Implications of linear phase dispersion on OFDM and Fast-OFDM systems

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Abstract: *This paper investigates a new proposed modulation scheme based on Orthogonal Frequency Division Multiplexing. Fast-OFDM (FOFDM) utilises the MSK principle which makes a more efficient use of the available bandwidth. This new scheme applies only in one dimensional modulation techniques (eg. BPSK). In this paper FOFDM is compared with OFDM under the effects of linear phase distortion. The distortion effect is compensated by using pre-distortion techniques. The evaluation is performed by comparing the constellations at the receiver.*

I. INTRODUCTION

Increasing demand for wireless data communication and services, introduced new technologies capable of handling higher data rates. The 3^{rd} generation wireless mobile system, UMTS, is widely expected to cope with such demands. Research has also been carried out for future generations of wireless systems. One of the proposed modulation formats is Orthogonal Frequency Division Multiplexing (OFDM). It has been already adopted in single frequency networks, such as DVB and DAB, as well as, in indoor wireless systems, such as IEEE 802.11 and Hiperlan [1].

OFDM is a promising technique because it alleviates most of the disadvantages encountered in other modulation formats when used for wireless systems. The division of the spectrum into many orthogonal sub-carriers, with the addition of a cyclic prefix guard interval, makes the signal robust to multipath delay spread allowing the use of more complex mapping techniques, thus higher data rates.

Most recently a variation of OFDM has been proposed [2]. Fast OFDM (FOFDM) is actually an OFDM modulation with twice the bandwidth efficiency. This is achieved by applying the Minimum Shift Keying principle to OFDM [3]. In other words the spacing of the sub-carriers in FOFDM is twice as dense compared to OFDM. A disadvantage of FOFDM is that it could only be used in conjunction with one dimensional modulation schemes. A comparison between those two modulation techniques is outlined in [2].

Channel estimation, in FOFDM systems, using common OFDM equalisation techniques is not possible since the dispersive nature of multipath fading channels will cause misalignment of the received sub-carriers leading to severe ICI. One way to overcome this is by using time-domain equalisation which is not practical. This paper examines the method of pre-distortion which requires the channel response to be a-priori known. Both OFDM and FOFDM systems are tested under linear phase distortion where an IIR all-pass filter with constant group delay is used for that purpose. In order to compensate this distortion the data is pre-distorted at the transmitter. The performance of the two modulation techniques is evaluated by looking at the received constellation diagrams.

Section II of this paper outlines the OFDM and FOFDM concept. Section III describes the system model and section IV presents the results. Finally, section V summarises the results.

II. OFDM AND FOFDM CONCEPT

OFDM stands for Orthogonal Frequency Division Multiplexing. Its basic concept is the division of the available bandwidth into a number of overlapping sub-carriers, orthogonal to each other. The N low rate data streams modulate the N orthogonal sub-carriers. In order for the sub-carriers to be orthogonal, their frequency separation must be $1/T\,Hz$, where T is the duration of the signalling interval in each sub-carrier. The orthogonality of the sub-carriers will ensure that the signal can be recovered at the receiver without any inter-carrier interference by using correlation techniques.

The complex envelope representation of an OFDM signal is given by:

$$S_{tx}(t) = \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} a_{n,k} g_n(t-kT) \qquad (1)$$

where, $a_{n,k}$, are a set of complex symbols transmitted on the n^{th} sub-carrier at the k^{th} signalling interval.

N, is the number of OFDM sub-carriers

$$g_n(t) = \begin{cases} \frac{1}{\sqrt{T}} e^{j2pf_n t \frac{2pnt}{T}}, t \in [0,T] \\ 0, t \notin [0,T] \end{cases} \qquad (2)$$

A major property of the OFDM principle is that it can be implemented using Discrete Fourier Transformation [4]. In practical systems, the IFFT is utilised to generate OFDM signals thereby simplifying design and avoiding the limitations imposed by analogue signal generation. Conversely, at the receiver the FFT is used to recover the signal.

The difference between OFDM and FOFDM is in the frequency separation of the sub-carriers, as Figure 1 demonstrates. OFDM uses a $1/T$ frequency separation, whereas in the case of FOFDM a $1/2T$ frequency separation is employed.

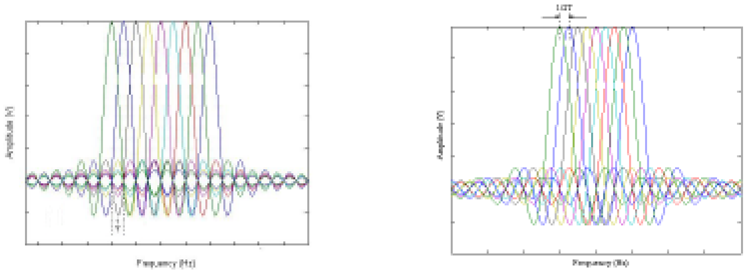


Figure 1 – OFDM (left) and FOFDM (right) sub-carrier separation (N=8)

The complex envelope representation of an FOFDM signal is expressed as:

$$S_{tx}(t) = \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} a_{n,k}\, g_n(t-kT) \qquad (3)$$

$$g_n(t) = \begin{cases} \frac{1}{\sqrt{T}} e^{j\frac{2\pi nt}{2T}}, & t \in [0,T] \\ 0, & t \notin [0,T] \end{cases} \qquad (4)$$

where, $a_{n,k}$, represents the complex data conveyed in time slot k and sub-channel n and $g_n(t-kT)$ represents the complex waveform (the complex sub-carrier) used to convey the complex data in the same time slot and sub-channel.

