Stable Distributed Power Control with High SIR Target for Cellular Wireless Communication Systems

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Abstract – In this paper we analyse the convergence properties of distributed power control (DPC) and propose an improved algorithm to overcome the weakness of DPC. The DPC algorithm can only work in the region of low signal to interference ratios (SIRs). When the user SIR target is high, the transmit power will diverge to infinite. Furthermore, before the system converges, the DPC algorithm may cause SIR fluctuations which makes the system unstable. Our proposed algorithm DPCH is designed for high SIR environment and can solve the limitations of DPC. Specifically, there is no SIR fluctuation in DPCH before the system converges. The predicted convergence properties of DPCH are verified by computational experiments. The numerical results demonstrate that DPCH can offer better performance than DPC in terms of stability and power saving, especially in the high SIR environment, while the convergence rate of DPCH is slightly slower than that of DPC in the low SIR environment.

Keywords - **Power control, SIR, convergence properties**

I. INTRODUCTION

Transmit power control [1] allows communication links to be established in a signal channel as well as guarantee the user required Signal to Interference Ratio (SIR). By using the minimum transmit power, multi-user interference is mitigated so that the system capacity is increased through higher channel reuse.

Power control can be categorised into two types: centralised power control and distributed power control. Centralized power control is also known as optimum power control that is concerned with the upper performance bounds with the control algorithms. In a centralized power control scheme, a central station is needed to control all the links in a cellular system. A good paper on centralized power control was given in [2], which investigated optimum transmit power control for cellular wireless communication systems. In distributed power control (DPC), each base station controls and updates the transmitted powers based on the information from the local mobile stations. Thus power control is "distributed" to all base stations without the need of a centralized controller. The advantages of distributed control are simplicity and the elimination of the link gain matrix between cells. The only information required for distributed control is the knowledge of local SIRs and local link gains between each mobile node and the base station. One of the early pioneering works in DPC was given in [3], where the author presented an initial interesting work on distributed power control based on proportional control concept, which has a stability problem when the desired SIR is high. The authors in [4] presented another interesting result using a distributed autonomous power control algorithm with the proven convergence property. Their work demonstrates that the power control algorithms are universal in the sense that the local link gain matrix does not affect the convergence to the optimum power vector. The algorithm can incorporate the user-specific SIR requirements and yield minimal transmit powers to satisfy them. Further works based on DPC were also reported in [6, 7, 8] for cellular wireless communication applications.

In this paper, we focus on distributed power control and investigate how to improve the standard DPC algorithm when the user SIR target is high. In order to achieve this goal, we first go through an in-depth analysis of the standard DPC algorithm and identify the key parameters that affect the convergence properties. Based on that, we propose and analyse an improved algorithm, called DPCH, for the high SIR environment. Our numerical results show that as long as the transmit power is within a feasible range, neither the initial power setting nor the number of users will affect the convergence properties. Moreover, the proposed DPCH algorithm can offer better performance than DPC in terms of system stability and power saving, especially in the high SIR environment.

The remainder of the paper is organised as follows. The following section introduces a network model illustrating the fundamental requirements for power control and some notations used in the algorithm development. In Section III, we analyse the basic DPC algorithm first, and then introduce an improved distributed power control for high SIRs (DPCH) algorithm. Section IV gives numerical results, which compares the performances of the DPC and DPCH. Conclusions and further work are summarised in Section V.

II. SYSTEM MODEL

The system model in our study consists of N mobile nodes (i.e., cell phones) that are randomly dispersed in several cells, each cell containing a base station, as illustrated in Figure 1. One transmitter and receiver pair comprises just one mobile node and one base station. Channels operating at specific carrier frequency bands are reused in different cells to ensure that the network has sufficient capacity to support the current set of operating links. A particular channel can be used by only one link (i.e., pair of communicating nodes via a base station) in each cell at a time. Other links in that cell must use other channels when they communicate via the same base station. This maintains the Quality of Service (QoS) by reducing, but not completely removing, the multi-user interference in the same cell.

