A Channel Estimation Method for MIMO-OFDM Systems

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Abstract: This paper proposes a simple and efficient method for MIMO-OFDM channel estimation using parameters similar to HIPERLAN/2. Both preamble and pilot structures are compared in a 2 transmit-2 receive Space Frequency Trellis Coded system and the Mean Squared Error is used as a metric for comparing the results. It is shown, via simulation, that the proposed methods incur a maximum loss of approximately 1 to 1.5dB as compared to perfect channel knowledge.

1 Introduction

Channel estimates for Multiple Input Multiple Ouput-Orthogonal Frequency Division Multiplexing (MIMO-OFDM) systems can be obtained by transmitting a training sequence from one antenna at a time while the remaining transmit antennas are idle. This method, however, becomes inefficient when the number of transmit antennas are large.

In this paper, we extend the channel estimation technique proposed in [1] to a MIMO system. Firstly, a brief description of the method used in [1] is provided in Section 2. Section 3 extends this technique to a MIMO system and proposes two specific methods for channel estimation. A 2 transmit-2 receive antenna system is examined as an example in Section 4. Simulation results for channel estimates, the Mean Squared Error (MSE), and the system performance of this technique when combined with trellis codes are shown in Section 5. Finally, Section 6 draws several conclusions from these results.

2 Iterative Channel Estimation Algorithm

In [1] a hardware efficient algorithm shows how the frequency response of null subcarriers can be interpolated by exploiting the frequency correlation of limited time excess delay channels. The algorithm is briefly described as follows:

1. Obtain initial channel estimate (typically performed using the Least Squares (LS) method).
2. Convert channel estimate to the Time Domain (TD) and window significant taps.
3. Convert this TD signal back to the frequency domain.
4. Replace the values of the known subcarriers with the initial estimate in step 1 (ignore this step for the last iteration).
5. Repeat steps 2-4.

Although the original application for this technique was single antenna IEEE 802.11a systems, it can be easily extended to MIMO systems. Consider a MIMO-OFDM system with M transmit, N receive antennas, and K subcarriers. Let $c_{m,k}$ be a pilot symbol transmitted from antenna m for subcarrier k. The received signal at receive antenna n can be modelled as:

$$r_{n,k} = \sum_{i=1}^{m} h_{n,m,k} c_{m,k} + \eta_{n,k} \tag{1}$$

where $h_{n,m,k}$ is the frequency response of subcarrier k between transmit antenna m and receive antenna n, and $\eta_{n,k}$ represents additive white Gaussian noise with zero mean and variance $\frac{\sigma_n^2}{2}$ per dimension. Hence, the received symbol at each receive antenna is a linear combination of transmitted symbols that are modified by channel gains and noise.

3 Proposed Channel Estimation Technique

Armed with this knowledge, we can construct a simple yet effective method of channel estimation. Note that (1) can be reduced to:

$$r_{n,k} = h_{n,m,k} c_{m,k} + \eta_{n,k} \tag{2}$$

if only one antenna is transmitting on that subcarrier. Then, $h_{n,m,k}$ can be estimated by simply dividing $r_{n,k}$ by the known training symbol, $c_{m,k}$. Thus, $\hat{h}_{n,m,k} = r_{n,k}/c_{m,k}$, where $\hat{h}_{n,m,k}$ is the channel estimate for subcarrier k between transmit antenna m and receive antenna n. The training sequence can then be composed of a number of pilot symbols on select subcarriers while the rest can be set to zero. The frequency response of the remaining subcarriers can then be interpolated using the algorithm described in [1].

A simple way of designing such training sequences is by sending a pilot symbol on subcarrier k of antenna m for $m \in [1, M]$ and zeros on subcarrier k for all other antennas. For the next subcarrier $k+1$, antenna $m+1$ can send a pilot symbol, while all other antennas transmit zeroes as seen in Figure 1.

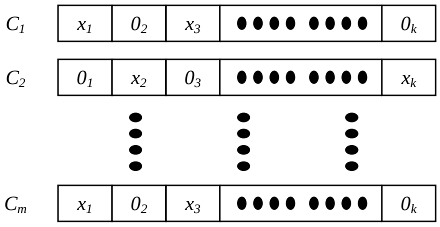

Figure 1: Training sequence design example, x_k = pilot symbol on subcarrier k, C_i = training sequence for transmit antenna i

Therefore, using this technique only one OFDM symbol per antenna is needed to estimate the entire MIMO channel at the expense of some added complexity.

These training sequences can be structured in two ways: a preamble structure or a pilot structure. In the preamble structure, the first OFDM symbol sent is composed strictly of pilot symbols while in the pilot structure, the first OFDM symbol sent is composed of both training and information data[1]. The advantage of using the preamble structure over the pilot structure is that the larger number of subcarriers in the preamble structure dedicated to pilot symbols results in better channel estimates. The pilot structure however, allows for tracking of a fast moving channel.

