

Large Signal Analysis of Mach-Zehnder Modulator Intensity Response in a Linear Dispersive Fiber

J M B Oliveira^{1,2}, H M Salgado^{1,2} and M R D Rodrigues³

¹Instituto de Engenharia de Sistemas e Computadores do Porto, Porto, Portugal

²Faculdade de Engenharia da Universidade do Porto, Porto, Portugal

³Computer Laboratory, University of Cambridge, Cambridge, U.K.

Abstract: Two novel theoretical models that give the intensity of the optical signal generated by a Mach-Zehnder modulator through a dispersive medium are reported. These models include the chirp and dispersion factors and can be used for large-signals and for large values of the chirp parameters. Theoretical results are in good agreement with the simulation.

1 Introduction

The Dual-Electrode Mach-Zehnder Modulator (DE-MZM) is an external electro-optic modulator that is widely used in Radio over Fiber (RoF) systems to generate millimeter-waves (mm-waves). By varying the drive signals applied to the electrodes, i.e., adjusting the chirp (α parameter), it is possible to extend the fiber-link distance without dispersion compensation [1].

In this article, two novel expressions for the optical intensity at the output of the dispersion medium are presented. These expressions, that include the chirp and the dispersion factors, are compared with the complete model simulated in MatLab/Simulink and with the model proposed by Devaux *et al.* [2], that has been widely used.

2 Theory

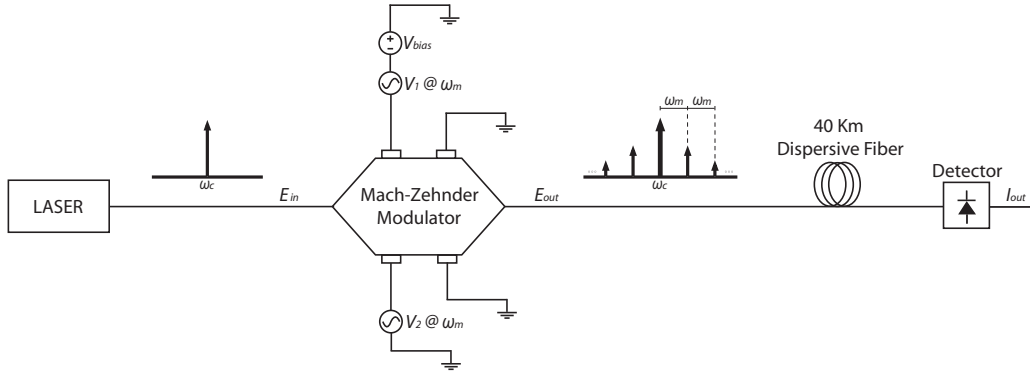


Figure 1: Scheme used for MatLab/Simulink simulation

Figure 1 represents the scheme used to mathematically analyse the effect of the dispersion in modulated optical signals. The electrical field of a lightwave at the output of a MZM is given by [1]

$$E_{OUT}(t) = \frac{1}{2} \left(e^{j\Delta\phi_1(t)} + e^{j\Delta\phi_2(t)} \right) e^{j\omega_c t} \quad (1)$$

with $\Delta\phi_1(t)$ and $\Delta\phi_2(t)$ being the phase changes in arm “1” and “2”, respectively, due to the Pockels effect originated by the electric fields applied to the electrodes; ω_c is the optical carrier angular frequency. These phase changes are given by

$$\Delta\phi_i(t) = \frac{\pi}{V_\pi} v_i(t), \quad i = 1, 2 \quad (2)$$

where $(\pi/V_\pi) = \eta$ is the index change per volt, V_π is the half-wave voltage, and v_1 and v_2 are the voltage signals applied to the arms of the MZM.

If we consider that the MZM is biased at *quadrature*, i.e., the operating point is at the center of the quasi-linear region of the MZM characteristic, offering a maximum signal excursion, then a bias voltage $V_b = (2n-1)V_\pi/2$, $n = 1, 2, 3 \dots$, must be applied between the arms. If we apply V_b at arm "1" of the MZM, sinusoidal signals with amplitudes V_1 and V_2 and modulation angular frequency ω_m at arms "1" and "2", respectively, then (2) is given by

$$\begin{cases} \Delta\phi_1(t) = \eta V_1 \sin(\omega_m t) - \frac{\pi}{2} \\ \Delta\phi_2(t) = \eta V_2 \sin(\omega_m t) \end{cases} \quad (3)$$

Equation (1) can be rewritten using the Euler's formula given by

$$e^{\Delta\phi_i(t)} = \cos(\Delta\phi_i(t)) + i \sin(\Delta\phi_i(t)), \quad i = 1, 2 \quad (4)$$

Substituting (3) in (4) a sum of terms is obtained that can be individually expanded in a series of Bessel functions. Thus, the electrical field at the MZM output given by (1) can now be expressed as a sum of Bessel functions series, i.e., a sum of multiple side-bands (SB) given by

$$\begin{aligned} E_{OUT}(t) = & \left[\frac{1}{2} (J_0(\eta V_2) - j J_0(\eta V_1)) \right. \\ & + \sum_{n=1}^{\infty} (J_{2n-1}(\eta V_1) + j J_{2n-1}(\eta V_2)) \sin((2n-1)\omega_m t) \\ & \left. + \sum_{n=1}^{\infty} (J_{2n}(\eta V_2) - j J_{2n}(\eta V_1)) \cos(2n\omega_m t) \right] e^{j\omega_c t} \end{aligned} \quad (5)$$

where $J_n(x)$ is the Bessel function of the first kind of order n . Note that the first two side-bands correspond to the lower and upper bands of a double side-band (DSB) signal.

