Performance of Block Turbo Coded OFDM with 16-QAM Modulation in Rayleigh Fading Channel

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Abstract: In this contribution the performance of Reed-Solomon (RS) Block Turbo Coding (BTC) is studied in a 16-QAM modulated OFDM system when operating on a Rayleigh fading channel. BER and PER performances are shown and compared to the performance of Convolutional Turbo Coding (CTC). BTC proves to mitigate the irreducible error floor that limits the performance of CTC at high signal-to-noise ratios (SNR).

1. Introduction

Coded OFDM has been the core technology in the physical layer of many wireless communication standards, including WLAN standards such as IEEE802.11g and HIPERLAN/2, as well as digital broadcasting systems such as Terrestrial Digital Video Broadcasting (DVB-T) [1]. It is also a very powerful candidate for fourth-generation mobile communications systems either by itself or by employing the OFDM principles in other solutions such as Multicarrier CDMA (MC-CDMA) or Multicarrier DS-CDMA (MC-DS-CDMA). Simplified equalisation, mitigation of ISI and ICI as well as the exploitation of the frequency-domain diversity are among other advantages of using OFDM.

On the other hand, Turbo codes have been extensively studied and employed since they were reintroduced by Berrou in 1993 [2] following the spirit of Bahl's work in 1973 [3]. These codes extract some extrinsic information in the decoding process; this information is then used in subsequent decoding iterations to improve the decoder performance. Different methods have been proposed in the literature to apply the concept to both convolutional and block codes, leading to the terms Convolutional Turbo Codes (CTC) and Block Turbo Codes (BTC), respectively.

Among the BTCs, the one that is probably of most interest is that introduced by Pyndiah [4]. Pyndiah's work is based on a simplified Chase algorithm [5] applied to product codes. They were shown to reach a capacity efficiency of as high as 98% in Gaussian channels using high code rates.

In this paper, the latter BTC is applied to a general OFDM system with a Rayleigh fading channel, and its performance is compared with that of CTC for QAM-modulated transmission.

The rest of the paper is organised as follows: section 2 describes the OFDM system model used to obtain the performance results presented in the paper. Section 3 follows with a description of the BTC scheme under study. The computer simulation results are shown and discussed in section 4, while section 5 concludes the paper.

2. OFDM system model.

At the transmitter side, N symbols each representing m coded bits are mapped by an m-ary mapper and the output symbols are multiplexed into N parallel branches and modulated each by a subcarrier through the normal OFDM modulation (IFFT). The transmitter output consists of the superposition of N signals in the time domain.

At the receiver, the received signal of a generic subcarrier after the FFT stage can be written as:

$$r(n) = h(n)e(n) + w(n) \tag{1}$$

Where $r(n)$, $e(n)$, $h(n)$ and $w(n)$ are the received signal, transmitted signal, complex flat-fading channel response and additive white Gaussian noise (AWGN) all at subcarrier (n), where $n = 1,2,\ldots N$, respectively. The channel is assumed to be perfectly known at all subcarrier positions. The data recover process involves equalisation, de-mapping and decoding of the received signal. In this paper, the encoder and decoder are based on either a CTC or a BTC, as described in the following section.

3. Block Turbo Coding (BTC).

As mentioned earlier, the BTC being considered is a Reed-Slomon (RS) product code. A RS code is a BCH code with non-binary elements belonging to the Galois Field GF($q = 2^m$) which represent m binary elements or bits. An RS code is defined with the parameters (n, k, δ), where n is the code word length, k the number of information symbols, and δ its minimum Hamming distance.

In a product code two linear block codes $\varphi^1(n_1, k_1, \delta_1)$ and $\varphi^2(n_2, k_2, \delta_2)$ are used. The information symbols are arranged in a $(k_2 \times k_1)$ array. The k_2 rows are coded using φ^1 and the resulting n_1 columns are coded each using φ^2 [7]. In general the output of the encoder is broken down into its binary elements followed by interleaving (if any) and modulation mapping. In our simulation we have the special case that the code order m is equal to the *m*-ary order of the QAM constellation and thus each RS code symbol maps directly on to one corresponding point in the QAM constellation.

