Throughput and delay properties of the OSSS+RED protocol for the MAC function in UTRA-FDD systems

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Abstract: In this paper we present an analysis of the OSSS+RED medium access control protocol of [1], under the assumption that the backlogged packets are considered as a separate traffic flow to the newly arriving packets. By doing so, the delay properties of the protocol can be obtained, and the performance of OSSS+RED still shows a superior performance over existing protocols. In addition to that, the new analysis separates the interactions of OSSS+RED at the time-slot level, with the characteristics of the offered load; hence we can extend our study beyond the simple assumption of Poisson arrivals to the complexity of traffic sources exhibiting the long range dependence (LRD) property.

1. Introduction

The Overload Signal Spread Spectrum with Random Early Discarding (OSSS+RED) protocol was presented in [1] as an enhancement of the OSSS protocol of [2]. In [1] the RED mechanism from IP systems [3] was used to compute the permission probability parameter that stabilises the operation of the MAC function in a DS-CDMA system, as an alternative to the methods presented in [4]. The OSSS+RED protocol showed clear advantages over its predecessor, OSSS alone. The throughput performance showed stability under different latency scenarios and at offered load levels above the capacity of the channel. Those results, however, were obtained under the assumption that the arrival stream to the access interface includes both, newly arriving and backlogged packets. In this paper we present a new analysis and results for throughput and delay, where the backlogged packets are considered as a separate traffic flow, contributing independently to an increased "effective offered load". We expect that, under the same assumptions, OSSS+RED will still show a superior performance over OSSS alone and SS-ALOHA. The following section (2) presents a brief explanation of the OSSS+RED protocol. Section 3 then gives the assumptions and considerations for the new modelling scenario, and its mathematical analysis. Section 4 presents the simulation scenario, and sections 5 and 6 present our results and conclusions. Finally, a complementary explanation of the mathematical analysis of the protocol is given in appendix A.

2. The OSSS+RED protocol

OSSS+RED requires that the channel be divided into time-slots of certain duration (t_s), at the beginning of which there is an access window of duration t_{aw}, where the mobile terminals schedule the transmission of their packets (of duration t_p) – see Fig. (1).

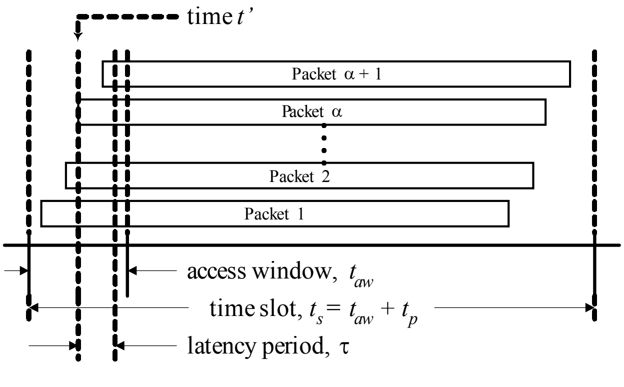

Fig. 1 Time-slots in OSSS+RED

From the start of the access window the base station listens to the channel for incoming transmissions, which gradually load the channel from zero to a certain level. When the channel capacity (c) is likely to be exceeded (at a given channel load $c\ell = \alpha$, at the time t') the base station turns on a congestion signal that is broadcast throughout its coverage area. The congestion signal carries with it a permission probability parameter $p(t')$ (dependent on t'), which the mobile terminals use to either randomly transmit or defer their transmissions. The base station calculates this permission probability, so that the change in the arrival rate reduces substantially the likelihood that the channel capacity c is exceeded. A criterion for the calculation of $p(t')$ was given in [1] (Eq. (5)). Here, we present an enhanced version of the equation, which takes into account some exception conditions in the value of t':

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$$p(t') = \begin{cases} 0, & \text{if} \quad 0 \le t' < \alpha\tau/(c-\alpha) \\ [ct' - \alpha(t'+\tau)] \,/\, [\alpha t_{aw} - \alpha(t'+\tau)], & \alpha\tau/(c-\alpha) \le t' < \min(t_{aw}-\tau,\ (\alpha/c)t_{aw}) \\ 1, & \min(t_{aw}-\tau,\ (\alpha/c)t_{aw}) \le t' \le t_{aw} \end{cases} \tag{1}$$

where τ is the latency in the channel[1].