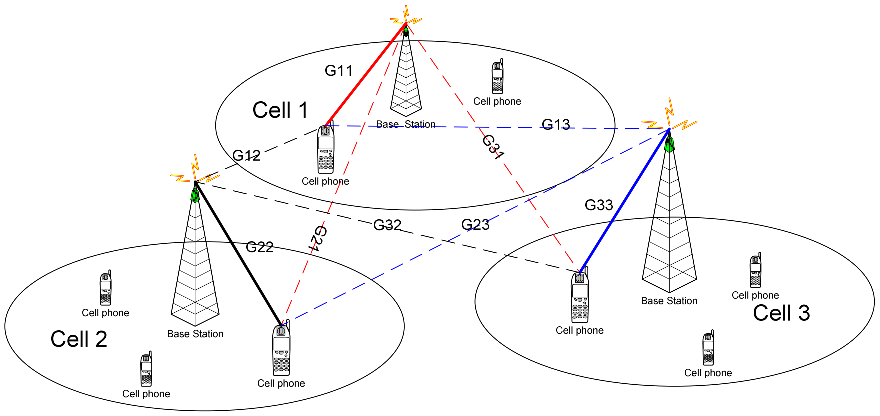


Fig. 1. System model.

Prior to introducing the power control algorithms and establishing their convergence properties in the following sections, some notational definitions in associated with the system model shown in Fig.1 are made as follows:

1. Base station to mobile propagation matrix: $G = [g_{ij}]$ The matrix element, g_{ij}, is the power gain (loss) from the sender (base station) of the j^{th} link to the receiver (mobile) of the i^{th} link. In reality, $\boldsymbol{G}$ suffers rapid stochastic fluctuations with time varying statistics. For simplicity, the propagation matrix assumed here is based on the free space model. For further discussion, we will consider its time-varying characteristics by using its expectation in the simulations. The dotted lines in represent the interference induced by links in different cells sharing the same channel.
2. Transmitter power vector: $P(t) = [p_1(t) \quad p_2(t) \quad \cdots \quad p_N(t)]$. The element $p_i(t)$ denotes the transmitter power in the i^{th} link. All the powers are updated synchronously at a fixed time interval, denoted by $t = 1, 2, 3, \ldots$
3. It follows from 1 and 2 that the power level received by the i^{th} mobile from its base station is $g_{ii} p_i(t)$.
4. Interference power at the i^{th} mobile: $\upsilon_i(t)$. This includes the inter-cell interference as well as the thermal noise, η , which is assumed to be the same for all the mobiles:

$$\upsilon_i(t) = \sum_{j \neq i} g_{ij} p_j(t) + \eta \tag{1}$$

5. Based on the foregoing, the SIR of the i^{th} link is defined as follows:

$$r_i(t) = \frac{g_{ii} p_i(t)}{\upsilon_i(t)} = \frac{g_{ii} p_i(t)}{\sum_{j \neq i} g_{ij} p_j(t) + \eta} \tag{2}$$

   The SIR level is the measurement of the QoS of the links. For each link, a user required lower SIR threshold, γ_i, is specified, reflecting a certain QoS that the links have to maintain in order to operate at a desired performance measure.

Hence:

$$r_i \geq \gamma_i \text{ for } i \in \{1,2,3,...,N\} \tag{3}$$

in order to allow links in different cells to share the same channel, while satisfying their QoS demands.

III. DISTRIBUTED POWER CONTROL WITH HIGH SIR TARGET

A. *The Standard DPC algorithm*

In order to improve the distributed power control for solving the stability problems when the SIR is high, and to understand its implication on convergence property, it is essential to find out the elements and parameters that affects the stability and convergence properties. We carry out this task by two steps: first we analyse a standard DPC and then extend it into an improved form. We build on the analysis of DPC in [4] and extend it into the improved form of DPC. A standard DPC can incorporate the user-specific SIR requirements and yield minimal transmit powers to satisfy them. The algorithm is expressed as follows,

$$\dot{r}_i(t) = -\beta[r_i(t) - \gamma_i], \quad i = 1,2,\ldots,n \tag{4}$$

where $\dot{r}_i(t) = \frac{d}{dt} r_i(t)$ and β is a constant determining the convergence rate of $r_i(t)$ towards the target SIR, γ_i. This closed-loop system may be represented by a Proportional and Integral (PI) controller with transfer function, $H(s)$, as shown in Figure 2.

