# Modelling and Analysis of WAN Traffic using Chaotic Intermittency Map Girum Aklilu, J.M. Pitts, R.J. Mondragon Department of Electronic Engineering, Queen Mary, University of London, U.K.

#### Abstract

Measurements of WAN traffic have prevailed the existence of multi-scaling features. Our study looks into measured TCP traffic from the wide area network in determining a single Hurst parameter does not describe the behaviour of the traffic at all time scales. But rather two distinct LRD behaviours are apparent at low time scales and large time scales. Based on the above analysis, we propose a new model that captures the variation of LRD behaviours at different time scales. Our model is a multiple intermittency chaotic map that expresses traffic generation based on non-linear dynamics. We show how this approach can produce traffic in a parsimonious, predictable and easily parameterisable manner. In validation, traffic traces from Lawrence Berkeley Laboratory are analysed to determine the multi-scaling features. We then make a statistical comparison between the synthetic traffic from our model and the real traffic traces.

#### 1 Introduction

Network dimensioning and performance evaluation relies heavily on the underlying characteristics of the traffic. Hence, it is essential to understand the range of behaviours that are exhibited such as heavy-tailed distributions, long-range dependence (LRD) and self-similarity. Analyses of high-quality traffic measurements have revealed the prevalence of long-range dependent features in various packet switching communication networks. Included are local area networks (LANs) [1], wide area networks (WANs) [2], variable-bit rate (VBR) video traffic [3], world wide web (WWW) traffic [4], and very-high-speed backbone network service (vBNS) traffic [5]. Collectively, these measurement works constituted strong evidence that burstiness was not an isolated, spurious phenomenon but rather a persistent feature existing across a range of network environments. The degree of correlation of the traffic and hence the LRD can be quantified using statistical scaling analysis with the Hurst parameter, H lying between  $\frac{1}{2}$  and 1 [6].

Studies based on measurements of Internet traffic [2,7,8] have implied that a single scaling does not describe the behaviour of the traffic at all time scales. Our study looks into measured TCP traffic from the wide area network [9]. These are measurements of wide area Internet traffic that connects Berkeley Labs with the rest of the world with time stamps accuracy of microseconds. We have analysed these traffic traces at low time unit in terms of milliseconds and argue that, a single LRD behaviour does not describe the measured Internet traffic over all time scales of interest but can be divided into at least two distinct ranges with different H values. Using the rescaled adjusted range, R/S statistical analysis [10], we have been able to show the two LRD behaviours at different timescales with a clear cross over point in between.

Furthermore we introduce a novel chaotic multi-map that has the capability of showing two different scaling for different time scales in line with the IP traffic shown from the traces. Chaotic map models have been applied as models of *ON/OFF* packet traffic [11,12]. They are accurate, predictable and computationally efficient. Additionally, these models have an intuitive relationship to the underlying physical *ON/OFF* process. But significant limitations were observed in their analytical tractability and poor aggregated modelling. Considerable achievement has been achieved in terms of overcoming the limits and extended the use of chaotic maps such that the mean traffic load, Hurst parameter and variance amplitude can be approximated analytically from the map parameters [13, 14].

# 2 Statistical analysis of traffic traces

### 2.1 Traffic traces studied

Since the collection and analysis of the original Bellcore traces [1], the area of traffic measurement has been tremendously active in determining the fact that network traffic exhibits self-similar scaling properties over a wide range of time scales. The measurement or traffic trace driven approach to networking research provides a balance to the more theoretical aspects of networking research. The traces that are looked at in this study are TCP traffic traces from the Wide Area Network (WAN) over which all traffic into or out of the Lawrence Berkeley Laboratory, located in Berkeley, California. The tracing was done on the Ethernet DMZ network using tcpdump on a Sun SparcStation using the BPF kernel packet filter. The measurements captured have packet arrival timestamps in microsecond precision. Even though, we have analysed three sets of traffic traces taken at different times; in this paper we have chosen one of the traces for detailed analysis. This trace (lbl-4) contains an hour worth of traffic data in which around 1.3 million packets were captured. These traces have been originally recorded and analysed by Paxson and Floyd [2].

## 2.2 Mathematical description of statistical analysis

The notion of self-similarity is not merely an intuitive description, but a precise concept captured by the following mathematical definition [10]. A self-similar time series has the property that when aggregated the new series has the same autocorrelation function as the original. That is, given a stationary time series  $X = (X_t: t = 0, 1, 2, ...)$  with constant mean  $\mathbf{m} = E[X_t](1)$ , finite variance  $\mathbf{s}^2 = E[(X_t - \mathbf{m})^2](2)$ , and an autocorrelation function

 $r(k) = E[(X_t - m)(X_{t+k} - m)](3);$  we define the *m*-aggregated series  $X^{(m)} = (X_k^{(m)} : k = 1, 2, 3, ...)$  by summing the original series X over non-overlapping blocks of size *m*. That is,

$$X_{k}^{(m)} = \frac{1}{m} \sum_{i=km-(m-1)}^{km} X_{i} \quad (4)$$

Then if X is exactly self-similar, it has the same autocorrelation function as the series  $X^{(m)}$  for all m, i.e.,  $r^{(m)}(k) = r(k)$  for  $k \stackrel{\circ}{=} 0$ . On the other hand, if  $r^{(m)}(k) \otimes r(k)$  for all k large enough, then X is called asymptotically (second-order) self-similar. Note that this means that the series is distributionally self-similar: the distribution of the aggregated series is the same (except for changes in scale) as that of the original. An interesting feature of the preceding definition is that the autocorrelation of the aggregated self-similar process does not go to 0 as  $m \otimes \Psi$ . The Hurst parameter expresses the speed of decay of the series' autocorrelation function such that  $r(k) \stackrel{\sim}{=} k^{(1+H)}$  (5). For a self-similar series,  $\frac{1}{2} < H < 1$  whereby as H approaches 1, the degree of self-similarity increases.

