Multiple Target Localisation Using Time of Arrival Information From Multistatic Radar

M S Cole† and I K Proudler†

† QinetiQ

Abstract: Time-of-arrival localisation is a technique common to a variety of sensors and scenarios, including multistatic radar. In this paper a single target localisation algorithm is presented that meets the Cramer-Rao lower bound. The issues that arise when there are multiple targets are discussed and an algorithm is introduced that successfully performs multiple target localisation. This algorithm uses only one set of measurements but is shown to be robust to significant levels of noise.

1. Introduction

Multisite radar systems in general and multistatic radar in particular can perform information fusion at many levels. Time-of-arrival (TOA) information fusion, where the time of a pulse to travel the round trip from the transmitter to the target to a receiver is used, is one of the lowest fusion levels. Although there is performance degradation when compared with radio signal fusion, the advantage is reduced cost, a lower demand on the communications channel and a simpler system [1]; making it suitable for systems with large baselines.

Using a monostatic radar for long range detection provides a poor instantaneous positioning performance at detection ranges: although the down-range accuracy is only tens of metres, the cross-range accuracy is of the order of tens of kilometres.

A multistatic radar system can utilise the good down-range accuracy to improve positioning performance and the long baselines required to protect a large area suggest TOA information fusion is suitable. The system envisaged in this paper performs target detection at the receivers and the fusion of the TOA information at a central processing station.

TOA systems are not unique to radar, similar systems can be found in sonar [2] and acoustics [3] and there are also mathematical similarities with other systems [4]: time-difference-of-arrival (TDOA) or multilateration systems; and satellite positioning systems such as GPS. The issue of multiple targets is not widely considered in passive systems, such as GPS or TDOA systems, as target identification information is often encoded into the signal. For example, the aircraft number is transmitted in airport multilateration, a mobile phone user identification is encoded into the signal. With active systems, required for non-cooperative targets, the association between the measurements and the targets is unknown (see section 3 for further details), and is a non-trivial problem to solve.

A naïve approach to the association problem would be to calculate all possible targets from all possible measurement associations, unfortunately this generates a lot of potential targets. There are techniques that reduce the number of potential targets, such as [5] but this technique uses a suboptimal method of data fusion. Data association can be performed using multiple snapshots [6], i.e. using a tracker, to eliminate incorrect, ghost, targets. This approach relies upon knowledge of the properties of the target, such as its allowable speed or acceleration, which can restrict the usability of the system.

The work presented in this paper however presents a single snapshot solution. In order to eliminate ghost targets, the algorithm utilises knowledge of the system geometry and its noise levels, information that is known, or at least determined, during system construction and operation.

The remainder of the paper is structured as follows. Section 2 discusses the single target positioning problem and section 3 presents an overview of the multiple target algorithm. Finally, in section 4 conclusions are drawn.

2. Single Target Localisation

Whether the scenario consists of multiple targets or just one, at some point it is necessary to determine accurately and efficiently the position of a target from a set of measurements. The target position can be found with a multistatic system through the use of the bistatic range equation. For a target positioned at $\mathbf{x}$, the bistatic range, r_k, is given by

$$r_k = r + d_k, \tag{1}$$

where r is the range from the transmitter to the target and d_k is the distance from the target to the k^{th} receiver. Without loss of generality, the transmitter is assumed to be at the origin.

As $r = \|\mathbf{x}\|$ and $d_k = \|\mathbf{x}_k - \mathbf{x}\|$, where $\mathbf{x}_k$ is the location of the k^{th} receiver and $\|\cdot\|$ is the euclidean norm, equation (1) can be reformed as

$$\mathbf{x}_k{}^T\mathbf{x} - r_k r = \tfrac{1}{2}\left(\mathbf{x}_k{}^T\mathbf{x}_k - r_k{}^2\right). \tag{2}$$

The difficulty of the single target localisation problem comes from the non-linear relationship of $\mathbf{x}$ and r, the unknown parameters, in equation (2). There are a number of approaches to tackling this problem.

