Direct Semi-Blind Equalization for Space-Time Block Codes

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Abstract: The key difference between space time block codes (STBCs) and a general multiple-input multiple-output system is that the transmitted signals of STBC systems are not mutually independent. In this paper we develop an algorithm that exploits the implicit structure of STBCs to achieve direct equalization without knowledge of the channel impulse responses. The performance of the algorithm is demonstrated through simulation results and compared with an existing scheme.

1. INTRODUCTION

Recently there has been much interest in semi-blind equalization and detection approaches for space time block codes (STBCs). One reason is that the performance of semi-blind techniques is superior to that of training sequence or blind techniques separately, as they incorporate information of both known symbols and the unknown sequence. Another reason is that for certain STBCs (such as the Alamouti code [2]) it is impossible to achieve blind equalization due to implicit ambiguity [3].

Most current approaches for equalization of STBCs require the estimation of the channel as the first step to achieve equalization (e.g., [1]), thus their computation is inevitably inefficient. A new framework for STBCs, called generalized space-time block codes (GSTBC), was recently presented [3], in which a direct estimation of blind and semi-blind zero-forcing equalizers was developed. However, the extension of this technique to frequency selective channels is limited to certain types of STBC codes. In this paper we exploit the structure of the GSTBC framework to develop a more general semi-blind equalization algorithm for STBCs. The proposed algorithm is designed for frequency selective channels by nature and is computationally efficient.

2. DATA MODEL

Consider a noiseless system with $M > 1$ transmitters and $K \geq 1$ receiver (although we initially neglect noise for clarity in developing the algorithm, we later assess its performance in environments with additive noise). Let the block length be N, the channel length be $L_h + 1$, the length of equalizers be $L_g + 1$, and the length of the training symbols be L_{ts}. The signal received at the jth receiver can be written

$$y_j(n) = \sum_{m=1}^{M} \sum_{k=0}^{L_h} h_{m,j}(k) s_m(n-k), \quad n = 0, \dots, N-1, \tag{1}$$

where $s_m(n)$, $n = 0, \dots, N-1$, is the nth symbol transmitted from the mth transmitter and $h_{m,j}(k)$ is the kth tap of the channel from the mth transmitter to jth receiver. Note that $s_m(n)$, $n < 0$, are training symbols or appropriate symbols from the previous block.

Stacking the received data over $N_t = N + L_{ts} + L_g$ successive observations gives

$$\mathbf{y}_j = \mathbf{H}_{1,j} \begin{bmatrix} \mathbf{s}_1^T & \mathbf{t}_1^T & \tilde{\mathbf{s}}_1^T \end{bmatrix}^T + \cdots + \mathbf{H}_{M,j} \begin{bmatrix} \mathbf{s}_M^T & \mathbf{t}_M^T & \tilde{\mathbf{s}}_M^T \end{bmatrix}^T \tag{2}$$

where

$$\mathbf{H}_{m,j} = \begin{bmatrix} h_{m,j}(0) & \cdots & h_{m,j}(L_h) & 0 & 0 \\ 0 & \ddots & & \ddots & 0 \\ 0 & 0 & h_{m,j}(0) & \cdots & h_{m,j}(L_h) \end{bmatrix}$$

is the $N_t \times (N_t + L_h)$ channel filtering matrix from the mth transmitter to the jth receiver, $\mathbf{y}_j = \begin{bmatrix} y_j(n) & \cdots & y_j(n - N_t + 1) \end{bmatrix}^T$, $\mathbf{s}_m$ is a column vector of N symbols in the current block, $\mathbf{t}_m$ is a column vector of L_{ts} training symbols and $\tilde{\mathbf{s}}_m$ is a column vector of $L_g + L_h$ symbols from the previous block, each transmitted from the mth antenna.

