Coding and Signal Design for Multiple Channels

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Abstract – **We propose a coded Discrete Multitone (DMT) modulation. The coded DMT is constructed using Reed Solomon (RS) code, combined with Berlekamp Massey algorithm, and Discrete Fourier Transform (DFT). The fundamentals of RS code and DFT are outlined. We discuss several implementation issues and potential benefits of the coded DMT.**

1. Introduction

Multi Carrier Modulation systems rely on segmenting the total available spectrum and then employing the multiple channels in combination. The transmitted signal is the aggregate of the signals on these individual channels. It is appropriate that signal design and coding should take account of this aggregation.

Since Shannon published a paper on channel capacity [1], there have been many advances in coding theory. We consider a block code, called Reed Solomon code [2], combined with a decoder, known as Berlekamp Massey algorithm. In traditional applications, block codes are defined over finite fields. Recent paper shows that they can be defined over the real-number and complex number fields [4].

We present the fundamentals of Discrete Fourier Transform (DFT) [3], and examine the relationship between the DFT and error control codes. Under certain conditions, discrete time sequences carry redundant information which then allow for the detection and correction of errors [5].

Areas of recent activity on multi carrier modulation include designing codes with spectral nulls [6], application to asymmetric digital subscriber line [7], investigating the distribution of intermodulation distortion [8], and controlling peak to average power ratio [9].

Discrete Multitone (DMT) modulation is a particular form of multi carrier modulation, where Inverse Fast Fourier Transform (IFFT) and Fast Fourier Transform (FFT) are used for modulation and demodulation.

2. Reed Solomon code and Berlekamp Massey algorithm

2.1 Reed Solomon code

The set of Reed Solomon (RS) codes is a subset of BCH codes [2].
For this set of codes, the block length $n = q^m - 1$ divides the number $q-1$ of non-zero elements in the multiplicative field of the symbol alphabet. That is to say, the symbol field $GF(q)$ and the error locator field $GF(q^m)$ are the same. Let $b \in GF(q)$, then the minimal polynomial of b over $GF(q)$ is

$$f_b(x) = (x - b) \tag{1}$$

Hence the generator polynomial $g(x)$ of a t error correcting RS code is

$$g(x) = (x - a)(x - a^2)\cdots(x - a^{2t}) \tag{2}$$

where a is a primitive element.

RS codes are optimum in the sense of Singleton bound. The optimality, the existence of efficient encoding decoding algorithms, flexibility offered by the wide range of codelengths, and the existence of hardware to perform encoding decoding operations justify our interest in this set of codes.

2.2 Berlekamp Massey algorithm

Peterson Gorenstein Zierler decoder [2] is a conceptually clear algorithm for decoding BCH codes, but Berlekamp Massey algorithm provides computationally more efficient decoder, by circumventing matrix inversion. Most of the computations required to decode BCH codes using Peterson Gorenstein Zierler decoder centres on the solution of a matrix equation. The matrix equation is equivalently represented by the equation

$$S_j = -\sum_{i=1}^{v} \Lambda_i S_{j-i} \quad (j = v+1, \cdots, 2v) \tag{3}$$

For fixed Λ, this is the equation of an autoregressive filter. Seen in this way, the problem becomes one of designing the linear feedback shift register that will generate the known sequence of syndromes S_i's. Denote a minimum length shift register for producing $S_1, \cdots, S_r$ by $\left(L_r, \Lambda^{(r)}(x)\right)$, where L_r is the shift register length, $\Lambda^{(r)}(x)$ is the feedback connection polynomial, and $\deg \Lambda^{(r)}(x) \le L_r$. Berlekamp Massey algorithm provides a way to compute a shortest length shift register $\left(L_r, \Lambda^{(r)}(x)\right)$ that generates the sequence $S_1, \cdots, S_{r-1}, S_r$, given $\left(L_i, \Lambda^{(i)}(x)\right) \quad \forall\, 1 \le i \le r-1$.