Furthermore, in designing the training sequences the minimum number of pilot symbols needed per transmit antenna is dependent on the channel order, L. Hence, for a training sequence, the total number of subcarriers needed for pilot symbols are, $\kappa \geq MC_P$, where C_P is the size of the Cyclical Prefix of the OFDM symbol. Ideally, κ should be determined by L. However, since systems are designed with a predetermined C_P length and subcarrier orthogonality is lost when $L > C_P$, it is only sensible to design a training sequence for the worst case. Obviously, for a fixed C_P size, M is limited to K/C_P.

Another design consideration is the subcarrier spacing between pilot symbols. Close spacing of pilot symbols will result in better channel estimates. However, the pilot symbols should also be spread out over the entire range of subcarriers in order to distribute the estimation error equally across the entire bandwidth of the system.

4 Case Study: 2 Transmit-2 Receive MIMO–OFDM System

As a specific example, a 2 transmit-2 receive antenna system with OFDM parameters similar to [2] was considered. The total number of subcarriers was 64 where only 52 of the subcarriers were actually used to transmit data[2]. The 12 remaining subcarriers (including one DC subcarrier) were nulled and $C_P = 16$. Thus, $\kappa \geq 32$ for a 2 transmit antenna system.

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[1] Here, the term *pilot structure* refers to an OFDM symbol while the term *pilot symbols* refers to known symbols used for channel estimation within an OFDM symbol.

[2] In a small deviation from [2], all 52 subcarriers are used for data symbols.

Table 1: Subcarrier Index of Non-Zero Pilot Symbols for Preamble and Pilot Structures

Preamble Structure	
C_1	7 9 11 13 15 17 19 21 23 25 27 29 31 34 36 38 40 42 44 46 48 50 52 54 56 58
C_2	8 10 12 14 16 18 20 22 24 26 28 30 32 35 37 39 41 43 45 47 49 51 53 55 57 59
Pilot Structure	
C_1	7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52
C_2	8 11 14 17 20 23 26 29 32 35 38 41 44 47 50 53
Information Data	9 12 15 18 21 24 27 30 36 39 42 45 48 51 54 55 56 57 58 59

Using the design considerations discussed in the previous section, both the preamble and pilot structures were constructed. The index of non-zero pilot symbols is summarised in Table (1), where C_i is the training sequence from transmit antenna i. All other subcarriers ($k \in [1, 64]$)[3] not specified in Table (1) are assumed to be zero.

It is easy to see that when the subcarrier of transmit antenna 1 transmits a pilot symbol, the same subcarrier on antenna 2 transmits a zero and (2) holds. Note that for the pilot structure, 20 subcarriers are used to transmit information data.

5 Simulation Results

The simulation results for the iterative channel estimation algorithm for a single antenna system can be seen in Figure (2). The '$\diamondsuit$' represents the initial channel estimate while the '$\circ$' represents the actual channel. Solid lines are the estimated channel after each iteration where a total of 15 iterations were performed. The channel had a zero mean random complex Gaussian distribution with variance $1/(2L)$, where $L = 3$.

Using the preamble and pilot structure described above, we also simulated the performance of this channel estimation method for a MIMO system with the same channel characteristics as previously described. The frequency response using the pilot structure is shown in Figure (3) where the solid curve is the estimated response while the dotted curve is the actual frequency response. It is easy to see that the estimated channel closely approximates the actual channel response.

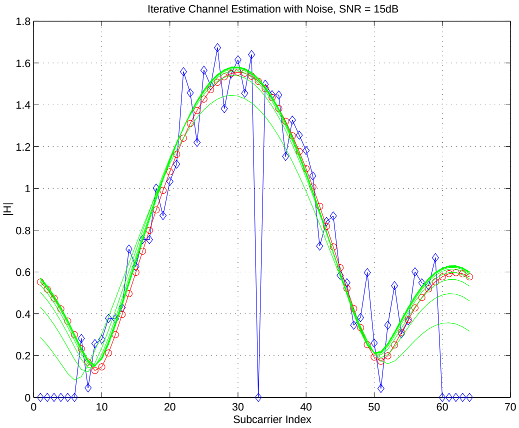

Figure 2: Iterative Channel Estimation ($\diamondsuit$ = Initial Estimate, $\circ$ = Actual Channel, $-$ = Estimated Channel after Iteration)