Algorithms have been developed that solve the generic class of problems [4], i.e. one algorithm can solve TOA, TDOA and GPS. The drawback of the adaptability is that the generic algorithm does not attain the Cramer-Rao lower bound (CRLB) for a TOA system.

An elegant and exact solution originally designed specifically for GPS [7] can be adapted to TDOA systems [8], through solving what are known as Bancroft equations. These equations can also be adapted for the TOA problem but, the solution does not meet the CRLB for the overdetermined case.

Non-closed form solutions can be found and they can be effective and efficient. If the algorithm is initiated with an approximate solution then the risk of non-convergence can be minimised.

Extending (2) to include the information for all of the receivers, the equations can be written in matrix form as

$$\mathbf{A}\mathbf{x} - \mathbf{b}r = \mathbf{c} \tag{3}$$

where $\mathbf{A} = \left[\mathbf{x}_1{}^T, \ldots, \mathbf{x}_K{}^T\right]^T$, $\mathbf{b} = \left[r_1, \ldots, r_K\right]^T$, $\mathbf{c} = \left[c_1, \ldots, c_K\right]^T$ and $c_k = \tfrac{1}{2}\left(\mathbf{x}_k{}^T\mathbf{x}_k - r_k{}^2\right)$.

The initial solution for an iterative algorithm can be provided by treating the range as a nuisance parameter and ignoring the non-linear relationship between $\mathbf{x}$ and r:

$$\begin{bmatrix}\mathbf{x}^{(0)} \\ r^{(0)}\end{bmatrix} = \tilde{\mathbf{A}}^{+}\mathbf{c}, \tag{4}$$

where $\tilde{\mathbf{A}} = \begin{bmatrix}\mathbf{A} & -\mathbf{b}\end{bmatrix}$ and $^+$ denotes the pseudo-inverse.

An equation that has been found to be effective for generating subsequent iterations of the target position is given by

$$\mathbf{x}^{(n+1)} = \left(\mathbf{A} - \mathbf{b}\hat{\mathbf{x}}^{(n)T}\right)^{+}\mathbf{c} \tag{5}$$

where $\hat{\mathbf{x}} = \mathbf{x}/\|\mathbf{x}\|$.

In the left hand plot of Figure 1 an example scenario is shown. The transmitter is positioned at the origin (shown by a circle) and there are four receivers (shown by crosses): positioned to the North, East and South, 100 km away, and one 25 km above the transmitter. The system performance is shown for target positions along the x-axis from 200 km up to 1500 km away at an altitude of 50 km.

For this scenario, it is assumed that the measured bistatic range error has a Gaussian distribution with a standard deviation of 60 m, this error includes the error from a number of sources such as range resolution, timing synchronisation and atmospheric propagation. Positioning errors of the radars are ignored as surveying GPS can provide an accuracy of milimetres. The simulation consists of 10 000 Monte Carlo runs.

The right hand plot of Figure 1 shows the algorithm positioning accuracy as well as the CRLB. The accuracy of the algorithm initial solution is shown by the dashed line, the solution after 3 iterations is shown by the dash-dot line with square markers, and the CRLB is shown by the solid line.

The plot shows the, not unexpected, result that the further away a target is, the larger the positioning error of a multistatic system is. At 1500 km there is approximately 4.5 km error. Whilst this sounds large, it is worth noting that cross-range distance of a radar with a 1° beamwidth is 26 km.

The plot additionally shows that the initial solution of the algorithm has an error approximately 0.5 km larger than the CRLB and within three iterations the algorithm solution meets the CRLB.