From (5) it is possible to obtain expressions that give us approximations for the output electric field, depending on the number of side-bands considered. If we choose four side-bands to perform the mathematical analysis, (5) can be approximated by

$$E_{OUT,4SB}(t) \approx [E_{DC} + E_{\omega_m} \sin(\omega_m t) + E_{2\omega_m} \cos(2\omega_m t)] e^{j\omega_c t} \quad (6)$$

where

$$\begin{cases} E_{DC} = \frac{1}{2} [J_0(\eta V_2) - j J_0(\eta V_1)] \\ E_{\omega_m} = J_1(\eta V_1) + j J_1(\eta V_2) \\ E_{2\omega_m} = J_2(\eta V_2) - j J_2(\eta V_1) \end{cases} \quad (7)$$

When the electric field propagates through a dispersive medium, like an optical fiber, each component will propagate with different velocities, thus, suffering different phase shifts. The optical detector is a square-law device, meaning that its output is proportional to the optical intensity. The RF beat signal amplitude, at angular frequency ω_m , is proportional to the optical intensity and, from (6), is equal to

$$\begin{aligned} I_{\omega_m,4SB} = & \left| 2|E_{DC}||E_{\omega_m}| \cos\left(\theta_{E_{DC}} - \theta_{E_{\omega_m}} + \frac{1}{2}\beta_2\omega_m^2 L\right) \right. \\ & \left. - |E_{\omega_m}||E_{2\omega_m}| \cos\left(\theta_{E_{\omega_m}} - \theta_{E_{2\omega_m}} + \frac{3}{2}\beta_2\omega_m^2 L\right) \right| \end{aligned} \quad (8)$$

where θ_E is the phase of the electric field of each component, L is the fiber-link distance, and β_2 is the second-order Taylor series expansion of the propagation constant [3]. With this result we can perform a large-signal analysis of this type of systems.

Note that if we consider only the first term of (8),

$$I_{\omega_m, 2SB} = \left| 2|E_{DC}||E_{\omega_m}| \cos \left(\theta_{E_{DC}} - \theta_{E_{\omega_m}} + \frac{1}{2}\beta_2\omega_m^2 L \right) \right| \quad (9)$$

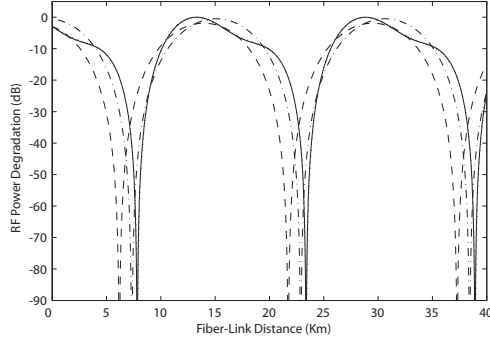
we obtain a new 2SB model. On the other hand, the conventional 2SB model proposed by Devaux *et al.* is based on a different two side-band electric field approximation and is given by [2]

$$I_{\omega_m} = I_0 m \sqrt{1 + \alpha^2} \left| \cos \left(-\frac{1}{2}\beta_2\omega_m^2 L + \arctan(\alpha) \right) \right| \quad (10)$$

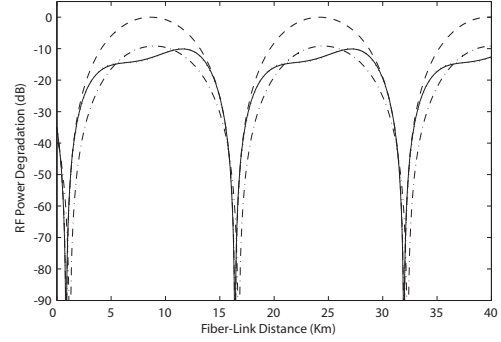
with $\alpha = (V_1 + V_2)/(V_1 - V_2)$ being the chirp parameter [4] and m the modulation depth. In the next section the advantages of each model and their range of validity are discussed.

