Structured Parallel Concatenated LDPC Codes

J. C. Serrato†and T. O'Farrell‡

†University of Leeds

Abstract: The concatenation of two or more Low Density Parity Check (LDPC) codes has proved to be an effective way to improve the bit error rate (BER) performance by increasing the redundancy whilst reducing the decoding complexity when compared to a LDPC of the same length. The concatenation of two LDPC codes with different Mean Column Weights (MCWs) through a bipartite graph in the decoder, allows the use of the Maximum A-Posteriori (MAP) decoding algorithm to obtain a better performance than that of the individual codes. This paper shows the performance of two concatenated LDPC codes obtained through well structured procedures with MCW1=2 and MCW2=3. The results are compared with a previous work considering two randomly created LDPC codes with MCW1=2 and MCW2=2.66 and approximately the same length. As all the individual LDPC codes are of rate 1/2, the concatenation results in a 1/3 rate code.

1 Introduction

H. Behairy and S. Chang presented in [1] a procedure to concatenate two LDPC codes, using the turbo principle. They concluded that several benefits could be obtained from considering this approach, such as, less complexity, less delay, less memory required and a good BER performance when compared to a single LDPC code with the same length, simply because the individual LDPC codes are shorter. To implement the concatenation no interleaver between the codes had to be used, as the performance is the same when considered.

To test their procedure, two half rate randomly created LDPC codes with different MCW were proposed, and concluded that the concatenation provided the best overall BER performance compared to the BER performances of the individual codes. The MCWs considered in the previous work are 2 and 2.66, in order to have a low MCW code to perform better in the low EbNo region, and a high MCW code to perform better in the high EbNo region.

Structured LDPC codes have been recently under development [2]. The main goal of these construction methods is to design LDPC codes with a girth as large as possible, as a function of some variables such as the length, MCW and rate. This paper analyses the performance of two structured, half rate, parallel concatenated LDPC codes, based on Cayley graphs and Graphical models.

2 Half rate (n,2,k) LDPC codes with girth 16.

Haotian Zhang and Jos M. F. Mora describe the construction of a class of (n,2,k) LDPC codes with girth 16 and code rate 1/2, based on a cylinder structure [3,4]. Assume the number of parity check equations $v = 8p$, and divide these points into 8 subsets of equal size, where points in each subset are aligned in a vertical line. These subsets $X_0, X_1, , X_7$, comprise a loop, and each point within subset X_k can only connect to points in the previous or next subset, and cannot connect to points in the same subset. The overall structure looks like a cylinder.

A cycle is formed either by consecutive lines passing all the 8 subsets around the cylinder structure, or within points in some of the 8 subsets. To achieve LDPC codes with girth 16, no cycles with less than 8 points on the cylinder structure should take place. To create the LDPC code, two edges (lines) should connect each single point in the subset X_k, with two different points in the next subset X_{k+1}. All the edges between two neighbouring subsets X_k and X_{k+1} in a cylinder structure compose a section S_i. The slope s is the edge connecting the point a_i in the subset X_k with the point b_j in the subset X_{k+1}. An Admissible Slope Pair (ASP) contains the pair of slopes that connect all the points in a section, and that has fullfilled the necessary conditions to avoid cycles shorter than 8 considering the previous ASPs assigned. To create a LDPC code of rate 1/2, each section should be assigned two different ASPs.

The corresponding H matrix is well structured and completely determined by p and the Admissible slope pairs $ASP_0,, ASP_7$. The codes used for this paper have the following characteristics:

- (240,2,120) p=15 and ASP={(0,0)(+2,-13)} {(0,0)(+3,-12)} {(0,0)(+4,-11)} {(0,0)(+6,-9)} {(0,0)(+1,-14)} {(0,0)(+3,-12)} {(0,0)(+7,-8)} {(0,0)(+6,-9)}
- (672,2,336) p=42 and ASP={(0,0)(+2,-40)} {(0,0)(+3,-39)} {(0,0)(+4,-38)} {(0,0)(+5,-37)} {(0,0)(+2,-40)} {(0,0)(+6,-36)} {(0,0)(+1,-41)} {(0,0)(+8,-34)}

3 Half rate (n,3,k) LDPC codes.

G.A. Margulis describes the construction of a class of (n,3,k) LDPC codes with code rate 1/2, based on Cayley graphs [5,6]. For this code consider a finite group G. Let $A \subset G$ be a subset of G satisfying $A = A^{-1}$. The Cayley graph $X(G, A)$ is the graph having as vertices the elements $g \in G$. The vertices $g, h \in G$ are connected by an edge whenever there is an $a \in A$ such that $h = ga$.