At the receiver, the input representing one word (row or column) to the BTC decoder can be written as:

$$R = H \cdot E + N \tag{2}$$

where

$$R = \begin{bmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{bmatrix} \tag{3}$$

is the received word, and n can be either n_1 or n_2,

$$E = \begin{bmatrix} e_{11} & \cdots & e_{1n} \\ \vdots & \ddots & \vdots \\ e_{m1} & \cdots & e_{mn} \end{bmatrix} \tag{4}$$

is the transmitted code word, and

$$H = \begin{bmatrix} h_{11} & \cdots & h_{1n} \\ \vdots & \ddots & \vdots \\ h_{m1} & \cdots & h_{mn} \end{bmatrix} \tag{5}$$

is the frequency domain representation of the Rayleigh fading channel at the subcarrier on which the OFDM symbol is placed. In general, $h_{ji},\ j = 1,2...m$ will have different values, however in our arrangement each column of H will have elements of the same value. N is the additive white Gaussian noise (AWGN) with standard deviation σ.

The optimum decoding method for the RS code is the maximum likelihood (ML) rule given by

$$D = C^i \text{ if } \Pr(E = C^i | R) > \Pr(E = C^l | R)\ \forall l \neq i \tag{7}$$

where

$$C^i = \begin{bmatrix} c^i_{11} & \cdots & c^i_{1j} & \cdots & c^i_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c^i_{m1} & \cdots & c^i_{mj} & \cdots & c^i_{mn} \end{bmatrix} \tag{8}$$

is the i^{th} code word of code φ (φ^1 or φ^2), and

$$D = \begin{bmatrix} d_{11} & \cdots & d_{1j} & \cdots & d_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ d_{m1} & \cdots & d_{mj} & \cdots & d_{mn} \end{bmatrix} \tag{9}$$

is the decision made based on R. Equation (7) can be rewritten using the Euclidean distance metric between R and the code word members of code φ as a measure of probability. That is:

$$D = C^i \text{ if } \left|R - C^i\right|^2 < \left|R - C^l\right|^2\ \ \forall l \neq i \tag{10}$$

where $$\left|R - C^i\right|^2 = \sum_{j=1}^{n}\sum_{f=1}^{m}(r_{jf} - c_{jf}^i)^2 \tag{11}$$

In a ML decoder, the search for the nearest code among all the q^k possible codewords in φ introduces prohibitive complexity to the system implementation. Driven by the aim of reducing the complexity of the ML decoder, the Chase algorithm was introduced [5] as a flexible decoding criterion that takes advantage of the low complexity of algebraic decoders and the optimal performance of the ML decoder. The idea behind it is to extend the correction capability of the conventional algebraic decoder by increasing the viewing range of the decoder using the following steps:

1. Decode the hard decision of R (known as Y^0) using an algebraic decoder to get C^0.
2. Find the p least reliable binary symbols in R and mask them (flip their values from (+1 to -1 and vice versa) to obtain a set of $2^p - 1$ new words Y^l, $l = 1,2,\ldots 2^p - 1$.
3. Decode each word in Y^l to get a subset C^l of candidate code words (instead of the single codeword identified by a conventional decoder).
4. The Euclidean distance metric is then used to find the nearest code word in C^l to the received word R; the decision D will also be called $C^{\min(i)}$.

The next step is to extract the extrinsic information to update the soft input for the following decoding iteration, which is the heart of the turbo concept. To achieve this, first the reliability of each decoded bit is calculated using the Log Likelihood Ratio (LLR) of each element of D based on R, defined by:

$$LLR_{jf} = \ln\frac{\Pr\{e_{jf} = +1/R\}}{\Pr\{e_{jf} = -1/R\}} \tag{12}$$

which was shown in [6] to simplify after expansion and normalisation and approximation to:

$$r'_{jf} = \frac{\sigma^2}{2}LLR_{jf} = r_{jf} + w_{jf} \tag{13}$$

where w is the extrinsic information needed to update the soft input going into the BTC decoder in the next decoding iteration.

