Low Sidelobe Difference Beams for Small Phased Arrays

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Abstract: Controllable and accurate difference beams are vital for the successful function of a tracking radar employing monopulse techniques to estimate target bearing. Traditional difference beam formation techniques that generate a continuous weight function break down when applied to small phased arrays due to insufficient sampling of the weight function. This paper presents some alternative techniques for forming far-field difference beams with equal level sidelobes for linear and planar phased arrays.

1 Introduction

Monopulse angle estimation techniques require the formation of sum and difference beams. An angle estimate is determined by dividing the difference beam by the sum beam. The ideal sum beam has a narrow mainlobe (for high angular accuracy) and no sidelobes (to stop returns from unwanted directions). This is not possible in practice, and a trade off exists between mainlobe width and sidelobe level. Low sidelobe sum beams for small phased arrays were examined by the author in [1]. Difference beams with low sidelobes are highly desirable, to minimise unwanted returns that would degrade the angular estimate of the target's location. Narrow mainlobes are also desirable, to increase angular accuracy. This paper examines techniques for forming difference beams with constant sidelobe levels for linear and planar arrays.

2 Difference Beams

Difference beams may be formed in a number of ways. The traditional continuous aperture methods of amplitude comparison and phase comparison are the most common. These techniques can be simulated for use in a fully digitised system with a phased array antenna, but the use of other techniques, such as the sampling of continuous weight functions are also possible.

When the phased array is small, care must be taken when sampling a continuous weight function. In a previous paper [1], the author reported the importance of treating each element of the phased array as being at the centre of a unit cell. This means, for a linear array of N elements equally spaced by d, the equivalent length aperture extends by a distance d/2 at each end of the array. This equivalent aperture length is the one that should be used when sampling against a continuous aperture weight function to obtain sidelobes closer to the desired level.

3 Linear Arrays

In this section on linear arrays, the difference beams that result from sampling a Bayliss continuous weight function are compared with a method for generating equal sidelobes.

3.1 Bayliss continuous line aperture weight function

In his 1968 paper [2], Bayliss concentrates on deriving the weight function for a continuous circular aperture that will yield a difference beam with all sidelobes below a predefined level. In the Appendix to the paper, he gives a similar analysis applied to a continuous line source.

The key parameters that are involved in deriving the weight function are the sidelobe level requested (SLR), and N, which gives the number of the null that defines the boundary between the central (approximately) constant sidelobe region and the far sidelobe region. Thus (N-1) sidelobes are held approximately constant.

The effect of sampling a continuous Bayliss weight function and applying the weights to an eleven element linear array is shown in Figure 1. The beams have been plotted with u on the x-axis, where $u = \sin(\theta)$, and θ is the angle measured from boresight to the look direction in the plane containing the array. The extent of real space is covered by $-1 \le u \le 1$: to examine the sidelobes that will appear when the beam is steered, we can extend the plot to $u = \pm 2$. That grating lobes are present at $u = \pm 1.8$ shows us that the element spacing is just over half a wavelength.

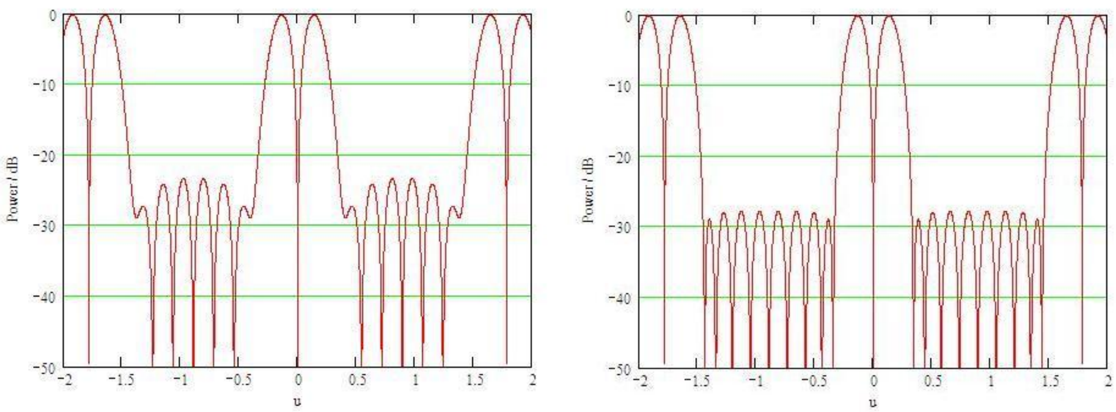

Figure 1: The two Bayliss difference beams that result from sampling a Bayliss weight function, SLR = -30 dB, N = 10 for an 11 element array, elements equally spaced by d, assuming the discrete array aperture is of length 10d on the left picture, and 11d on the right picture

The impact on sampling the weight function correctly is quite dramatic – the sidelobe level drops from -23 to -27 dB, and the widths of the mainlobes are decreased. However, the sidelobe level is not at the -30 dB level requested.