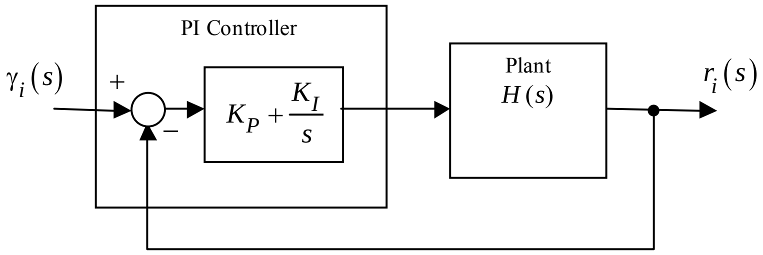


Fig.2. DPC algorithm represented by the PI controller.

Equation (4) could be achieved by choosing K_p and K_I such that $\frac{K_I}{1+K_p} = \beta$, for example by setting $K_p = 1$ and $K_I = 2\beta$. By analysing the system characteristic equation, $s + \beta = 0$, we know that the system stability problem can be characterised by a closed-loop pole at $s_i = -\beta$, and therefore exponential convergence of $r_i(t)$ towards γ_i with time constant, $1/\beta$, ultimately as $\dot{r}_i(t) \to 0, r_i(t) \to \gamma_i$. This conclusion is consistent with the other mathematical treatments in [3,4].

We now address the interference problem. Assuming the interference level stay as a constant, we convert the dynamics to the transmission power, which yields

$$\dot{\boldsymbol{P}}(t) = -\beta \boldsymbol{B}\boldsymbol{P}(t) + \beta\boldsymbol{\Gamma} \tag{5}$$

where

$\dot{p}_i(t) = \frac{d}{dt} p_i(t)$, $\boldsymbol{\Gamma} = \left(\frac{\gamma_1 \eta}{g_{11}}, \frac{\gamma_2 \eta}{g_{22}}, \ldots, \frac{\gamma_n \eta}{g_{nn}} \right)'$ is the column vector of normalised noise power, and $\boldsymbol{B}$ is the matrix with entries $\boldsymbol{B} = \begin{cases} 1 & i = j \\ -\frac{\gamma_i g_{ij}}{g_{ii}} & i \neq j \end{cases}$.

Equation (5) is a continuous time differential equation. In order to obtain a more realistic physical meaning of this algorithm, we then implement it in a discrete time difference equation so that the algorithm is easier to compute. Thus (5) becomes

$$\boldsymbol{P}(k+1) = (\boldsymbol{I} - \beta \boldsymbol{B}) \boldsymbol{P}(k) + \beta \boldsymbol{\Gamma} \tag{6}$$

where

$$\boldsymbol{C} = (\boldsymbol{I} - \beta \boldsymbol{B}) = \begin{cases} 1 - \beta & i = j \\ \beta \frac{\gamma_i g_{ij}}{g_{ii}} & i \neq j \end{cases}$$

The algorithm will converge as long as the maximum eigenvalue (Spectral Radius) of $\boldsymbol{C}$ is strictly less than unity.

By setting $\beta = 1$ we obtain the well known simplified form of DPC algorithm in terms of current transmit power, SIR level and user target:

$$P_i(k+1) = \frac{\gamma_i}{R_i(k)} P_i(k) \tag{7}$$

Therefore each link measures autonomously its current SIR, $R_i(k)$, and tries to achieve its target γ_i in the next step, by boosting its power when the current SIR is below its target and lowering it otherwise. As a result, this is a distributed and adaptive power control algorithm.

B. The DPCH algorithm

The knowing weakness of a distributed control approach based on the proportional controller is the stability problem when the SIR target is high. The instability problem could be intuitively reflected as divergence of transmitter powers, or fluctuation of SIRs. It is important to find out elements that determine the convergence property so that an appropriate strategy can be made to tackle the stability problem.

Based on the above analysis we have made in DPC, we know that the Spectral Radius (SR) of matrix $\boldsymbol{C}$ determines the algorithm convergence property. Furthermore, since the SR has relationships with the row (column) sum of the matrix [5], once we increase the value of SIR target, the row (column) sum will increase, thereby causing the SR exceeding of the range. Hence we propose an improved algorithm specifically for the conditions where high SIR may be required, named as DPCH, which we hope the system will still be stable and with a proven convergence. The implication or trade-off corresponding to a high SIR with regarding to a convergence rate will also be dealt with. The improved algorithm can be expressed as follow:

$$\dot{r}_i(t) = \beta'(1 - \frac{r_i(t)}{\gamma_i}) \tag{8}$$

Comparing (8) with the basic DPC algorithm (4) reveals that the DPCH algorithm is simply obtained by dividing the right hand side of (4) by γ_i and hence the basic action (power control) of the new algorithm is similar, that is, when $r_i(t)$ is below its target γ_i, $\dot{r}_i(t) > 0$, and the transmit power is boosted to increase $r_i(t)$, otherwise, the transmit power will decrease to just meet the SIR target so as to minimise the power consumption.