Now, stacking again over the K receiver branches we finally have:

$$\mathbf{y} = \mathbf{H}_1 \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{t}_1 \\ \tilde{\mathbf{s}}_1 \end{bmatrix} + \cdots + \mathbf{H}_M \begin{bmatrix} \mathbf{s}_M \\ \mathbf{t}_M \\ \tilde{\mathbf{s}}_M \end{bmatrix}, \tag{3}$$

where $\mathbf{y} = \begin{bmatrix} \mathbf{y}_1^T & \cdots & \mathbf{y}_K^T \end{bmatrix}^T$ and $\mathbf{H}_m = \begin{bmatrix} \mathbf{H}_{m,1}^T & \cdots & \mathbf{H}_{m,K}^T \end{bmatrix}^T$.

Before we continue developing our algorithm, we first give a brief description of GSTBC which will be used in the following sections.

3. GENERALIZED SPACE-TIME BLOCK CODES

A new code framework for space-time block codes, termed Generalized Space-Time Block Codes (GSTBCs), was given in [3]. It was shown that for most STBCs, the data transmitted from the mth antenna can be expressed as either

$$\mathbf{s}_m = \mathbf{U}_m \mathbf{s}, \tag{4}$$

or (for certain codes) $\tilde{\mathbf{s}}_m = \mathcal{U}_m \tilde{\mathbf{s}}$, where $\mathbf{U}_m$ (or $\mathcal{U}_m$) is a square precoding matrix for the mth transmitter, $\mathbf{s}$ is the block of information bearing symbols, and $\tilde{\mathbf{s}} = \begin{bmatrix}\text{real}(\mathbf{s})^T & \text{imag}(\mathbf{s})^T\end{bmatrix}^T$ (and $\tilde{\mathbf{s}}_m$ has the same form as $\tilde{\mathbf{s}}$). Two specific examples from [3] that we will consider in this paper are:

(i) The Alamouti coder for the $M = 2$ STBC in [2] is

$$\mathcal{U}_1 = \begin{bmatrix}\mathbf{I}_{N/2} \otimes (\mathbf{J}\tilde{\mathbf{I}}) & 0 \\ 0 & \mathbf{I}_N\end{bmatrix}, \quad \mathcal{U}_2 = \begin{bmatrix}\mathbf{I}_{N/2} \otimes (\tilde{\mathbf{I}}) & 0 \\ 0 & \mathbf{I}_{N/2} \otimes (\mathbf{J})\end{bmatrix}, \tag{5}$$

where $\tilde{\mathbf{I}} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\mathbf{J} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $\mathbf{I}_n$ is a $n \times n$ identity matrix, and $\otimes$ denotes the Kronecker product.

(ii) The diagonal coding scheme in [4] uses a $\mathbf{U}_m$ which is diagonal with nonzero entries drawn at random from the unit circle.

In the following sections, for notational convenience we will only use the structure (4). It is straightforward, however, to modify the appropriate equations to suit the alternate formulation required for the Alamouti code.

4. SEMI-BLIND EQUALIZATION

Although STBCs are usually viewed as a special case of MIMO, there is implicit structure in STBCs which mean that these multiple inputs are not mutually independent. It is this redundant information in STBCs that we will exploit to achieve equalization by using only a small number of training symbols.

Let $\mathbf{g}_{m,j}$ denote the zero-forcing equalizer for the channel $\begin{bmatrix}h_{m,j}(0) & \cdots & h_{m,j}(L)\end{bmatrix}$, and $\mathbf{G}_{m,j}$ be the $(N + L_{ts}) \times (N + L_{ts} + L_g)$ Sylvester matrix formed by $\mathbf{g}_{m,j}$. The zero-forcing equalizer should satisfy

$$\mathbf{G}_m \mathbf{y} = \begin{bmatrix}\mathbf{s}_m^T & \mathbf{t}_m^T\end{bmatrix}^T \tag{6}$$

where $\mathbf{G}_m = \begin{bmatrix}\mathbf{G}_{m,1} & \cdots & \mathbf{G}_{m,K}\end{bmatrix}$.