The value r' has been given in [6] as:

$$r'_{jf} = \frac{M^{\min(-i)} - M^{\min(i)}}{4}\cdot c_{jf}^{\min(i)} \tag{14}$$

where for each symbol in the code word $C^{\min(i)}$, $M^{\min(i)}$ represents the Euclidean distance between R and $C^{\min(i)}$ i.e. D, and $M^{\min(-i)}$ represents the Euclidean distance between R and the code word $C^{\min(-i)}$, which is the next closest code word in subset C^l when its value at position jf is different from that in codeword $C^{\min(i)}$. If $C^{\min(-i)}$ cannot be found, then r'_{jf} is defined as:

$$r'_{jf} = \beta\cdot c_{jf}^{\min(i)} \tag{15}$$

where β is a weighting factor that can either be set as an increasing constant or approximated as the following LLR [4]:

$$\beta \approx \ln\left(\frac{\Pr\{d_j = e_j\}}{\Pr\{d_j \neq e_j\}}\right) \tag{16}$$

After obtaining r', w is calculated, and from there R can be updated using the equation:

$$R(g) = R + \alpha(g) \cdot w(g) \tag{17}$$

Where g is the index of the decoding iteration, and α is a weighting factor that is meant to reduce the dependency on w at early stages of the decoding process when its values are not reliable enough to make decisions.

4. Simulation results

The BTC performance was evaluated by the method of computer simulation for an OFDM system affected by frequency selective fading. The Code used is the RS(15,11,2) code. The modulation scheme is 16-QAM and the frequency selective channel is modelled by i.d.d. Rayleigh flat-faded channel taps per OFDM subcarrier. The guard interval is assumed to be longer than the delay spread of the channel. The results are compared with the performance of a rate ½ CTC code operating in the same channel conditions.

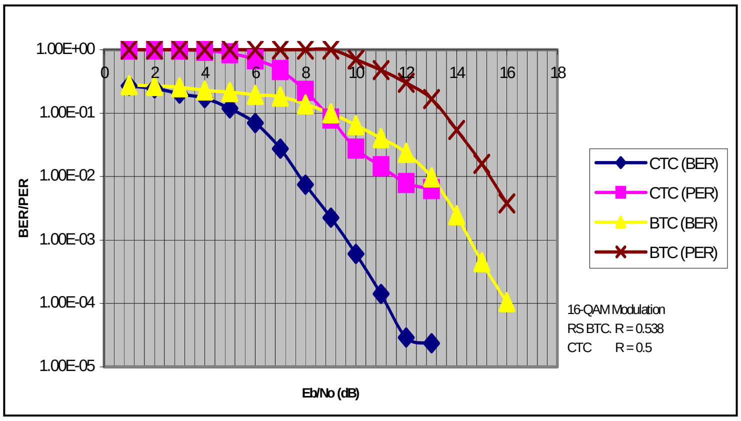

Fig. 1. BER/PER performance of BTC vs. CTC in 16-QAM modulated OFDM system with Rayleigh fading channel (4 iterations)

The BER and PER performances are depicted in Figure (1). From the results it can be seen that the BTC exhibits a poor system performance compared to CTC at low values of Eb/No. However, as Eb/No increases the CTC performance reaches an irreducible error floor while the BTC performance continues to improve. The BTC outcome is advantageous for services that require very low packet error rates such as real time video streaming. Providing sufficient SNR is available then a target quality of service can be reached as desired.

7. Conclusions.

In this work the performance of BTC was studied for an OFDM system with 16-QAM modulation and a Rayleigh frequency selective channel. The system BER and PER performances for BTC were compared with those for CTC. It was shown that while BTC requires higher SNR levels before reaching lower BER and PER levels, it does not suffer from an irreducible error floor as in the case of CTC. This property of BTCs in OFDM make them suitable candidates for applications that require very low PERs such an real time video streaming.

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