3 Results and Discussion

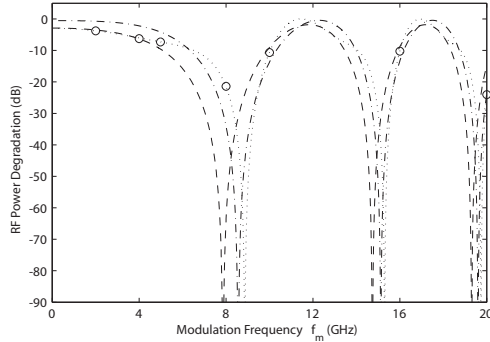
To validate and compare our two models with the one proposed by Devaux *et al.* we used MatLab/Simulink to numerically simulate the system represented in Figure 1. In this kind of analysis it is common to express the results in terms of RF beat signal power, which is proportional to the optical intensity squared. Using small input signals we observed that our two models are in agreement with the one proposed by Devaux *et al.* and with the numerical results for small and large chirp configurations.



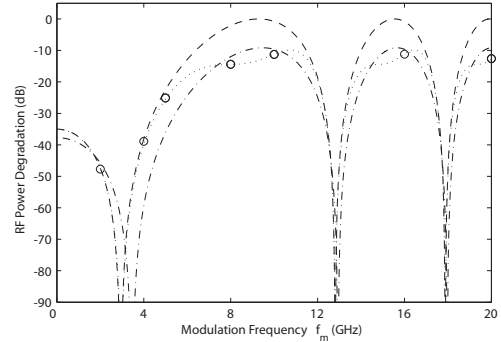
(a) *Small chirp configuration*: $V_1 = 2.0$ Volt; $V_2 = -1.0$ Volt; $V_\pi = 5$ Volt; $f_m = 20$ GHz



(b) *Large chirp configuration*: $V_1 = 2.0$ Volt; $V_2 = 1.4$ Volt; $V_\pi = 5$ Volt; $f_m = 20$ GHz



(c) Power degradation as a function of modulation frequency for the *small chirp configuration* and $L = 40$ Km



(d) Power degradation as a function of modulation frequency for the *large chirp configuration* and $L = 40$ Km

Figure 2: Power degradation as a function of fiber-link distance and modulation frequency for large values of the modulating signal: ‘———’, and ‘o’ numerical simulation; ‘.....’ 4SB model; ‘- · - · -’ 2SB model; ‘- - -’ Devaux *et al.* model.

The results for large signals and for the different models, will now be compared and discussed.

3.1 Large Signal Results

Figures 2(a) and 2(b) show the predicted RF beat signal power degradation due to dispersion as a function of fiber-link distance for small and large chirp configurations, respectively. Figures 2(c) and 2(d) show the predicted RF beat signal power degradation due to dispersion as a function of modulation frequency for small and large chirp configurations, respectively. The new 4SB model perfectly matches simulation and can be used to predict the behavior of the RF beat signal power for small and large signals and for all values of the chirp parameter.

In Figures 2(a) and 2(b) the solid line and dotted line, corresponding to the simulation and 4SB model are in excellent agreement as they are almost indistinguishable. In Figures 2(c) and 2(d) the numerical simulation were obtained only at specific points (dots). As in the previous case the 4SB results match very well the simulation results.

Another interesting result is the new 2SB model behavior. As we can see, the new and simple 2SB model still gives a good approximation for small and large chirp configurations and for a large range of modulation frequency. Like the one proposed by Devaux *et al.*, this model is conducted by a simple cosine law, but in general produces better results in particular for predicting the “null-intensity”. The results for negative chirp values are similar.

4 Conclusions

An analytical model to predict the RF beat signal power degradation through a dispersive fiber, considering an electrical field of four side-bands at the MZM output, was presented and simulated, for the first time. This model is valid for small and large signals driving the modulator as well as for large values of the chirp parameter and a large range of modulation frequencies. We also presented an alternative model based on the two side-bands electric field approximation, at the MZM output, which still gives good results (in both regimes) with added simplicity. These methods may be applied for measuring accurately the chirp parameter of a MZM and to predict more precisely the behavior of RF beat signal power degradation due to dispersion and large signals configurations.

Acknowledgments

This work was supported in part by “Fundação para a Ciência e a Tecnologia” (FCT) under the programme “Programa Operacional Ciência Tecnologia e Inovação” - POCTI/FEDER with grant REEQ/1272/EEI/2005 - Fibre Optic Supported Broadband Communication Networks. J. M. B. Oliveira also acknowledges support from FCT through a PhD grant.

References

- [1] H. Kim and A. H. Gnauck, “Chirp characteristics of dual-drive, Mach-Zehnder modulator with a finite DC extinction ratio,” *IEEE Photonics Technology Letters*, vol. 12, pp. 298-300, March 2002.
- [2] F. Devaux, Y. Sorel and J. F. Kerdiles, “Simple measurement of fiber dispersion and of chirp parameter of intensity modulated light emitter,” *Journal of Lightwave Technology*, vol. 11, pp. 1937-1940, December 1993.
- [3] G. P. Agrawal, *Nonlinear Fiber Optics*, 3rd edition, Academic Press, 2001.
- [4] G. H. Smith, D. Novak and Z. Ahmed, “Overcoming chromatic-dispersion effects in fiber-wireless systems incorporating external modulators,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 45, pp. 1410-1415, August 1997.