A Block Refinement Scheme for Tracked MIMO Channel Estimates

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Abstract: In a wireless cellular phone system, the channel impulse responses between the mobile terminal and the base station are time varying due to the movement of the mobile terminal or changes in the environment. These channels are frequently tracked using adaptive algorithms to assist in the decoding of the transmitted symbols. However, the tracked estimates are often noisy due to the inherent noise of the system. In this paper, we present a block refinement algorithm for the tracked channel estimates which exploits the smooth varying characteristic of the channel by using polynomial fitting.

1 Introduction

Time varying channel impulse responses have always posed a problem for wireless communications system design. The movement of the mobile terminal coupled with the changing environment often results in rapid fluctuation of the channel response. Often, the coherence time is much shorter than the frame period of the system, so the channel cannot be assumed to be stationary within the frame. As a result, procedures to compensate for the time varying condition have to be incorporated in the system design.

There are several methods to counteract the effect of time varying channels. One of the common techniques used is differential encoding of the source [1, 2]. This technique is very effective especially in very fast fading environments. However, the method incurs a 3dB degradation in the bit error rate performance. One other method is to track the channel responses with an adaptive algorithm [3, 4, 5], which is the case we are focusing on in this paper. The channel estimates of the tracker, however, tend to be noisy due to the inherent noise of the system.

In this paper, we propose a block refinement scheme to refine the channel estimates of the tracker. The method exploits the smooth varying property of the channel responses and constrains them using polynomial fitting.

2 System Model

In a wireless system with M transmit antennas and N receive antennas, the $N \times 1$ receive vector, $\mathbf{y}(n)$ at time instance n, can be written as,

$$\mathbf{y}(n) = \mathbf{H}(n) \begin{bmatrix} \mathbf{x}(n) \\ \mathbf{x}(n-1) \\ \vdots \\ \mathbf{x}(n-L) \end{bmatrix} + \mathbf{v}(n), \tag{1}$$

where $\mathbf{x}(n)$ is the $M \times 1$ transmit vector, $\mathbf{v}(n)$ is the $N \times 1$ noise vector, and

$$\mathbf{H}(n) = \begin{bmatrix} \mathbf{H}_0(n) & \dots & \mathbf{H}_L(n) \end{bmatrix} \tag{2}$$

is the channel matrix.

Each of the matrices $\mathbf{H}_i(n), \forall\, i = 1 \ldots L$, are defined as the $N \times M$ time varying channel matrix at delays of i. For flat fading channels, $L = 1$.

We assume that the channels are of the wide sense stationary, uncorrelated scattering (WSSUS) type [6], which is valid in many practical channels and is mathematically tractable. Such channels have tap coefficients which are uncorrelated.

The autocorrelation of each channel coefficient is [7]

$$\phi_{rr}(k) = \Omega J_0(2\pi k f_d T), \tag{3}$$

where Ω is the total average power of all multipath components, $J_0(\cdot)$ is the zero-order Bessel function of the first kind and f_dT is the normalized doppler frequency. The variation of the tap coefficients are modelled using Jakes's model [7].

3 Tracking Method

To formulate the channel tracking problem in the form appropriate for use by state tracking filters (eg. Kalman [8] or particle [9]), we let the state matrix at time n be the channel matrix $\mathbf{H}(n)$. Since the variation of the channel coefficients can be approximated by an autoregressive (AR) process of order 1, the state transition can be written as [4]

$$\Phi(n|n-1) = \alpha \mathbf{I}_N$$

where α is the AR(1) coefficient, and $\mathbf{I}_N$ is an $N \times N$ identity matrix.

To track the time varying channels, we use the algorithm in [3] and extend it for multiple-input multiple-out systems [4]. We start off with the initial channel responses which can be obtained by training sequences [3], or methods described in [10].

First, the current channel states are estimated from the previous state using

$$\tilde{\mathbf{H}}(n) = \Phi(n|n-1)\hat{\mathbf{H}}(n-1). \tag{4}$$

This give a rough estimate of the current channel assuming a zero mean process noise. Using this estimate, we can use detection methods to obtain initial estimates of the transmitted symbols, $\tilde{\mathbf{x}}(n)$. These estimated symbols are then used in the tracking filter to obtain a more refined channel estimate, $\hat{\mathbf{H}}(n)$.

Finally, the refined channel estimates are used to give a more accurate set of transmitted symbol estimates, $\hat{\mathbf{x}}(n)$. The procedure is repeated to estimate the next symbol.

4 Block Refinement

In this section, we describe a simple block technique for refining the channel estimates obtained via the tracking algorithm described above.

The refinement is achieved by fitting polynomials to the channel estimates previously obtained. To be able to do this, we first assume that the channel variation is smooth and continuous. This assumption holds in real situations since it is commonly known that the fading channels can be modelled as a sum of exponentials [7, 11].

Suppose we have obtained m estimates of the time varying channel coefficient $h_{ij}(n)$, which is the $\{i,j\}$ element of the matrix $\mathbf{H}(n)$, at times $\tau_1 \ldots \tau_m$, and want to fit a $(k-1)$th order polynomial to them. We can form the equation

$$\begin{bmatrix} \tilde{h}_{ij}(\tau_1) \\ \tilde{h}_{ij}(\tau_2) \\ \vdots \\ \tilde{h}_{ij}(\tau_m) \end{bmatrix} = \begin{bmatrix} 1 & \tau_1 & \tau_1^2 & \ldots & \tau_1^k \\ 1 & \tau_2 & \tau_2^2 & \ldots & \tau_2^k \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \tau_m & \tau_m^2 & \ldots & \tau_m^k \end{bmatrix} \begin{bmatrix} \alpha_{ij}^{(0)} \\ \alpha_{ij}^{(1)} \\ \vdots \\ \alpha_{ij}^{(k-1)} \end{bmatrix} \tag{5}$$

where $\alpha_{ij}^{(p)}$ is the pth order coefficient of the polynomial. The order of the polynomial is determined by the Doppler spread of the channel and the block length. For a block length of 100 symbols, and normalized Doppler spread of 5×10^{-3}, a 3rd order polynomial is found to be sufficient. For lower Doppler spreads, a 2nd order polynomial will suffice.

