nodes, it will seriously affect the result of our estimation. We also pointed out that a large density or propagation will degrade the performance of nodes in the same propagation coverage, since the resource like the bandwidth has to be shared. Also a large propagation radius requires more energy consumption of nodes, which is not economic to the battery life in the mobile node. On the contrary, a low density or small propagation radius will destroy the integration of the ad-hoc network. We can see when propagation radius $r = 1$, the simulation of average hop count suddenly drops down. It is because many nodes have to lose connections with others and become lonely islands. Therefore, a reasonable node distribution in the plane is important to the performance of a multi-hop ad-hoc network.

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