Let q be an odd prime. Let F_q be a finite field of q elements and consider the Special Linear Group over F_q ($SL_2(F_q)$) as the set consisting of all 2 x 2 matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

with entries in F_q and having determinant $ad - bc = 1$. One readily verifies that $SL_2(F_q)$ is a group of order $q^3 - q$.

Consider the subset $A := \left\{ A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, A^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \right\}$.

Now, let $G = SL_2(F_q)$ and let A be as above. Then the Cayley graph $X(G, A)$ is a 4-regular graph with $q^3 - q$ vertices. From this graph, it is possible to construct a (3,6) regular LDPC code. As left vertices two copies of G are taken (i.e. G and $\tilde{G}$). The right vertices of the bipartite graph will consist of the set G. An element $g \in G$ on the left will be connected with the right vertices gA^2,$gABA^{-1}$,gB. An element $\tilde{g} \in \tilde{G}$ on the left will be connected with the right vertices $\tilde{g}A^{-2}$,$\tilde{g}AB^{-1}A^{-1}$,$\tilde{g}B^{-1}$.

The bipartite graph described above describes a (3,6) regular LDPC code of block length $2(q^3-q)$ and girth $c \geq log_{(1+\sqrt{2})}\left(\frac{q}{2}\right)$. The corresponding H matrix is well structured and completely determined by q and the bipartite graph. The codes used for this paper have the following characteristics:

- (240,3,120) q=5
- (672,3,336) q=7

4 Parallel Concatenation.

The encoder requires the two different generator matrices to generate systematic codewords in order to send the information bits just once, together with the redundancy obtained from each generator matrix.

The decoder follows the turbo decoding principle without the use of an interleaver. Each LDPC decoder computes the a posteriori probability as described in [7] using the sum-product algorithm with modifications to accommodate the a priori information. The first decoder computes the a posteriori probability using the received sequence with no a priori information. The process of exchanging information between decoders continues until one or both decoders converge to a valid codeword, or the maximum allowed number of iterations is reached. LDPC codes are defined by a bipartite graph; therefore, the concatenation of two LDPC codes can also be defined by a bipartite graph in such a way that the variables corresponding to the information bits and the parity bits are related to the checks in both codes.

5 Simulation Results.

The characteristics of the structured LDPC codes considered in the investigation mentioned previously, belong to a category of the best designs known so far with MCWs of 2 and 3 and rate=1/2, in terms of BER performance. Fig. 2 and Fig. 3 show the BER performance of two structured versus two random parallel concatenated LDPC codes under AWGN channel conditions, respectively. The average MCW of the proposed structured parallel concatenated LDPC code is $\frac{MCW1+MCW2}{2} = \frac{2+3}{2} = 2.5$. The additional redundancy is only improving the BER performance at high Eb/No values and actually performs worse than the individual codes at low Eb/No values. The performance of the structured parallel concatenated LDPC codes is similar to that of the random codes.

6 Conclusions.

The BER performance of the parallel concatenated approach is worse than the performance of the individual codes at low Eb/No values; therefore, it is recommended to assign thresholds, in order to assign more iterations to the MCW=2 decoder if the Eb/No value of the received signal is low, to assign more iterations to the MCW=3 decoder if the Eb/No value of the received signal is medium and to assign an even number of iterations to both decoders if the Eb/No value of the received signal is high. As the length of the individual codes is increased, the Eb/No value at which they will be outperformed by the parallel concatenated code is increased and therefore, it is less attractive to use this approach. It can be concluded that for short length, low complexity, small delay, and high Eb/No values, this approach is an attractive option and could replace a single LDPC code of the same length.

References.

[1] H. Behairy and S. Chang, "Parallel concatenated Gallager codes," Electronic Letters, vol. 36, issue 24, 23 Nov 2000, pp. 2025-2026.

[2] J.M.F. Moura, Jin Lu, H. Zhang, "Structured low-density parity-check codes," IEEE Signal Processing Magazine, vol. 21, issue 1, Jan 2004, pp. 42-55.