As can be seen from (6), there are two parts of information that we can use to estimate the equalizers. One is the redundant information among the multiple inputs ($\mathbf{s}_m$) and the other is the training symbols that we know at the receivers ($\mathbf{t}_m$). In the following two subsections, we will derive two cost functions from these two parts of information.

4.1 Blind cost function

To extract the information symbols from (6), we can write

$$\mathbf{B}\mathbf{G}_m \mathbf{y} = \mathbf{s}_m = \mathbf{U}_m \mathbf{s} \tag{7}$$

where $\mathbf{B} = \begin{bmatrix}\mathbf{I}_N & \mathbf{0}_{N \times L_{ts}}\end{bmatrix}$, $\mathbf{0}_{m \times n}$ is a $m \times n$ zero matrix and $\mathbf{I}_n$ is a $n \times n$ identity matrix.

Then for any two different equalizers, we have

$$\mathbf{U}_m^{-1} \mathbf{B}\mathbf{G}_m \mathbf{y} = \mathbf{U}_k^{-1} \mathbf{B}\mathbf{G}_k \mathbf{y}, \quad m \neq k. \tag{8}$$

Since $\mathbf{G}_{m,j}\mathbf{y}_j$ represents a convolution operation, we can rewrite this operation as:

$$\mathbf{G}_{m,j}\mathbf{y}_j = \mathbf{Y}_j \mathbf{g}_{m,j} \tag{9}$$

where $\mathbf{Y}_j$ is an appropriate matrix of dimension $(N + L_{ts}) \times (L_g + 1)$ formed from the elements of $\mathbf{y}_j$. Hence, (8) can be written as

$$\mathbf{U}_m^{-1} \mathbf{B}\mathbf{Y}\mathbf{g}_m - \mathbf{U}_k^{-1} \mathbf{B}\mathbf{Y}\mathbf{g}_k = \mathbf{0}_{N \times 1}, \quad m \neq k, \tag{10}$$

where $\mathbf{Y} = \begin{bmatrix}\mathbf{Y}_1 & \cdots & \mathbf{Y}_K\end{bmatrix}$, and $\mathbf{g}_m = \begin{bmatrix}\mathbf{g}_{m,1}^T & \cdots & \mathbf{g}_{m,K}^T\end{bmatrix}^T$.

Our first proposed cost function (which is based on the structure of STBCs) is formulated by minimizing the squared norm of the left-hand term of (10) for all equalizer pairs. This can be written as

$$J_{\text{blind}} = \mathbf{g}^H \mathbf{C}^H \mathbf{C} \mathbf{g} \tag{11}$$

where

$$\mathbf{C} = \begin{bmatrix} \mathbf{U}_1^{-1}\mathbf{BY} & -\mathbf{U}_2^{-1}\mathbf{BY} & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{U}_1^{-1}\mathbf{BY} & 0 & 0 & \cdots & 0 & -\mathbf{U}_M^{-1}\mathbf{BY} \\ 0 & \mathbf{U}_2^{-1}\mathbf{BY} & -\mathbf{U}_3^{-1}\mathbf{BY} & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \mathbf{U}_2^{-1}\mathbf{BY} & 0 & \cdots & \cdots & -\mathbf{U}_M^{-1}\mathbf{BY} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \mathbf{U}_{M-1}^{-1}\mathbf{BY} & -\mathbf{U}_M^{-1}\mathbf{BY} \end{bmatrix}$$

and $\mathbf{g} = \begin{bmatrix} \mathbf{g}_1^H & \cdots & \mathbf{g}_M^H \end{bmatrix}^H$.

It would appear that minimizing (11) with respect to $\mathbf{g}$ (with an additional constraint such as $\|\mathbf{g}\| = 1$) would be a sufficient criterion to estimate the equalizers. However, we note that there is a family of $\mathbf{g}$ that satisfy (8), yet do not necessarily satisfy (7). This ambiguity can be resolved by introducing the semi-blind term described below.