3. DFT and redundancy of DFT

3.1 Discrete Fourier Transform

Discrete Fourier Transform (DFT) is the Fourier representation of a finite length sequence, and it corresponds to samples equally spaced in frequency of the Fourier transform of the signal.
Given a finite duration sequence $x(n)$ of length N, defined over the range $0 \le n \le N-1$, its DFT is given by

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \qquad , \qquad 0 \le k \le N-1 \tag{4}$$

and Inverse Fourier Transform (IDFT) is given by

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn} \qquad , \qquad 0 \le n \le N-1 \tag{5}$$

where $W_N = e^{-j(2p\ /\ N)}$. Properties of DFT include linearity, circular shift, symmetry, and circular convolution [3]. DFT plays a central role in the implementation of a variety of digital signal processing algorithms, as a result of the existence of efficient algorithms for the computation of the DFT. The fundamental principle that these algorithms, collectively known as Fast Fourier Transform (FFT) algorithms, are based upon, is that of decomposing the computation of the DFT of a sequence of length N into successively smaller DFTs [3].

3.2 Redundancy of DFT

We describe the relationship between DFT and error control codes. Under certain conditions, discrete time sequences carry redundant information which then allow for the detection and correction of errors. Let $\underline{x} = (x(0), x(1), \cdots, x(N-1))$ be a vector denoting a data sequence, let $R_N(k)$ denote the remainder of the integer k when divided by N, let a be any integer in the range $0 \le a \le N-1$, and let K be a positive integer less than N. We consider the data sequence having the property that $X(R_N(k)) = 0$ for $k = a, a+1, \cdots, a+(N-K-1)$. That is, the sequence with $N-K$ consecutive zero spectral components. For this set of data sequences, the vector $\underline{x}$ can be reconstructed from any K of its N components. Assuming a received data sequence $y(i)$ is obtained by passing the original data sequence $x(i)$ over an additive noise channel where T or fewer values of the transmitted data sequence are corrupted by noise, then the original data sequence $x(i), i = 0,1,\cdots, N-1$ always can be computed from the received data sequence $y(i)$, $i = 0,1,\cdots,N-1$ if $T \le (N-K)/2$ [5].

4. Discrete Multitone Modulation

A simplified block diagram of the DMT transmitter is presented in figure 1.

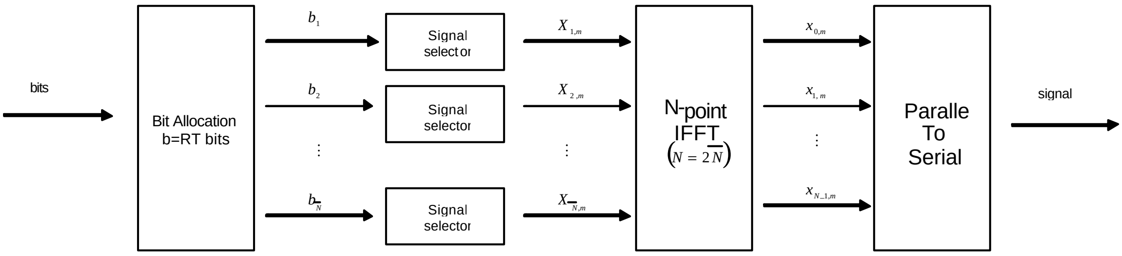


Figure 1. Baseline DMT transmitter

At the input to the system, the bit stream is partitioned into blocks of size $b = RT$ bits, where R is the input bit rate, T is the DMT symbol period, and b is the number of bits contained in one DMT symbol. The bits collected during the m^{th} symbol interval are allocated among $\overline{N}$ subchannels or tones in a manner determined during system initialization with b_i bits assigned to tone i, and $\sum b_i = b$. On subchannel i, the b_i bits are mapped to a constellation point $X_{i,m}$ in a constellation size 2^{b_i}, and the collection of constellation points $\{X_{i,m} : i = 1,\cdots,\overline{N}\}$ serves as the input to an Inverse Fast Fourier Transform (IFFT) block. The time domain signal that is transmitted over the channel is obtained by performing a length $N = 2\overline{N}$ IFFT on the complex constellation points, where constrains are imposed to ensure a real valued signal.

The receiver corresponding to the transmitter illustrated in Figure 1 is presented in Figure 2.