Therefore, the discrete time version of the DPCH algorithm is

$$P_i(k+1) = (I - \beta' \mathrm{B}')P_i(k) + \beta' \Gamma' \tag{9}$$

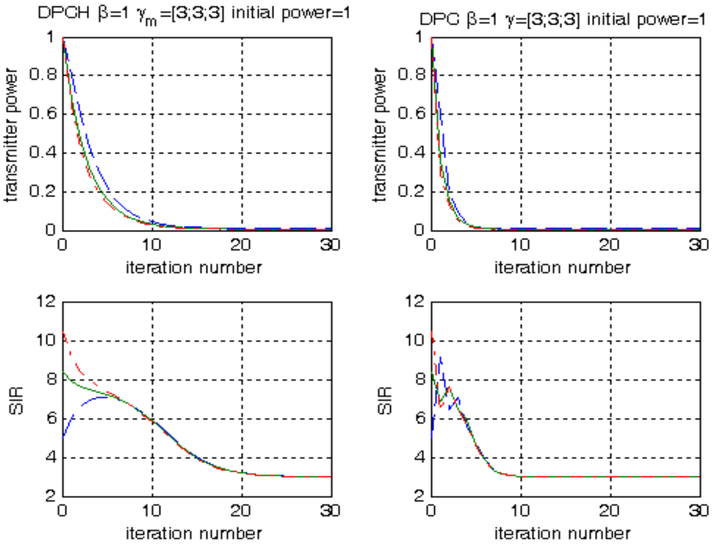
where

$$\boldsymbol{C'} = \boldsymbol{I} - \beta' \boldsymbol{B'} = \begin{cases} 1 - \dfrac{\beta'}{\gamma_i} & i = j \\ \beta' \dfrac{g_{ij}}{g_{ii}} & i \neq j \end{cases}$$

It is intuitive that although we set the SIR requirement, γ_i, in a high level, the matrix $\boldsymbol{C}$ ' can still guarantee the sum (column) row within unity. Therefore, the algorithm can still converge.

IV. NUMERICAL RESULTS

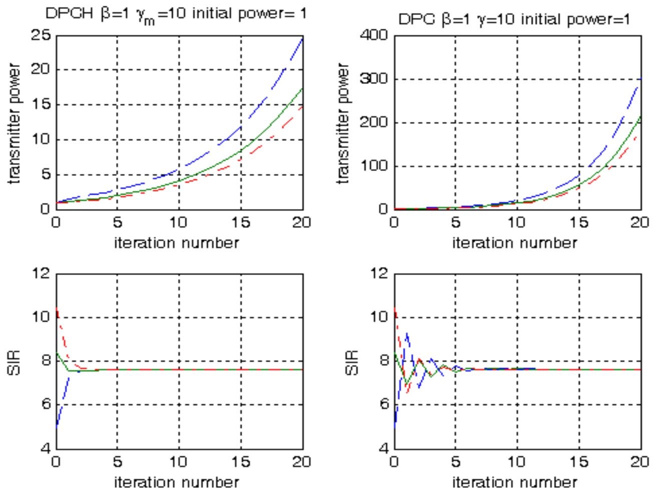
This section gives computation results and comparisons between DPC and DPCH. Figure 3 shows the general comparison between the two algorithms under normal (low) SIR environment for three mobile users. The fluctuations of SIRs in DPC are replaced by the monotonic convergence in DPCH. This means the system operated under DPCH will be more stable; links could evolve their SIR continually without dropping down. Although the convergence speed in DPCH is slightly slower than that of DPC, this can be adjusted by flexibly setting of β'.



(a) DPCH (b) DPC

Fig.3. Comparison of DPCH and DPC in general condition.

Figure 4 shows the network dynamics under high SIR requirements. Although the power level in both algorithms diverges to infinite, this could be avoided by setting a saturation threshold in real systems. The most important feature is that the power consumption has been significantly reduced in DPCH compared with DPC.



(a) DPCH (b) DPC

Fig.4. Comparison of DPCH and DPC under high SIR requirements.