4.2 Semi-blind cost function

Similarly to (7), we can extract the training information from (6) as

$$\tilde{\mathbf{B}}\mathbf{G}_m\mathbf{y} = \mathbf{t}_m \tag{12}$$

where $\tilde{\mathbf{B}} = \begin{bmatrix} \mathbf{0}_{L_{ts}\times N} & \mathbf{I}_{L_{ts}\times L_{ts}} \end{bmatrix}$. Again by using the commutative property (9) we have

$$\tilde{\mathbf{B}}\mathbf{Y}\mathbf{g}_m = \mathbf{t}_m. \tag{13}$$

Stacking over M transmitters, we obtain the second cost function:

$$J_{\text{training}} = (\mathcal{V}\mathbf{g} - \mathbf{t})^H(\mathcal{V}\mathbf{g} - \mathbf{t}), \tag{14}$$

where $\mathcal{V} = \mathbf{I}_M \otimes (\tilde{\mathbf{B}}\mathbf{Y})$ and $\mathbf{t} = \begin{bmatrix} \mathbf{t}_1^T & \cdots & \mathbf{t}_M^T \end{bmatrix}^T$.

Combining (11) and (14), the coefficients of equalizers can be estimated by solving the following optimization problem

$$\min_{\mathbf{g}} \|\mathbf{C}\mathbf{g}\|^2 + \alpha(\mathcal{V}\mathbf{g} - \mathbf{t})^H(\mathcal{V}\mathbf{g} - \mathbf{t}) \tag{15}$$

where α is a weight coefficient which determines the contribution of the blind and training components of the cost function. The estimated equalizer is then

$$\hat{\mathbf{g}} = (\mathbf{C}^H\mathbf{C} + \alpha\mathcal{V}^H\mathcal{V})^{-1}\mathcal{V}^H\mathbf{t}. \tag{16}$$

Using $\hat{\mathbf{g}}$ we form $\hat{\mathbf{G}}_m^H$ in (7) and the block of data symbols is then estimated as

$$\mathbf{s} = \mathbf{U}^{\dagger}\tilde{\mathbf{G}}\mathbf{y}, \tag{17}$$

where $\mathbf{U} = \begin{bmatrix} \mathbf{U}_1^T & \cdots & \mathbf{U}_M^T \end{bmatrix}^T$, $\tilde{\mathbf{G}} = \begin{bmatrix} (\mathbf{B}\hat{\mathbf{G}}_1^H)^T & \cdots & (\mathbf{B}\hat{\mathbf{G}}_M^H)^T \end{bmatrix}^T$, and $\dagger$ denotes pseudoinverse..

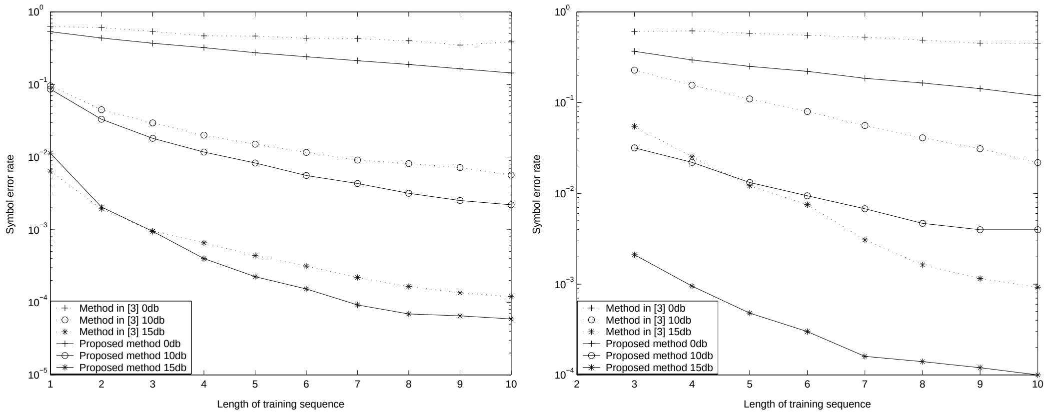
A question remains as to how many training symbols are required to ensure (15) has a unique solution. During simulation, we find that this is dependent on the specific kind of code. For the diagonal code, our method requires one training symbol to eliminate the ambiguity. Whereas for the Alamouti code, at least L_h training symbols are required. We are yet to find a formal proof of this, although our simulations indicate that it is the case. We note the same difficulty is met by the algorithm in [3].