Using the linear least squares criteria, we have the cost function

$$\mathcal{L} = [\mathbf{h}_{ij} - \Psi \mathbf{a}_{ij}]^T [\mathbf{h}_{ij} - \Psi \mathbf{a}_{ij}], \tag{6}$$

where Ψ is the Vandermonde matrix in (5), $\mathbf{h}_{ij}$ is the channel coefficient vector, and $\mathbf{a}_{ij}$ is the vector of polynomial coefficients in (5).

The least squares estimate of the coefficients is

$$\hat{\mathbf{a}}_{ij} = [\Psi^T \Psi]^{-1} \Psi^T \mathbf{h}_{ij}. \tag{7}$$

On obtaining the polynomial vector $\hat{\mathbf{a}}$, a refined set of the channel coefficients, $\mathbf{H}(n)$, at times $\tau_1 \ldots \tau_m$ is obtained by

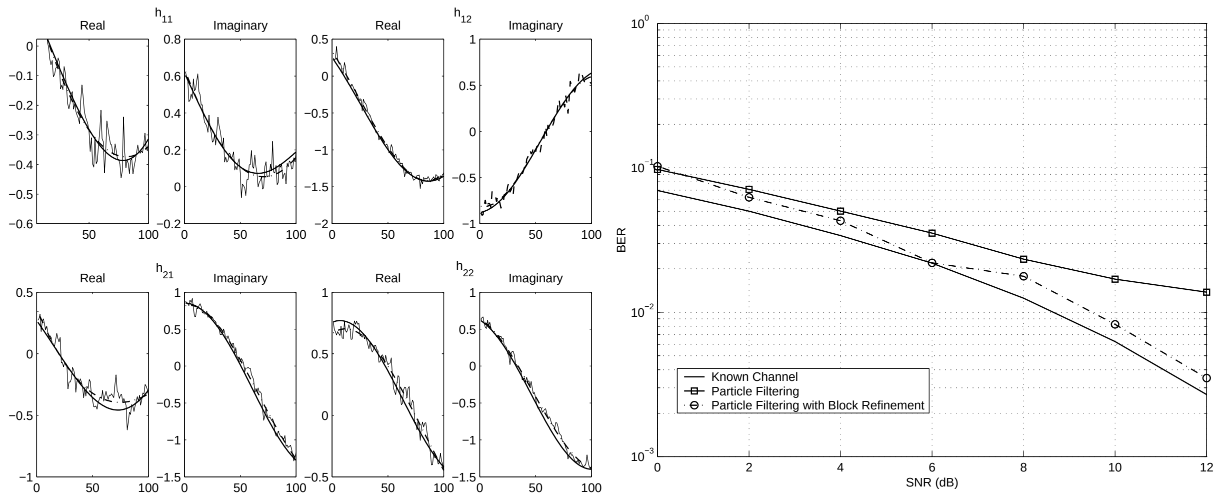
$$\hat{h}_{ij}(n) = \sum_{p=0}^{k-1} \alpha_{ij}^{(p)} n^p, \qquad \forall\, n = \tau_1 \ldots \tau_m, \tag{8}$$

where $\hat{h}_{ij}(n)$ is the $\{i, j\}$ element of the refined matrix $\hat{\mathbf{H}}(n)$. Using the new channel coefficient estimates, we can then compute the transmitted symbol estimates using block equalization algorithms such as the block minimum mean squared error (MMSE), or least squares estimators.

5 Simulations and Performance Analysis

In this section, we present the performance of systems employing the block refinement scheme. In all the simulations, we employ auxiliary based particle filters [9] with 100 particles under mixed Gaussian noise [12] with power ratios of 100.

We assume that the initial channel information is previously estimated and known in all cases. We also insert one pilot symbol between every 20 symbols to prevent the divergence of the algorithm.



(a) (b)

Figure 1: (a) Tracking of channel coefficients, (bold) actual channel, (thin) particle filter tracked, (-·-) block refined (b) Comparison of the BER curves with and without the block refinement scheme.

In Fig. 1(a), we demonstrate the process of block refinement using a 2×2 system with flat fading ($L = 1$) channels with normalized Doppler spreads of 5×10^{-3}. The variation of the channel coefficients over time are shown in the figure, along with the particle tracked estimates, which is represented by the thin line. The block refined estimates (-·-) are obtained from the block of particle tracked estimates by fitting 3rd order polynomials to the estimates and evaluating the polynomial functions at the appropriate time instances.

In Fig. 1(b), we show the bit error rate (BER) curves of a 2×4 system with known channel coefficients, tracking with particle filter, and particle filter tracking with block refinement. The time varying channels used have a normalized doppler spread of 10^{-3} with $L = 2$. A 2nd order polynomial is used for fitting the channel coefficients. We can see that we have an improvement of at least 1 dB in performance at low SNRs and over 2 dB at higher SNR by applying the block refinement scheme to the particle tracked channel estimates.

6 Conclusions

In this paper, we presented a block refinement scheme which can be applied to systems where the channel coefficients are tracked with adaptive algorithms (eg. Kalman or particle filters). We have shown that the scheme improves the bit error performance of the system when applied to estimated channel coefficients tracked by an adaptive filter.

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