3. Throughput and delay analysis of OSSS+RED

Let us consider a one-cell DS-CDMA system with N mobile terminals, each one producing an offered load of one packet per time-slot with probability φ. When a given packet is not successfully transmitted (due to the MAC function or the MAI problem) then the corresponding mobile terminal (in the backlog state) halts its packet production and re-attempts the transmission of the given packet for as many times as needed.

This system is modelled as an $(N+1)-$state Markov chain, where each state represents the number of backlogged mobile terminals. The usual approach to the solution of such a system is the formulation of the following system of equations:

$$\begin{aligned} &i)\quad \boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{A} \\ &ii)\ \sum_{i=0}^{N} \pi_i = 1 \end{aligned} \tag{2}$$

Where $\boldsymbol{\pi}$ is the vector of equilibrium-state probabilities (π_i) and $\mathbf{A}$ is the matrix of transition probabilities between states of the Markov chain ($A_{i,j}$). These probabilities are given by the following expression:

$$\begin{aligned} A_{i,j} &= \sum_{k=\max(0,j-i)}^{N-i} \binom{N-i}{k} \varphi^k (1-\varphi)^{N-i-k} \Pr(X_s = i+k-j \mid X_a = i+k) \\ &i = 0,\ 1,\ 2...\ N \qquad j = 0,\ 1,\ 2...\ N \end{aligned} \tag{3}$$

where X_a represents the number of transmission attempts at a given time-slot and X_s is the random variable denoting the number of successful packet transmissions (under a given value of X_a). Eq. (7) in appendix A gives an expression for $\Pr(X_s = s \mid X_a = a)$.

The solution of a system like the one described is given by solving the system of linear equations in (2). When the probabilities π_i are obtained, the throughput of the access interface is given by,

$$S = \sum_{i=0}^{N} \pi_i \sum_{k=0}^{N-i} \binom{N-i}{k} \varphi^k (1-\varphi)^{N-i-k} \tag{4}$$

Regarding the average access delay metric, it can be calculated by using the following expression:

$$E[AccessDelay] = \sum_{k=1}^{\infty} k(1-P_{succ,new})(1-P_{succ,blog})^{k-1} P_{succ,blog} \tag{5}$$

where $P_{succ,new}$ and $P_{succ,blog}$ are the probabilities of successful transmission for newly arriving and backlogged packets, respectively. $P_{succ,new}$ and $P_{succ,blog}$ are given by (6):

$$\begin{aligned} P_{succ,new} &= \sum_{i=0}^{N-1} \pi_i \sum_{j=0}^{N-i-1} \binom{N-i-1}{j} \varphi^j (1-\varphi)^{N-i-1-j} \sum_{n=1}^{i+1+j} \frac{n}{i+1+j} \Pr(X_s = n \mid X_a = i+1+j) \\ P_{succ,blog} &= \sum_{i=1}^{N} \pi_i \sum_{j=0}^{N-i} \binom{N-i}{j} \varphi^j (1-\varphi)^{N-i-j} \sum_{n=1}^{i+j} \frac{n}{i+j} \Pr(X_s = n \mid X_a = i+j) \end{aligned} \tag{6}$$

4. Simulation scenario

The OSSS+RED protocol is analysed on a system with parameters similar to those used in [1]: mobile terminals experiencing equal channel conditions (i.e. latency, power reception), channel capacity $c = 16$ packets, time-slot duration $t_s = 10$ ms, access window duration $t_{aw}/t_s = 0.05$, access window to latency quotient

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[1] In OSSS+RED the packets can suffer from multiple access interference (MAI) because the latency in the channel does not allow an immediate transmission of the congestion signal.

$t_{aw} / \tau = 5,\ 10,\ 50$ (high/medium/low latency), congestion threshold $\alpha = 12,\ 14,\ 15$ (high/medium/low latency), packet length $L = 424$ bits and spreading factor $S_f = 64$.

5. Results

The results on the throughput and delay metrics for OSSS+RED are presented in Fig. 2 and 3, respectively. As stated in section 3, OSSS+RED is evaluated under three latency cases, due to the dependency of the protocol on this channel property. Also, we present results of the SS-ALOHA protocol (with ideal load control) for comparison purposes.