3.2 Zolotarev approximation difference beams

Price and Hyneman [3] demonstrated that linear array difference patterns with equal sidelobe levels are ideal – ie they display the narrowest main difference lobe width and the largest slope on boresight for a specified sidelobe level.

McNamara [4] subsequently realised that a class of functions known as Zolotarev polynomials did possess exactly the properties required for ideal linear array difference beam synthesis. They have characteristics similar to the Dolph-Chebyshev sum beam. Zolotarev was, in fact, Chebyshev's student and developed much of his tutor's work. Full analysis of the Zolotarev polynomial requires knowledge of elliptic integrals, Jacobi moduli, Jacobi eta, zeta and elliptic functions. The author has developed a close approximation to the Zolotarev function that is easily implemented for linear arrays.

The general form of a continuous aperture difference beam weight function g(x) can be written as follows, where x is the normalised distance from the centre of the linear array, ie $-1 \le x \le 1$:

g(x) = (antisymmetric function of x) * (sum beam type expression for x) (1)

As we are trying to create a difference beam with all sidelobes equal, the Dolph-Chebyshev [5] linear array constant sidelobe sum beam weights will be used in equation (1), as per Stegen's method of calculation [6].

For the antisymmetric function, two functions were initially trialled: x and sin(x*π/2). Interestingly, neither gave equal sidelobe levels; using x gave sidelobes that decayed to a constant level, and sin(x*π/2) gave sidelobes that rose to a constant level. The function required to make all sidelobes equal needs to be between these two, and via the Taylor expansion of sin(x) in x

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \tag{2}$$

such a function can be found. The first term in the expansion is x. By adding on the first extra term, suitably scaled, we can create the following antisymmetric function

$$x - a * \frac{x^3}{3!} \tag{3}$$

which, when used with a suitable value for a and used as the antisymmetric term in equation (1), gives an excellent approximation to the ideal Zolotarev difference beam, as shown in Figure 2:

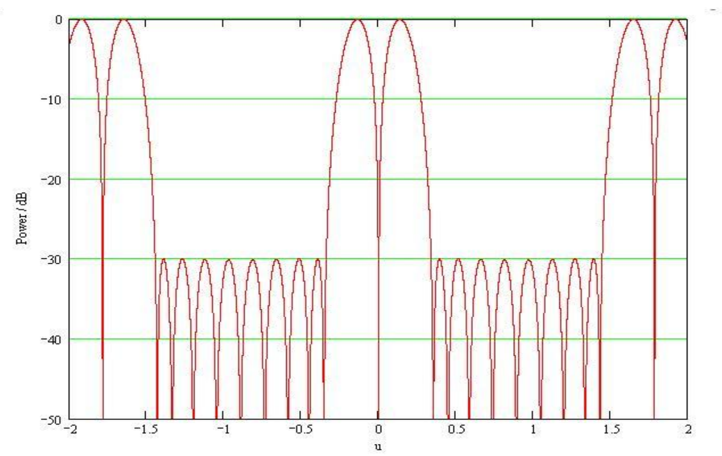

Figure 2: Zolotarev approximation difference beam, -30 dB sidelobes for an 11 element linear array

The value of a was 0.54, and the Dolph sum beam sidelobe level weights were generated for -39.5 dB. So this technique is successful in creating equal sidelobe difference beams, but is subject to a degree of trial and error in choosing the value of a and the associated sum beam sidelobe level.

4 Planar Arrays

Planar array difference beams with low sidelobes are usually formed by sampling the Bayliss continuous aperture weight function. This does not yield exactly the sidelobe level requested, so a technique that generates sidelobes below a desired level, and exact in one plane, is presented.

4.1 Difference beams with fixed sidelobe level in one plane

Consider the elevation difference beam for a nineteen element hexagonal array. The elevation plane of the far field elevation difference beam may be calculated by considering a linear array of five elements derived from the full array as shown in Figure 3:

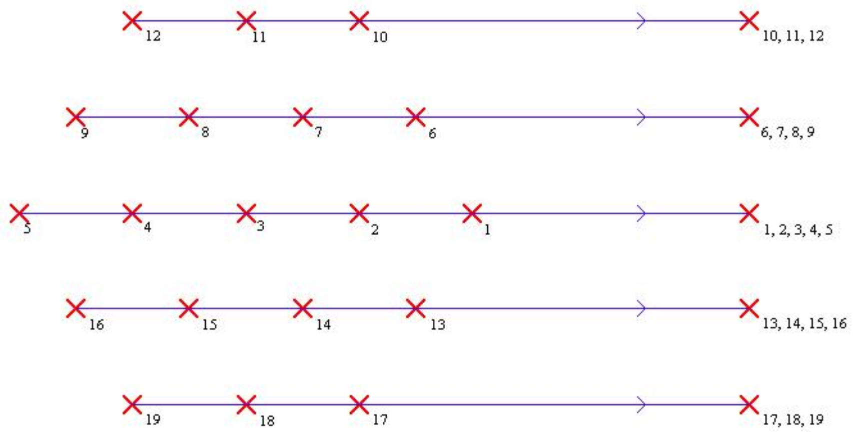

Figure 3: 19 element array condensed down to a 5 element array

The weight of a composite element in the five element array is given by the sum of the appropriate weights in the hexagonal array.

