A Multiplier-less CMA Adaptive Equaliser

S.M. El-Eraki, J.C. Batchelor, P. Lee and R.J. Langley

University of Kent at Canterbury

Abstract: This paper introduces a new type of adaptive filter for use in reducing multipath in analogue FM reception. The filter has potential benefits in terms of convergence time as its multiplication is carried out in the logarithmic domain. The Constant Modulus Algorithm (CMA) was used as the convergence test for the system. The filters were implemented in software.

1. Introduction

Analogue FM transmissions to vehicles suffer from multipath effects. These are more difficult to eliminate than in digital systems due to the receiver is incapable of determining uncorrupted signal content. In FM radio communication, the transmitted signal can be represented in a complex form as

$$S(t) = A_c \exp\{j(\omega_c t + P(t))\} \quad (1)$$

where A_c is the carrier amplitude; ω_c is the carrier frequency; and $P(t)$ is the modulating signal.

A_c is a constant value, the exponential form has a constant modulus so clearly the signal should have a constant envolope. In an ideal radio channel, the received signal would consist of a single direct path signal and would be a perfect representation of the transmitted signal. However, it is well known that in a real channel the transmitted signal is reflected and scattered by obstacles such as buildings, trees, mountains, etc. The channel can also introduce Doppler shifting if the transmitter or receiver is moving. The resulting signal at the receiver is the summation of the direct and the multipath signals.

The extent of the Doppler effect depends on the relative motion between the transmitter and receiver. The Doppler shift can be written as:

$$\Delta f = \pm \frac{v}{\lambda} \quad (2)$$

where λ is the transmitted signal wavelength; v is the moving receiver velocity; and

$\pm$ accounts for the receiver moving towards/away from the transmitter.

With a signal frequency of 98.5MHz and v = 100km/h, the maximum Doppler shift Δf is 9 Hz.

Referring to (1), a received signal $x(t)$ consisting of a direct and a single indirect ray can be represented mathematically as:

$$x(t) = A_c \exp\{j(\omega_c t + P(t))\} + A_c a \times \exp\{j(\omega_c(t-\tau) + P(t-\tau))\} \quad (3)$$

where a is multipath attenuation and τ is multipath delay.

The time delay τ represents extra phase variations in the combined signal and the FM receiver demodulates them as noise components such as high frequency spikes which cause clicks in the audio signal.

2. The Constant modulus algorithm adaptive equaliser

Spurious phase variations in the composite received signal cause an amplitude variation in the carrier envelope. Although this amplitude modulation may not directly lead to audio distortion, it is symptomatic of the presence of multipath. An adaptive equaliser can be implemented to restore the carrier modulus to a constant value by using the CMA to control the coefficient values of an adaptive digital filter. There are various search techniques suitable for converging on the correct coefficient values.

An N-Tap transversal was assumed as the basis for this adaptive filter. The value of N is determined by practical considerations, [1]. An FIR filter was chosen because of its stability. The use of the transversal structure allows relatively straight forward construction of the filter, Fig.1.

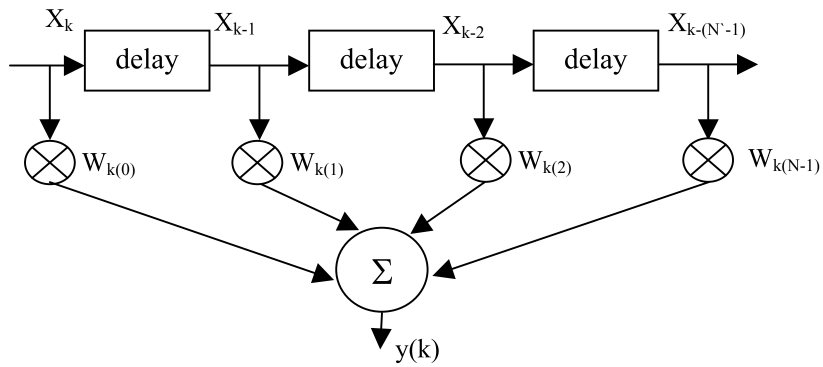

Figure 1. Transversal FIR Filter

As the input, coefficients and output of the filter are all assumed to be complex-valued, then the natural choice for the property measurement is the modulus, or instantaneous amplitude. If $y(k)$ is the complex valued filter output, then $|y(k)|$ denotes the amplitude. The convergence error $p(k)$ can be defined as follows:

$$p(k) = |y_k| - A \qquad (4)$$

where the A is the amplitude in the absence of signal degredations.