# 3 Multiple Intermittency map

# 3.1 Structure of the multi-map

Single Intermittency maps have already been used to model LRD behaviour in an ON/OFF traffic source [11,12]. The multiple intermittency map has a general structure that is similar to a single intermittency map but has multiple portions in the ON section. As two different scaling are exhibited in the Internet traffic traces that we looked at, we have devised the structure of the map so that it has two portions in the ON section as shown in Figure 3.1.



Figure 3.1 ON/OFF source model representation and multiple intermittency map structure

The multiple intermittency map has the following equations:

$$x_{n+1} = F(x_n) = \begin{cases} F_1(x_n) = e_1 + x_n + \frac{1 - e_1 - d_2}{d_2^{m_1}} x_n^{m_1} & 0 < x_n < d_1 \\ F_2(x_n) = e_2 + x_n + \frac{1 - e_2 - d_2}{d_2^{m_2}} x_n^{m_2} & d_1 \le x_n < d_2 \\ F_3(x_n) = x_n - d_2 \frac{1 - x_n}{1 - d_2} & d_2 \le x_n < 1 \end{cases}$$
(6)

Packets are generated as long as  $x_n$  is less than the discriminant value  $d_2$ . This is described by an indicator variable  $y_n$ 

$$y_n = \begin{cases} 1, & 0 < x_n < d_1 \\ 1, & d_1 \, \mathbf{\pounds} \, x_n < d_2 \\ 0, & d_2 \, \mathbf{\pounds} \, x_n < 1 \end{cases}$$
(7)

# **3.2** Parameterisation of the multi-map

## 3.2.1 Packet size distribution

On analysis of the trace packet sizes, we have determined a tri-modal distribution whereby three packet sizes and relative distribution sizes give approximation to the overall measured packet size distribution. Furthermore, we have looked at the "simple Imix" packet size mixture proposed after extensive studies of Internet measurement at the National Laboratory for Applied Network Research (NLANR) [15]. In agreement, the results of this study have shown that accurate approximation is achieved when using their proposed packet mixtures. Hence, the multi-map model produces one of the three specific packet sizes when it in the *ON* section.

# 3.2.2 Multi scaling analysis

We have chosen a finest time unit of 1milliseconds to perform the multi-scaling analysis. Hence, the number of packets/bytes over the Ethernet every 1ms is analysed and using R/S statistics (figure 4.1), the two scaling slopes that correspond to the Hurst parameters are calculated. Finding the Hurst parameters gives us a direct relationship with parameters  $m_1$  and  $m_2$ , H=(3m-4)/(2m-2) (8) [14]. We also find the cross over point which corresponds to a time unit (t) relative to the time unit of analysis. The cross over point corresponds to the e and  $d_1$  parameters that are effective in determining the time scales over which LRD is present. Further information can be referred to the work we have done in [16].

# 3.2.3 Time scale and load analysis

The load and variance of the traces in respect to the time unit of analysis is worked out. Parameter  $d_2$  and  $m_2$  determine the proportion of time increments that have a packet and hence the load; we follow a look up table whereby the two parameters versus the load can be referred to in comparison to the specific load of the trace. We further determine the time unit that corresponds to the iteration time of the multi-map model using the formula  $2t_2 = P_1/C$ ; where  $P_1$  is the maximum packet size and C is the maximum link rate.

The following parameters have been calculated:  $m_1 = 1.8$ ,  $m_2 = 1.6$ , e = 1\*10-6,  $d_1 = 1.9*10-5$ ,  $d_2 = 0.1$ ,  $?t_2 = 500\mu s$ . The packet size *s* are: s = 45 bytes, m = 520 bytes, l = 1451 bytes, Ps = 0.6515, Pm = 0.333, Pl = 0.0155

## 4. Comparison of real versus synthetic traces

### 4.1 R/S scaling analysis

The figures below show the R/S statistics achieved using the multi-map as a model compared to the analysis from the real TCP traffic traces from the wide area networks. These graphs correspond to synthetic traffic that is achieved by parameterising the multi-map model to have the same characteristics as real traces. Their multi-scaling feature similarity is clearly shown in the figure.



Figure 4.1 R/S analysis for lbl-4 Internet traffic trace and its equivalent trace from multi-map model

## 4.2 Other statistical comparisons

The multi-scaling feature seen in the traces with the two H values is not enough on its own merit to signify the traffic. Other statistical analyses need to be considered such as the actual variability of the traces as well as the mean load. We have measured the variance, mean load of the traces and compared them with the model's synthetic trace. Table 1 below shows the statistical comparison for the trace and its corresponding equivalent synthetic traces achieved from the multi-map.

	LBLTCP4		Multi-map		
	Packet count	Byte count	Packet count	Byte count	
Variance	0.527	167.8	0.5	137	
Mean load	0.2397	330	0.232	293	
H <sub>1</sub>	0.87	0.90	0.82	0.82	
H <sub>2</sub>	0.66	0.65	0.65	0.61	

Table 1:	The	lblTCP4	and its	equ	ivalent	synthetic	trace	statistical	analy	sis

## 5. Conclusions

We have introduced the multiple intermittency map, as a model capable of simulating multi-scaling traffic. A statistical comparison was made between the real traffic trace and the synthetic trace with similar results in terms of the multi-scaling features as well as mean traffic load and variance analysis. This model provides a parsimonious, fast and easily parameterisable model that depicts today's real internet traffic characteristics.

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