5. NUMERICAL STUDY

Consider a system with $M = 2$ transmit antennas and $K = 6$ receive antennas. The block length is 40. Both training and information symbols are chosen from a QPSK constellation. Channels are frequency selective with length $L_h + 1 = 3$, and the equalizer length is $L_g + 1 = 3$. The value of α in (16) was set as $\alpha = 1$. Two kinds of space time coders were implemented: (i) the Alamouti coder [2], and (ii) the diagonal coder [4]. The performance of our proposed approach is compared with [3].

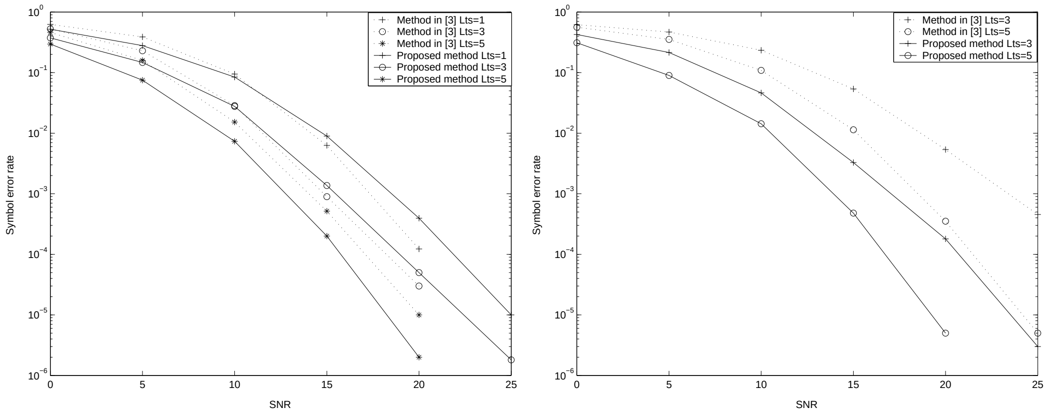
Figure 1 demonstrates how the length of training sequences influences the performance of the two algorithms when SNR is fixed. Figure 2 shows the performance of the two approaches versus SNR with fixed length of the training sequences.

The performance of our proposed approach is comparable to [3] for the diagonal coder, but shows significant improvement for the Alamouti coder. The reason for this may be that we divide the equalization into two steps. First the coefficients of the equalizers are estimated and then the sequence is calculated by using the equalizers estimated. In each step, the number of the unknown parameters is small and thus the estimation error is reduced.



(a) Diagonal code (b) Alamouti code

Figure 1: Semi-blind symbol error rate of s_1 versus the length of training sequence used.



(a) Diagonal code (b) Alamouti code

Figure 2: Semi-blind symbol error rate of s_1 versus SNR.

6. CONCLUSION

A new approach to semi-blind direct equalization of space-time block-coded systems was developed in this paper. By exploiting the redundant structure of space-time block codes and the commutative property of the Sylvester matrix, we were able to achieve equalization for STBCs without channel estimation. The scheme is suitable for most kinds of codes listed in [3]. Simulation results indicate that the proposed algorithm is able to achieve comparable or better performance than existing schemes (depending on the code used).

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