The error $p(k)$ should be zero when the envelope has the proper value, and non-zero otherwise. The error carries sign information to indicate which direction the envelope is in error. The adaptive algorithm is defined by specifying a performance/cost/fitness function based on the error $p(k)$ and then developing a procedure that adjusts the filter impulse response so as to minimise or maximise that performance function.

$$Y_k = \sum_{i=0}^{i=N-1} w_k(i)\, x_{k-I} \qquad (5)$$

The gradient search algorithm was selected to simplify the filter design. The filter coefficient update equation is given by:

$$W_{K+1} = w_K - \mu\, e_K\, x_K \qquad (6)$$

where X_K is the filter input at sample k, e_k is the error term at sample k $= p_k \,.\, y_k$ and μ is the step size for updating the weights value.

3. The multiplier-less adaptive filter

Conventional transversal filters require a multiplying element for each tap. Multiplication is a time consuming process and it was decided to investigate a filter that could carry out the multiplication in the logarithmic domain to save time. A multiplier-less FIR filter based on logarithmic algebra was implemented in code to enable a decrease in convergence time, Fig.2.

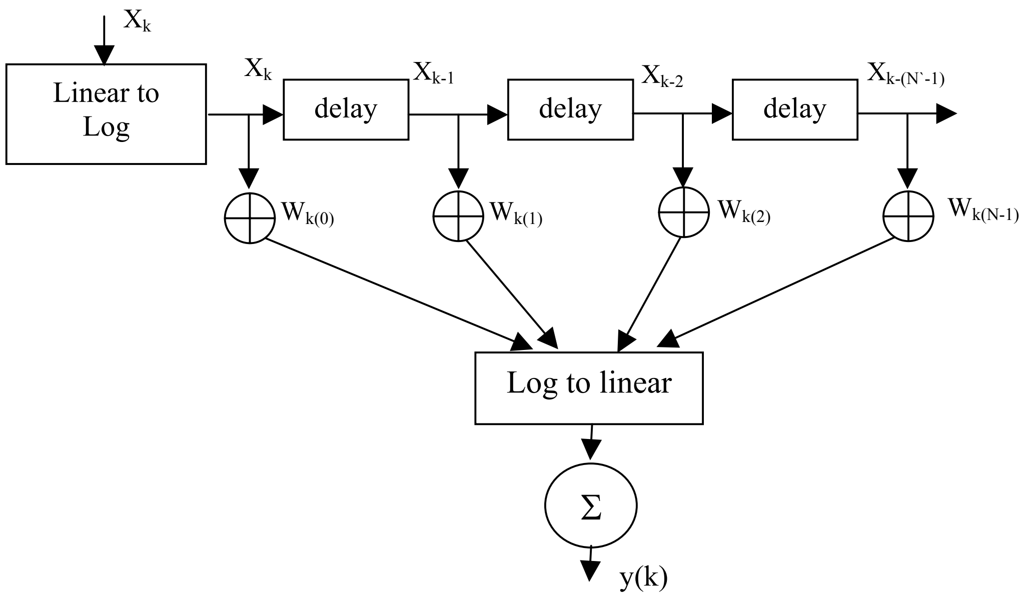

Figure 2. Multiplier-less Transversal FIR Filter

The filter architecture shown in Fig.2 converges more rapidly than an equivalent length conventional filter because no multiplication operations are necessary and the conversion from linear to logarithmic domains is approximate. The log/linear conversion technique is outlined below.

Any binary number N can be written as

$$N = 2^K (1+x) \qquad (7)$$

where k is the binary power of N's most significant bit. The part of the binary number to the right of the k^{th} order bit is shifted k places to the right to give x. For example, the decimal number 11 would in binary be $1011 = 2^3 (1+0.011)$.

Now, from (7) it can be shown that, $\log_2 N \approx k + \log_2(1+x)$, for $0 \leq x < 1$.

Therefore, the linear approximation of (7) is, [2]:

$$\log_2 N \approx k + x \qquad (8)$$

For example, the approximate binary value of $\log_2 11$ is 11 + 0.011 = 11.011. Converting back to linear is again done with shift and add functions. The conversion processes discussed in this paper are fast and convenient as they do not require the use of a look up table. It will be seen that their approximate nature is not significant providing the filter is appropriately designed.