Figure 5 shows the comparisons of both algorithms with the number of users increased in the systems. As expected, the increased number of links (from 3 to 7) does not affect the convergence property. However, due to the additional users, the interference among these users does increase, hence the transmission power will be boosted automatically to overcome the impact.

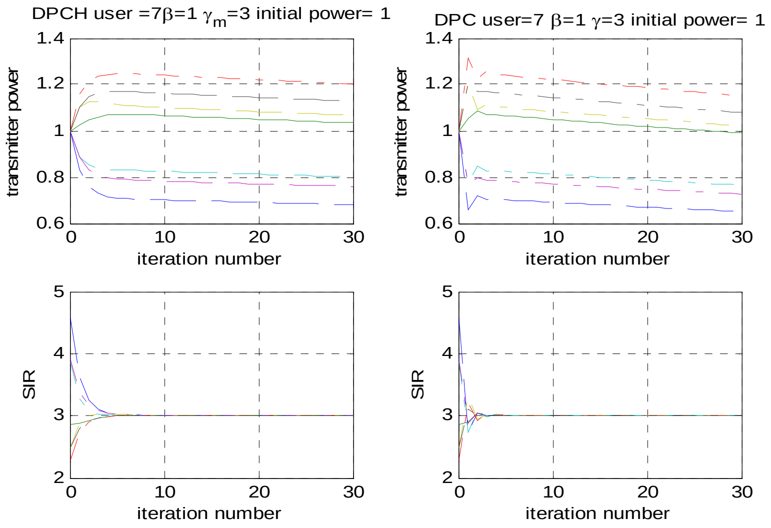


Fig.5. Comparisons of DPCH and DPC with increasing number of users.

As we can see from the simulations, DPC works well with low SIRs. The algorithm converges geometrically fast when the users have compatible SIR requirements. However, when we increase the link SIR, the algorithm causes a transmitter power increase and SIR fluctuations. This is due to the fact that for high SIRs, the row sums of the scaled normalised matrix has high probabilities to exceed one. This also means that the maximum eigenvalue exceeds unity. Furthermore, before the system converges, DPC algorithm may cause SIR fluctuations, which makes the system unstable. The convergence properties are mainly determined by propagation matrix. If the SR of the normalised propagation matrix is less than unity, the convergence of the algorithm is guaranteed. In theory, a good network should have a propagation matrix that the values of the off-diagonal elements are much less than those of the diagonal elements. In practise, such a network has low interference level, which results in the above mathematical property. We have also found that changing the initial power setting and increasing the number of users will not affect the convergence properties of the algorithm, as long as the transmission power is within the feasible power domain. Therefore, there is no need to increase the initial transmit power in order to attain a high SIR.

The improved algorithm is designed specifically for high SIR environment. From the matrix point of view, the probabilities of SR to exceed 1 is decreased. Hence the DPCH algorithm becomes easier to converge in high SIR environment, compared to DPC. As shown in Fig. 3-5, the DPCH algorithm has represented smoother curves on SIRs and transmission power along the iteration steps than that of DPC. There is no fluctuation before convergence. This means that the improved algorithm makes the system more stable. Another advantage of DPCH is its low power consumption in high SIR environment. It saves transmission power more than that of DPC. However, the trade-off is that the convergence rate of the DPCH is slower then that of the standard DPC algorithm for low SIRs.

V. CONCLUIONS AND FURTHER WORK

In this paper we have analysed the DPC algorithm and proposed the improved form of the algorithm when the SIR target is high. The standard DPC algorithm works well in the region of low SIRs. However, this approach has stability problems when the SIR target is high. The stability problems exhibit a transmitter power divergence and SIR fluctuations. In order to have a further proof on the stability problems of the DPC, we have also investigated network admission control using DPC [9]. However, the results are not included in this paper due to the length limit. Since the admission control is based on the DPC algorithm, thus it also suffers fluctuations before convergence, as with DPC. This is still under investigation although our initial finding of the stability problems for admission control with DPC is consistent with [10].

We have thus proposed the improved form of the distributed power control algorithm that is specifically for high SIR environment. We have proved in theory that DPCH can converge when the SIR target is high. In the computational experiments, there are no SIR fluctuations before the system converges. This means that the improved algorithm makes the system more stable. However, the convergence rate of the improved algorithm is slightly slower than that of the standard DPC algorithm when the SIR target is low. Further work is required in distributed power control for systems to have a wider range of the SIR target.

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