Application of Evolutionary Game Theory in Wireless Ad Hoc Networks

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Abstract: Evolutionary Game Theory originated as an application of Game Theory in modelling evolution of animals in biology. However, the subsequent developments in Evolutionary Game Theory have made it more readily applicable in the areas where Game Theory is applied. This paper investigates the applicability of Evolutionary Game Theory in wireless Ad Hoc Networks and uses its concepts to build a mathematical model of the Ad Hoc Network. This model is simulated under different traffic levels and the dynamics of the network are discussed. For this model only two pure strategies - altruistic and selfish strategies are considered and the utility function is based on the amount of power each node has to transmit.

1 Introduction

In wireless Ad Hoc Networks a group of nodes combine to form a temporary infrastructure-less network. The nodes communicate with far off nodes through intermediate nodes. In order for a wireless Ad Hoc Network to be efficient it is imperative that most nodes, if not all, should participate in relaying of data. Even though participating in relaying data improves the performance of the network, it is done at a personal cost of the relaying nodes. These nodes loose their limited battery power in transmitting for others. This conflicting situation of whether to participate or not and how much and when to participate can be best analysed using the concepts of Game Theory.

2 Motivation for Evolutionary Game Theory

The 1970s and 1980s witnessed the emergence of Evolutionary Game Theory which used the concepts of Game Theory to model the evolution of dispersal behaviour of animals [1]. This theory overcomes some of the shortcomings of Game Theory such as the criterion of rationality which is replaced by population dynamics and stability. The concept of Evolutionary Game Dynamics provides a dynamics which describes how the frequencies of strategies within a population change in time, according to the strategies' success [2]. The Nash Equilibrium is replaced by a new concept of equilibrium called Evolutionary Stable Strategy (ESS). ESS is a strategy which when all the members of a population follow it, no mutant strategy would be able to invade the population under the influence of natural selection [1]. These concepts provide the tools necessary to model and study the evolution of strategies in Ad Hoc Networks.

3 Modelling of the network game

The game is modelled to analyse the strategies of the nodes. It is assumed that there exists two pure strategies in the network - altruistic strategy and selfish strategy. It is also assumed that every packet requires the same amount of energy and each node tries to minimise the amount of energy it has to spend. In order to model the game for the network, a fitness function is defined. The fitness function describes the payoff each node will receive for the strategy it follows.

3.1 Fitness function

The fitness function based on the utility function described in [3] is totally dependent on the amount of power each node has to transmit and hence independent of the protocols used.

When a node participates to transmit data through the network its transmitted power is $-p_r$, the minus describes the loss of power. If the node decides to transmit direct then it would transmit $-p_d$ amount of power. If a node is altruistic then it would transmit $-p_o$ amount of power transmitting data for other nodes.

The selfish strategy can be further divided into two. A node which uses the network to transmit its own data but refuses to relay data for others can be termed as a malicious selfish node. The second type can be described as a node which when following honest selfish strategy refuses to participate in the network and doesn't worry about the benefit of the overall network.

The malicious selfish strategy would become the ESS and would successfully invade the network as it will be the best possible strategy in terms of saving power. The malicious selfish strategy reduces the overall performance of the network and hence needs to be discouraged. This can be achieved by the implementation of protocols such as CONFIDENT [4] and CORE [5]. Any node that joins the network to transmit its own data will have to transmit data for others too. Hence, for this model when a node follows the selfish strategy it refrains from participating in the network and transmits directly if within the range and if not either changes to altruistic strategy and participates in the network or refrains from transmitting.

Based on the above descriptions, the fitness function for the game is defined as:

$$F = -\alpha(p_r + p_o) - (1 - \alpha)p_d \tag{1}$$

Where

 $\alpha = \begin{cases} 1 & if node is altruistic \\ 0 & if node is selfish \end{cases}$

The required transmitted power is exponentially proportional to the distance between transmitter and receiver, and calculated using the path loss exponent [6]. This can be mathematically represented as

$$p \propto d^{\alpha}$$
 (2)

Where p is the transmitted power, d is the distance between the transmitting node and receiving node and α is the path loss exponent.

3.2 Game dynamics

The game dynamics are used to model and study the evolution of strategies over a period of time as a function of the fitness. At every iteration a node can change its strategy to altruistic or selfish depending on its previous fitness only. Applying the work done in [7] the following equations for the finite evolutionary game dynamics are derived.

$$P_a = \left(\frac{n_a f_a}{n_a f_a + n_s f_s}\right) \left(\frac{n_s}{N}\right) \tag{3}$$

$$P_s = \left(\frac{n_s f_s}{n_a f_a + n_s f_s}\right) \left(\frac{n_a}{N}\right) \tag{4}$$

Where, P_a is the probability to change to an altruistic strategy, P_s is the probability to change to a selfish strategy, n_a is the number of altruistic nodes in the network, n_s is the number of selfish nodes in the network, N is the total number of nodes in the network, f_a is the fitness of the altruistic node and f_s is the fitness of the selfish node.

4 Simulation

The above model is simulated for different traffic levels and the best strategy that would be required by the nodes to transmit least amount of power is analysed. The minimum energy routing is done through Djkstra's algorithm. The network is assumed to have 30 nodes at any stage of the simulation and are distributed randomly over a square area of dimension 500 meters. The path loss exponent is set as 4 for this simulation.

4.1 Traffic levels

The model is simulated for two different levels of traffic. In traffic level I, all the nodes transmit at least once and at least three nodes transmit data at any given iteration. In traffic level II the density of traffic is relatively low, at any given iteration between one to three nodes transmit their data and some of the nodes are assumed not to transmit any data.

5 Results

The following results were obtained from the simulation. For each traffic level the number of altruistic and selfish strategies are shown along with the average transmission power per iteration.

5.1 Traffic Type I



Figure 1: Network dynamics and Average Transmitted Power for traffic of type I.

It can be seen in Figure 1 that the number of altruistic nodes increases steadily after the initial transient period. During the initial period the nodes which have not yet started to transmit data turn to selfish as it would not be beneficial for them to transmit data for others as long as they have no data to transmit. Once all the nodes start their transmission, it is beneficial for all those nodes to participate in the network as the cost of being selfish is very high in this situation (as it would have to transmit directly). Hence, the best strategy for every node is to be altruistic as long as it has data to transmit. This results in the network becoming predominantly altruistic. The average transmitted power for every iteration reduces as the number of altruistic nodes increases.

5.2 Traffic Type II

In Figure 2 the number of altruistic nodes are relatively lower than in traffic level I. There are two reasons for this, first is due to the nodes having smaller amount of data to transmit do not save much power by remaining altruistic and transmitting for others. Hence, they change to selfish strategy to conserve their power and lifetime. Second, since some of the nodes do not have any data to transmit they immediately defect to selfish strategy since being altruistic they would have to transmit data for others without having any gain in return. Due to relatively lower number of altruistic nodes in the network the average transmitted power is higher compared to traffic level I.

6 Conclusion

In this paper it has been shown that Evolutionary Game Theory can be used to model and analyse the strategies of the nodes in a network. It can be positively concluded from this model



Figure 2: Network dynamics and Average Transmitted Power for traffic of type II.

that the dynamics of the network depend very much on the amount of traffic in the network. When all the nodes have large amount of data to transmit the altruistic strategy is the optimal strategy. As the traffic reduces, the number of selfish nodes increases resulting in the increase of average transmitted power. In such situations it may be necessary to provide positive incentives to encourage altruistic behaviour.

References

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