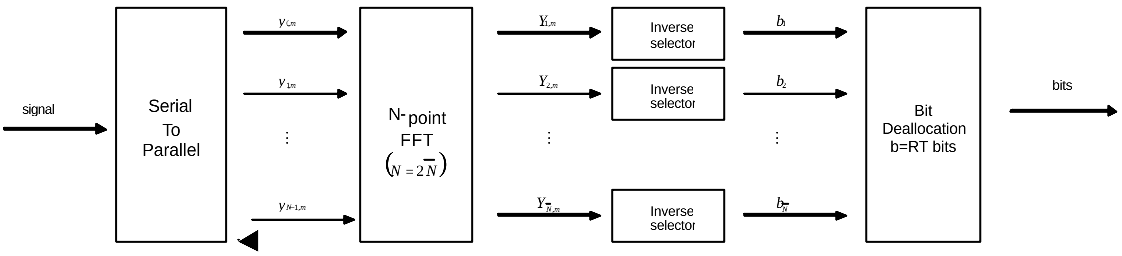


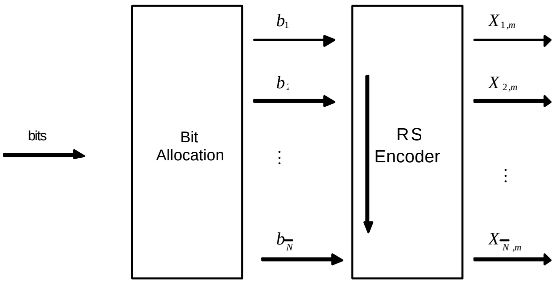
Figure 2. Baseline DMT receiver

This structure consists of the inverse operations of those performed in the transmitter. At the input to the receiver, N time domain samples $\{y_{i,m} : i = 0,\cdots,N-1\}$ are collected each DMT symbol period, and a Fast Fourier Transform (FFT) is performed to obtain a set of noisy constellation points $\{Y_{i,m} : i = 1,\cdots,\overline{N}\}$.

5. Coded Discrete Multitone Modulation

For the data rates and types of applications that we want to support, coding scheme should have the following desirable properties: 1. large coding gain, 2. reasonable implementation complexity, 3.flexibility, adaptable to data rate, 4. computational efficiency.

To achieve all the goals listed above, we propose a block coding scheme operating across the tones. Figures 3 and 4 illustrate how the proposed coding scheme is incorporated in the DMT system. Only those portions of the DMT transceiver involving the encoding of bits into complex symbols, and the decoding of complex symbols into bits are depicted, since the modulation and demodulation operations remain unchanged.



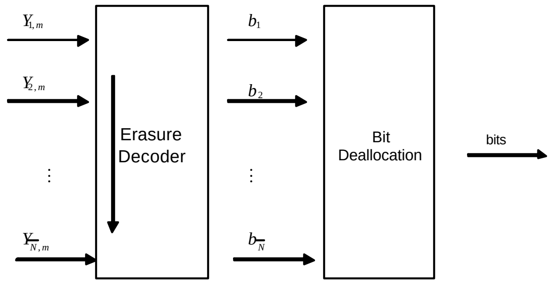


Figure 3. Coded DMT transmitter

Figure 4. Coded DMT receiver

The encoder in Figure 3 operates on the bits at the output of the bit allocation unit, producing a set of complex symbols that serves as the input to the IFFT block. One approach is to use multiple copies of the RS encoder. Another method is to encode across the subchannels as shown in Figure 3. In the latter case, the signal selector now becomes time varying, selecting points from different size constellations as the encoder operates across the block, and therefore introduces interdependencies among the signal points selected on different tones. As a result of the redundancy of the RS code, the signal selector will make the best tradeoff between bandwidth expansion, where additional tones are used for transmission, and signal set expansion, where the sizes of the constellation supported by a subset o f the tones are increased, in distributing the additional bits. This unique capability of the DMT system allows the blockcode to achieve respectable coding gains over bandlimited channels. The coded bits are allocated according to the spectral characteristics of the channel.

In the receiver, depicted in Figure 4, a single erasure decoder is used to operate across the subchannels on the noisy constellation points at the output of the FFT. The erasure decoder finds the closest code sequence to the received signal points under the assumption of essentially infinite constellation sizes in each of the subchannels, and then masks off the appropriate numbers of bits from the decoded constellation labels according to the information stored in the bit allocation table. The Berlekamp Massey algorithm, combined with the computation of erasure locator polynomial, can be used to correct both errors and erasures.

6. Conclusion

We proposed a coded Discrete Multitone (DMT) modulation. The coded DMT is constructed using Reed Solomon (RS) code, combined with Berlekamp Massey algorithm, and Discrete Fourier Transform (DFT). The fundamentals of RS code and DFT were outlined. We discussed several implementation issues and potential benefits of the coded DMT.

7. References

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