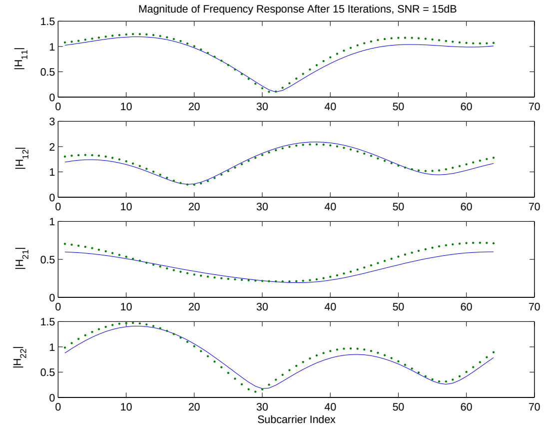

Figure 3: Channel Estimate (solid curve) and Channel Response (dotted curve), $L = 3$, 15 iterations

To quantify the performance of both structures, the MSE was used and is given by the equation:

$$MSE \triangleq E\left\{(\hat{\mathbf{h}} - \mathbf{h})^H(\hat{\mathbf{h}} - \mathbf{h})\right\} \tag{3}$$

where $\hat{\mathbf{h}}$ and $\mathbf{h}$ are both $KMN \times 1$ vectors[4]. The simulation results are shown in Figure (4). As

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[3] The indexing scheme used here differs slightly from [2] where negative integers are used as indexes. Here DC is set to index number 33.

[4] Note that the MSE is defined over all subcarriers over all possible channel paths.

expected, the MSE of the preamble structure is lower than the pilot structure. However, the MSE of the pilot structure indicates an error floor for high SNR values. This implies that at some high SNR value, the noise factor of the initial estimates become dominant in estimating the unknown subcarriers. Thus, increasing the number of iterations will only improve the estimates marginally. The SNR level where this effect becomes noticeable is dependent on the number of pilot symbols in the training sequence. This is evidenced by the fact that the preamble structure does not exhibit any error floor.

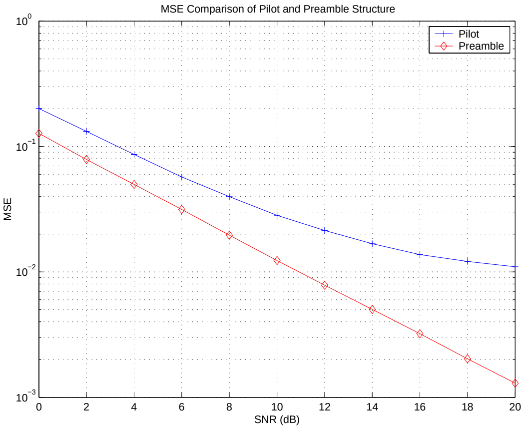

Figure 4: Mean Squared Error of Channel Estimates (15 iterations)

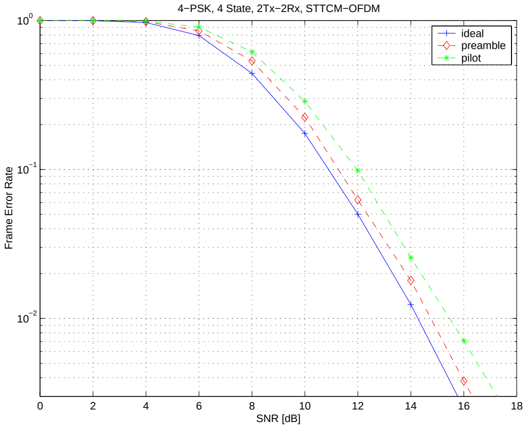

Figure 5: Simulated System Performance

The channel estimation method was also simulated for a real MIMO-OFDM system using Tarokh's 4-PSK, 4 state trellis code as described in [3]. The frame length was 130 symbols, and the channel was assumed to be static over one entire frame (quasi-static). Each frame began and ended with zeros to terminate the trellis appropriately, and no interleaving was used. The performance of the system using both structures and perfect channel knowledge can be seen in Figure (5).

Again, we can see that the preamble structure performs better than the pilot structure which incurs a loss of 1 to 1.5dB as compared to the ideal case. For a given frame error rate, we see that the difference in SNR between the ideal case and the pilot structure increases for high SNR values. This is not surprising since the MSE of the pilot structure does not decrease linearly as seen in Figure (4).

6 Conclusions

We have proposed a method of channel estimation that uses the algorithm proposed in [1] and extended it to MIMO-OFDM systems. The design of a training sequence for a 2 transmit-2 receive antenna system was shown and the simulation results indicate that the preamble structure incurs a loss of less than 1dB, while the pilot structure incurs a loss of 1.5dB as compared to perfect channel knowledge. Although we only showed the design of a 2 transmit-2 receive system, this technique can be easily extended to systems with more antennas at both the transmitter and receiver where the number of transmit antennas are limited by the ratio of total subcarriers to the cyclic prefix size. However, within this bound, this method uses only 1 OFDM symbol per antenna to estimate the entire channel repsonse.

Acknowledgments.

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