4. Results and Discussion

A model of 5 ray multipath Rayleigh channel with a Doppler shift of 9 Hz was created using Simulink. The 5 multipath rays were all indirect with maximum delay values of (2e-6 30e-6 3e-6

23e-6 10e-6) and attenuations of 3 dB. An audio file was used to create a modulated signal which was exposed to the multipath and fed to a Direct Digital Down-converter. The down converted baseband I and Q signals were input to an adaptive filter. The filtered I and Q outputs were then fed to an FM baseband demodulator and the resulting audio signals were played back and plotted. Fig.3 shows the demodulated audio containing a multipath induced spike at about 0.12s. This resulted in a characteristic click associated with multipath in mobile FM channels. Figure 4 shows the adaptively equalized audio output using a conventional 100 tap FIR filter, while Fig.5 shows the output of the non-multiplying filter using logarithmic algebra. The difference between the outputs of the two filters is shown in Fig.6. Inspection of Fig. 6 shows that after an initial short convergence period, the performance of the two filters is almost exactly identical and no difference is noticeable when the waveforms a played via a loudspeaker. Both filters converged in less than 3.7ms which is sufficient for eliminating multipath at FM frequencies.

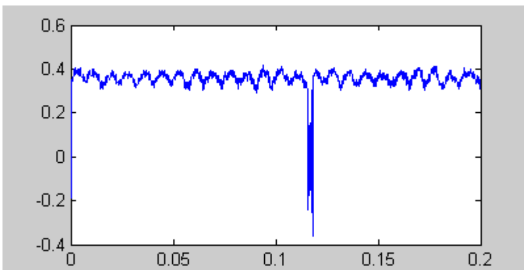

Figure 3. Audio with multipath (no equalization)

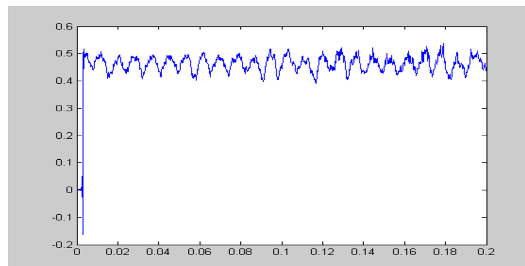
Figure 4. Audio filtered by conventional filter

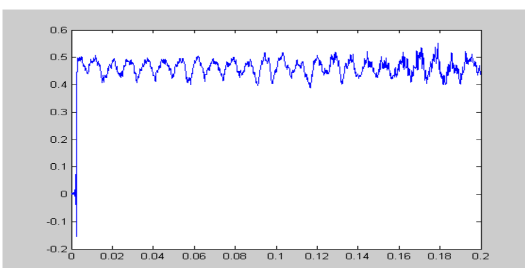
Figure 5. Audio filtered by logarithmic filter

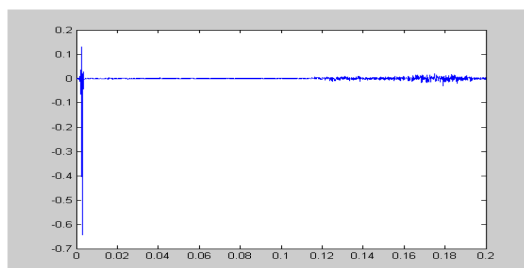
Figure 6. Difference in filter outputs.

The Logarithmic filter requires at least 60 coefficients to overcome the approximation errors in the conversion process. This is not problematic as a conventional filter would also use these number of taps.

5. Conclusion

Removal of multipath from analogue FM is problematic as it is a blind process. This paper used the Constant Modulus Algorithm and introduced and simulated a way of simplifying adaptive equalizer filter structure by replacing the multipliers with adders, therefore increasing convergence speed and potential for future deployment in vehicles.

References

[1] John R Treichler, C. R Johnson, and JR. Michael G. Larimore, Theroy and design of adaptive filters, A wiley –interscience publication, 1987.

[2] M. Combet,H Ban Zonneveld, and L. Verbeek, "Computation of the base two logarithm of binary numbers," IEEE trans. Electron. Coput., vol EV-14, pp 863-867 DEC 1965.