[3] H. Zhang and J.M.F. Moura, "The design of structured regular LDPC codes with large girth," in IEEE Globecom 2003, San Francisco, CA, Dec 2003.

[4] H. Zhang and J.M.F. Moura,"Large girth LDPC codes based on graphical models,"in Proc.IEEE Signal Processing and Wireless Communications(SPAWC) Workshop,Rome,Italy,June 2003.

[5] G. A. Margulis, "Explicit constructions of graphs without short cycles and low density codes," Combinatorica, vol. 2, no. 1, pp. 71-78, 1982.

[6] J. Rosenthal and P. O. Vontobel, "Constructions of LDPC codes using Ramanujan graphs and ideas from Margulis," in Proc. 38th Allerton Conf. Communication, Control, and Computing, Monticello, IL, Oct. 2000, pp. 248-257.

[7] F.R. Kschischang and B.J.Frey, "Iterative decoding of compound codes by probability propagation in graphical models," IEEE J. Select. Areas Commun.,vol. 16,no. 2,pp. 219-230,Feb.1998.

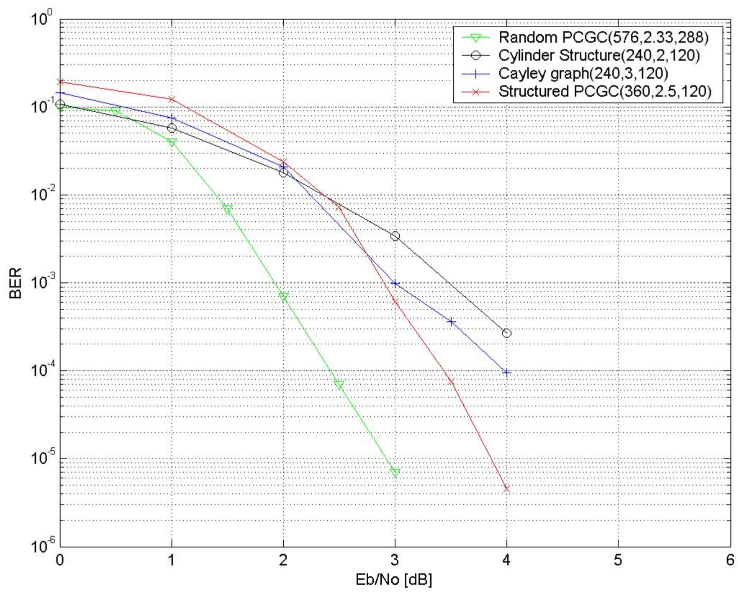

Figure 1: Structured (360,2.5,120) vs Random (576,2.3,192) Parallel Concatenated LDPC code + BPSK + AWGN

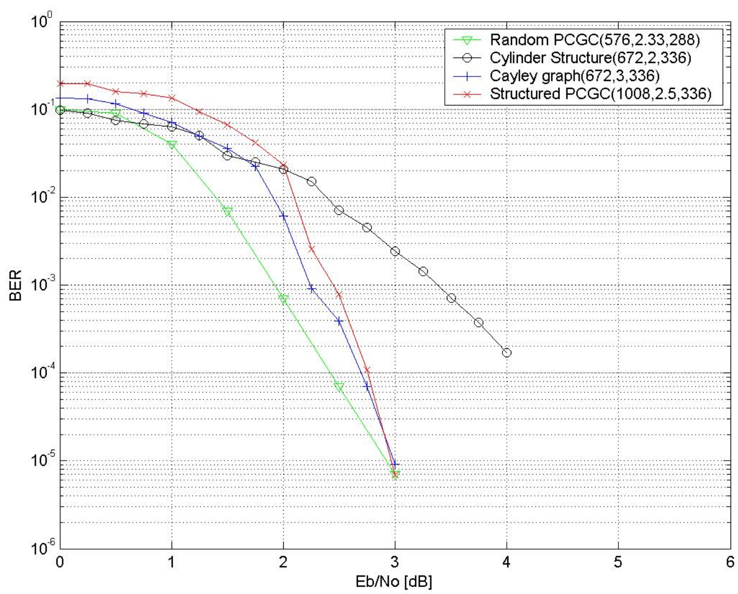

Figure 2: Structured (1008,2.5,336) vs Random (576,2.3,192) Parallel Concatenated LDPC code + BPSK + AWGN