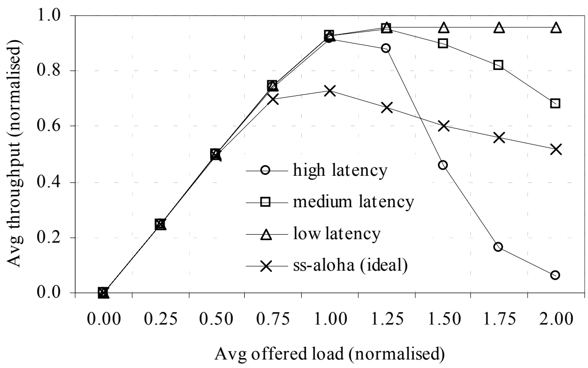

Fig. 2 Throughput versus offered load in OSSS+RED and ideal SS-ALOHA

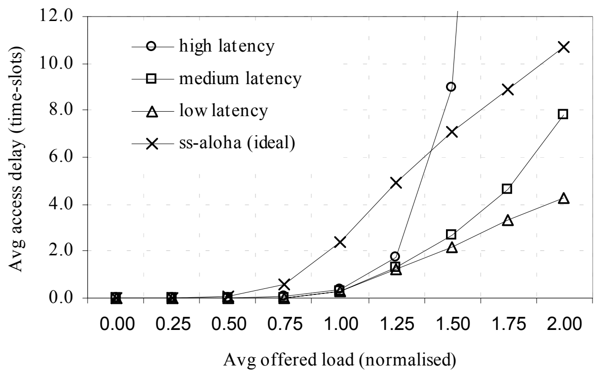

Fig. 3 Average access delay versus offered load in OSSS+RED and ideal SS-ALOHA

Figures 2 and 3 show that, even under the realistic scenario of backlogged packet transmissions (not considered in [1] and [2]) OSSS+RED still exhibits a good throughput and delay performance, and superior overall performance (in the medium and low latency cases) compared to its SS-ALOHA counterpart. Although the performance is still dependent on the latency property at high levels of the offered load, these excursions beyond the capacity of the channel are less likely to occur under this new model. It is worth noting that the results in [1] were obtained by treating both new and backlogged packets as new arrivals at the access interface. If this condition holds, the approach suggests that the OSSS+RED protocol can tolerate offered load excursions beyond the operating point without compromising protocol stability. However, treating the backlogged packets as a separate flow distinct from new arrivals at the input to the access interface is more realistic, therefore the new analysis is more accurate. In this case the key result is that OSSS+RED achieves an almost optimal throughput performance when the (real) offered load approaches channel capacity.

6. Conclusions

This paper presents a new analysis of the OSSS+RED protocol of [1], where the interactions between newly generated and backlogged packets are taken into account. Accordingly, the new analysis accurately predicts the throughput and delay performance metrics.

OSSS+RED shows a superior performance over its SS-ALOHA counterpart, and allows excursions of the offered load up to the capacity of the channel.

Another advantage of the analysis presented is that the interactions of the traffic at the access window level (Eq. (7)) have been separated from the specific characteristics of the offered load (Eq. (4)). This implies that the OSSS+RED protocol can now be analysed under different offered load cases, i.e. using traffic sources exhibiting long range dependence. This is the subject of further investigations by the authors.

Appendix A. Distribution of the number of successful packet transmissions

The number of successful packet transmissions when $X_a = a$ packets are attempted for transmission at a given time-slot is a random variable X_s that takes values in the range $s = 0, 1, 2... a$. The calculation of the $\Pr(X_s = s \mid X_a = a)$ probabilities is split into three cases: *i*) $0 \le a \le \alpha$, *ii*) $\alpha < a \le c$ and *iii*) $c < a$:

$$
\begin{array}{lll}
 & & \text{if} \\
i) & \Pr(X_s = s \mid 0 \le a \le \alpha) = \delta(a-s) & s = 0,\ 1,\ 2... a \\
ii) & \Pr(X_s = s \mid \alpha < a \le c) = \begin{cases} 0, & \\ \Pr(X_{B\alpha} = s-\alpha \mid \alpha < a \le c), & \end{cases} & \begin{array}{l} s = 0,\ 1,\ 2... \alpha - 1 \\ s = \alpha,\ \alpha+1... a \end{array} \\
iii) & \Pr(X_s = s \mid c < a \le N) = \begin{cases} \sum_{l=c-\alpha+1}^{a-\alpha} \Pr(X_{B\alpha} = l \mid c < a \le N) \Pr(X_M = s \mid c\ell = \alpha + l), \\ \Pr(X_{B\alpha} = s - \alpha \mid c < a \le N) + \\ \quad \sum_{l=c-\alpha+1}^{a-\alpha} \Pr(X_{B\alpha} = l \mid c < a \le N) \Pr(X_M = s \mid c\ell = \alpha + l), \\ \sum_{l=s-\alpha}^{a-\alpha} \Pr(X_{B\alpha} = l \mid c < a \le N) \Pr(X_M = s \mid c\ell = \alpha + l), \end{cases} & \begin{array}{l} s = 0,\ 1... \alpha - 1 \\ s = \alpha,\ \alpha + 1... c \\ \\ s = c+1,\ c+2... a \end{array}
\end{array} \tag{7}
$$

Where $\delta(n)$ is the discrete, impulse function, $X_{B\alpha}$ is the number of packets transmitted beyond the α limit (when $X_a = a,\ \alpha < a$) and X_M is the number of packets properly received at the receiver when the load in the channel ($c\ell$) reaches a certain value. The distributions of $X_{B\alpha}$ and X_M are given in (8) and (10), respectively:

$$
\begin{aligned}
P(X_{B\alpha} = k \mid \alpha < a) = {} & \frac{g(\lambda_a, \tau, k)}{\sum_{n=0}^{a-\alpha} g(\lambda_a, \tau, n)} \times \frac{1 - \sum_{l=0}^{\alpha-1} g(\lambda_a, t_l, l)}{1 - \sum_{l=0}^{\alpha-1} g(\lambda_a, t_{aw}, l)} + \\
& \frac{1}{\sum_{n=0}^{a-\alpha} g(\lambda_a, \tau, n)} \sum_{l=0}^{k} g(\lambda_a, \tau, l) \binom{a-\alpha-l}{k-l} \int_{t_l}^{t_h} \nu(t)^{k-l} (1-\nu(t))^{a-\alpha-k} f_{t'}(t) dt + \\
& \frac{\delta(k - [a-\alpha])}{1 - \sum_{l=0}^{\alpha-1} g(\lambda_a, t_{aw}, l)} \sum_{l=0}^{\alpha-1} \left[g(\lambda_a, t_h, l) - g(\lambda_a, t_{aw}, l) \right] \\
& k = 0,\ 1,\ 2... a - \alpha
\end{aligned} \tag{8}
$$

where $\lambda_a = a / t_{aw}$, $g(\gamma, t, n) = \exp(-\gamma t)(\gamma t)^n / n!$, $\nu(t) = (\alpha t_{aw} - \alpha[t+\tau]) / (c t_{aw} - \alpha[t+\tau])$, $t_l = \alpha\tau/(c-\alpha)$, $t_h = \min(t_{aw} - \tau,\ (\alpha/c) t_{aw})$ and $f_{t'}(t)$ is the probability density function of t', given by:

$$
f_{t'}(t) = \lambda_a g(\lambda_a, t, \alpha - 1) \big/ (1 - \textstyle\sum_{l=0}^{\alpha-1} g(\lambda_a, t_{aw}, l)) \tag{9}
$$

Regarding X_M, its probability mass function is given by,

$$
P(X_M = m \mid c\ell) = \binom{c\ell}{m} P_s(c\ell)^m (1 - P_s(c\ell))^{c\ell - m},\ m = 0,\ 1,\ 2... c\ell \tag{10}
$$

Where $P_s(c\ell)$ is defined as the packet success probability when the load in the channel is $c\ell$:

$$
P_s(c\ell) = \left[1 - Q\left(\sqrt{3 S_f / (c\ell - 1)} \right) \right]^L \tag{11}
$$

$Q(x)$ is the Markum Q function, S_f is the spreading factor in the CDMA transmission system and L is the length (in bits) of the packets, and it is assumed that no error correction capability takes place in the transmission system.

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