Figure 2, show the constellation points of an 8 carrier FOFDM signal using BPSK mapping. As the figure demonstrates the real part of the data is received with no ICI whereas the imaginary part of the data is not. Thus, the transmitted data can be recovered by taking the real part of the received signal. However when complex mapping is applied, for example QPSK, there is ICI both in the real part and the imaginary part of the data [2]. Thus, the signal cannot be recovered.

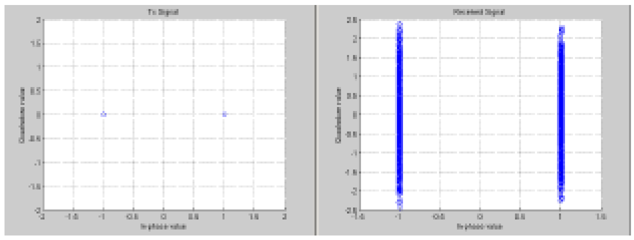


Figure 2 – Constellation diagrams of transmitted (left) and received (right) BPSK data using FOFDM

III. SYSTEM MODEL

a. OFDM system

The OFDM system block diagram is shown in Figure 3.

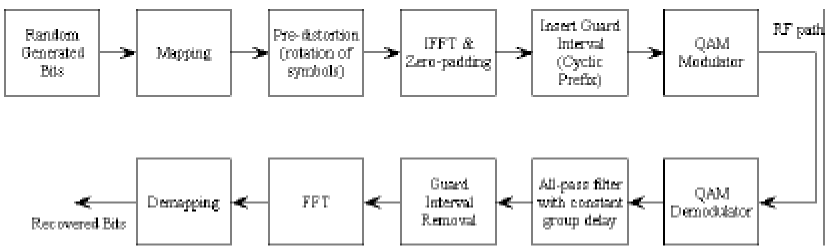


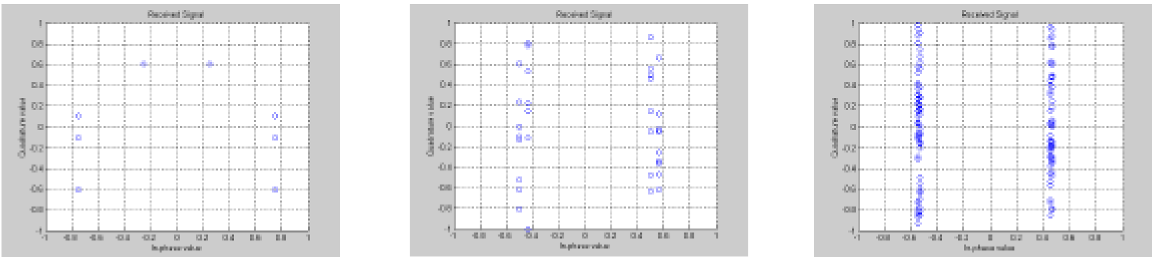
Figure 3 – OFDM system block diagram

The random binary sequence is fed to a mapping block that performs BPSK mapping. The mapped data is zero-padded and then inserted into a 64-point IFFT. Zero-padding is an operation in which more zeros are appended in the original sequence which provides an oversampled version of the signal. A guard interval is then added to the OFDM signal using the cyclic-prefix operation used to protect from intersymbol and intercarrier interference. In such a scheme, instead of having zeros in the interval between two consecutive OFDM symbols, a copy of the last samples of the next OFDM symbol is used as the guard interval.

b. FOFDM system

In [2], the FOFDM system is simulated in the continuous-time, that is, each symbol is multiplied by the corresponding orthogonal sub-carrier. In other words, no FFT is used. It is interesting to note that in [5], a multicarrier CDMA system with twice the bandwidth efficiency is proposed. The system proposed, uses the partial symmetry of the samples out of the IFFT. In every OFDM symbol, the first $N/2+1$ samples are transmitted with the rest discarded. At the receiver the remaining samples are reconstructed using the partial symmetry of the Fourier transform as explained in [5].

The FOFDM system constructed here uses this partial symmetry of the Fourier samples with the only difference that at the receiver instead of reconstructing the remaining samples, zeros are inserted. Figure 4 demonstrates the effect of *"zero-insertion"* on the received constellation when transmitting BPSK data.