Using the technique from section 3.2, the weights can be calculated to form a five element linear array difference beam with equal sidelobes. Given this result, and using the symmetry of the weights for a difference beam, the weights for all elements in the hexagonal array can be calculated in terms of two parameters. Whatever the parameter values, the sidelobes in the elevation plane will be equal. Through adjustment of these parameters, approximately equal sidelobes everywhere else may be obtained. A similar approach may be employed for the azimuth difference beam, which results in a single parameter calculation.

Example beams produced using this method are shown in Figure 4. The sidelobe levels along the dotted line planes are shown. The aim of having no sidelobes above -30 dB has been achieved.

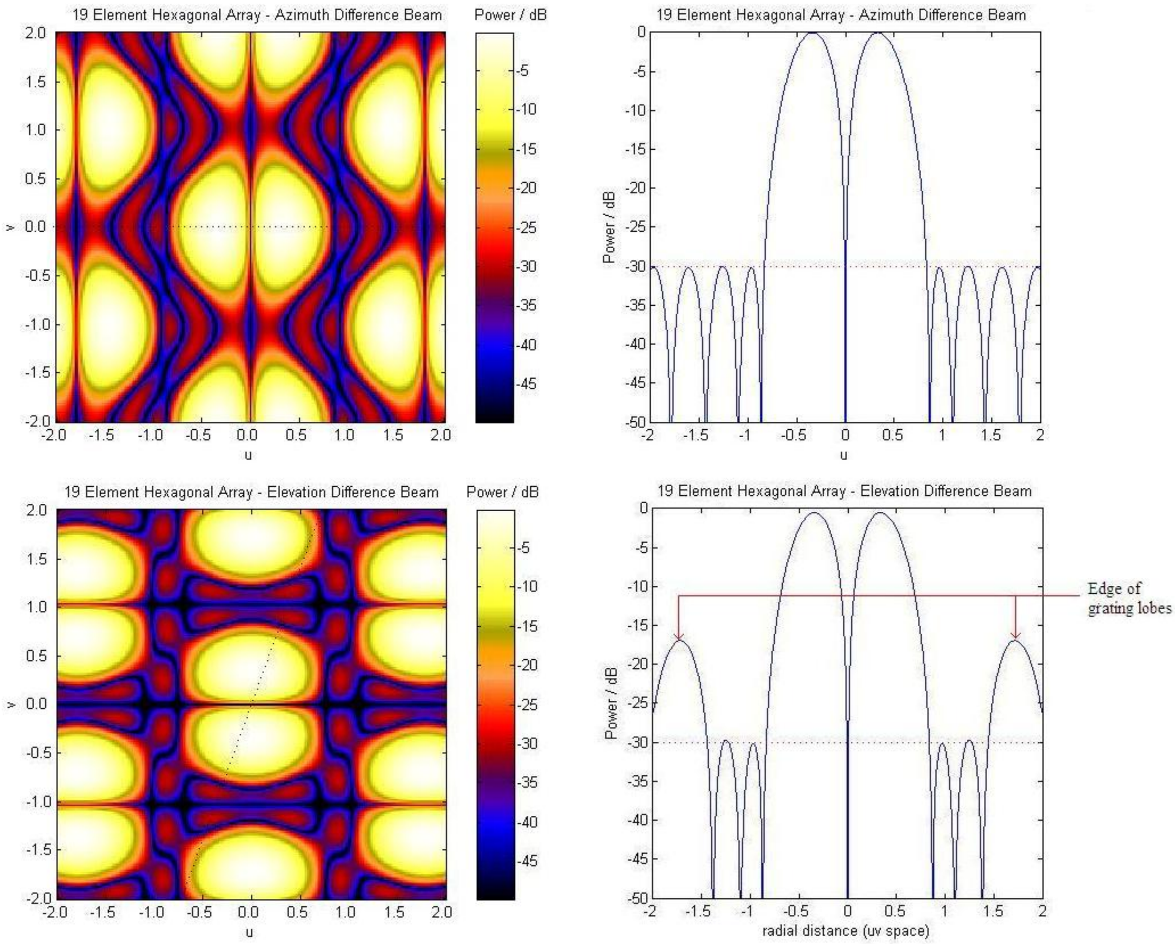

Figure 4: Azimuth and elevation difference beams with -30 dB sidelobes

5. Conclusions

A simple method for calculating equal sidelobe level difference beams for linear arrays has been presented. This technique may be applied to planar arrays to give equal sidelobes in one plane, and extended to adjust the beam so all sidelobes are below a desired level.

Acknowledgements

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