(a) (b) (c)

Figure 4 – (a) 8 samples FFT (b) 32 samples FFT (c) 128 samples FFT

The above figure demonstrates that as the number of FFT samples, or sub-carriers, is increased the system resembles that of the continuous-time FOFDM, with the constellation points gathering around ± 0.5 instead of ± 1. This is easily overcome by multiplying the received constellation by 2. The FOFDM system block diagram is identical to that of the OFDM system, shown in Figure 3, with the difference that only the first $N/2+1$ samples of each OFDM symbol are used.

c. Design of all-pass filter with constant group delay

An all-pass filter has the following transfer function:

$$H(z) = \frac{a_{N+1}Z^{N} + a_N Z^{N-1} + \ldots + a_1}{a_1 Z^{N} + a_2 Z^{N-1} + \ldots + a_{N+1}} = \frac{a_{N+1} + a_N Z^{-1} + \ldots + a_1 Z^{-N}}{a_1 Z^{N} + a_2 Z^{N-1} + \ldots + a_{N+1}} \qquad (5)$$

The magnitude response of such filter is unity and its phase response is linear as depicted in Figure 5.

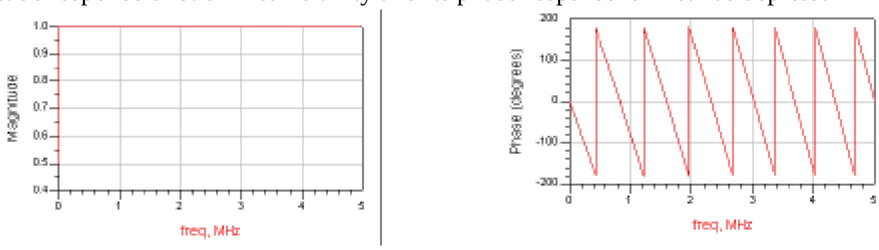
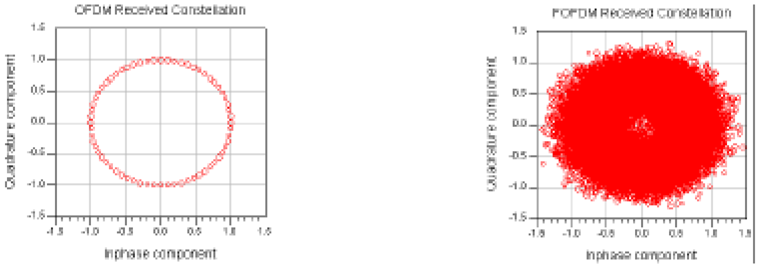


Figure 5 – Magnitude and Phase response of all-pass filter

The effect of this filter is to give a corresponding phase shift to each sub-carrier at the receiver.

IV. EFFECT OF LINEAR PHASE DISTORTION TO OFDM AND FOFDM CONSTELLATIONS

Both systems are first tested without pre-distorting the BPSK data. The effect from the linear phase distortion of the all-pass filter is shown in Figure 6.



(a) (b)

Figure 6 – Received BPSK constellation in (a) OFDM and (b) FOFDM systems

Figure 7, displays the received constellations when the data is pre-rotated at the transmitter according to the phase shift that the IIR filter infers to the symbols at the receiver.

(a) (b)

Figure 7 – Received pre-distorted BPSK constellation in (a) OFDM and (b) FOFDM systems

V. CONCLUSIONS

Figure 6a, demonstrates that in the OFDM case, under linear phase distortion, the received BPSK constellation points experience only a phase rotation that can be obviously compensated or corrected for at the receiver. In the FOFDM case, the linear phase shift inferred from the all-pass filter introduces inter-carrier interference to the received FOFDM signal. Due to ICI the received FOFDM constellation, shown in Figure 6b, is cluttered (phase rotation cannot be predicted) and it is not possible to correct for this effect using standard OFDM equalisation techniques.

When pre-distortion is used, both OFDM and FOFDM systems can be recovered adequately. Figure 7, clearly depicts that both the constellation points of the OFDM and FOFDM systems respectively, are distinguishable. One point worth to mention is that the received FOFDM constellation diagram using pre-distortion techniques, shown in Figure 7b, is obscure. In situations where the linear phase shift is even worse the constellation points of such signal will blend and demodulation will not be possible. This disadvantage emerges from the *"zero-insertion"* effect, shown in Figure 5. The use of more FFT points will alleviate this problem.

In the case of the FOFDM signal being affected by a multipath channel, where the signal is not only delayed, but also attenuated, it is expected that the pre-distortion technique will work. Nevertheless, in such cases, the a-priori knowledge of the channel impulse response is difficult.

This paper investigated the implications of linear phase distortion in OFDM and FOFDM systems. It has been observed that channel estimation, which is important for wireless systems, using standard OFDM techniques is not efficient in FOFDM systems. The disadvantage of FOFDM is that at the receiver the sub-carriers must be aligned, otherwise, intercarrier interference will occur. One method to overcome this problem is by pre-distorting the transmitting data which will make the signal to be orthogonal (and thus have aligned sub-carriers) at the receiver. The disadvantage of this technique is that the channel response must be known a-priori.

Acknowledgments

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