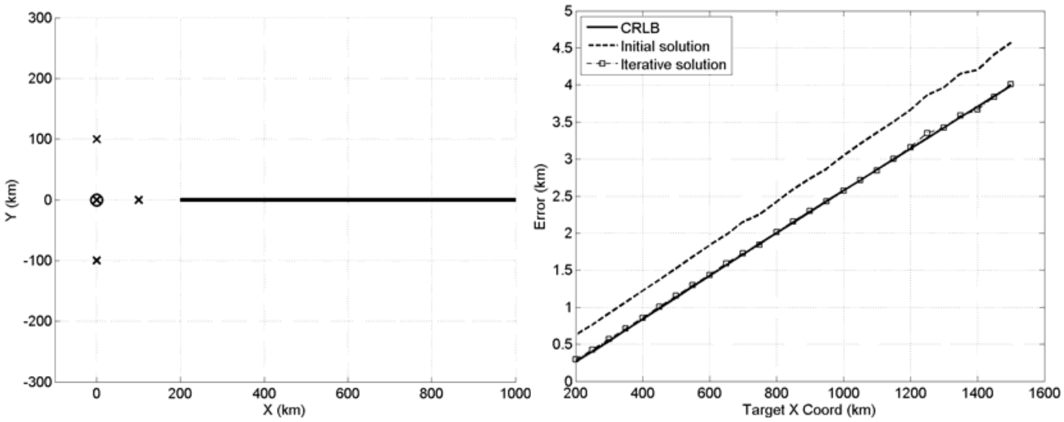

Figure 1: The example scenario is shown in the left plot. The target positioning accuracy is shown in the right plot alongside the CRLB for the system

3. Multiple Target Positioning

The key issue to be solved for the multiple target scenario is the data association problem: for a given pulse from the transmitter, it is not known which of the returns measured at the receivers are associated with which target. There can potentially be a large number of possible data associations to be considered. If there are N targets and M receivers, then there are N^M potential combinations. For example, a missile cloud could consist of dozens of reflectors, consisting of re-entry vehicles and debris, and the developed algorithm requires at least four receivers to be able to detect multiple targets.

The multiple target positioning algorithm consists of a number of stages. The initial stage reduces the number of potential data associations that need to be considered further through geometry – knowing the direction the transmitter is pointing. The computational cost of calculating the positions of all of the potential associations is thus reduced.

The second stage of the algorithm determines the validity of the candidate associations by calculating a metric, the residual of equation (3), for each of them. The potential target positions are also calculated during this process. A threshold is then used to identify the true associations/targets from the incorrect associations, known as ghost targets.

The threshold used is an approximation of the upper bound of the residual and is a function of the system error and the scenario geometry. In determining the scenario geometry, it is sufficient to use an estimate of the target position, such as that which used to cue the radar, or use the average of the receiver measurements to calculate a position.

The performance of the threshold in distinguishing between true and ghost targets is demonstrated for the system shown in the left hand plot of Figure 1. In this scenario there are three targets present, in addition to the target use in the single target scenario, additional targets are placed to the North and South at a distance of 100 m.

In the left hand plot of Figure 2, the residuals of all of the potential associations returned from the first stage of the algorithm are shown. The residuals of true targets are marked by a dot and ghost targets by a cross. The threshold used in the second stage of the algorithm is shown by the solid line. A key feature of the plot is that the residuals of the true targets are largely bounded by the threshold. Ghost target residuals are largely dependent on the scenario geometry: the residuals are clustered together and the clusters then trace out lines as the target position changes. In this scenario there is a cluster of ghost targets that have the same magnitude as the true targets.

The algorithm performance can be assessed using a Monte Carlo simulation; a 100 runs were used for the results shown in the right-hand plot of Figure 2. The probability of detection by the algorithm, shown by the solid line, is above 0.99 for most of the scenario. The probability of false alarm is defined as the ratio of the number of ghost targets selected to the total number of targets selected. Due to the excessive symmetry of the geometry – the radars and the targets are symmetrical – the performance is quite high at 0.4. More realistic scenarios would enable a better performance. The peak at 300 km is due to the effects of the geometry.

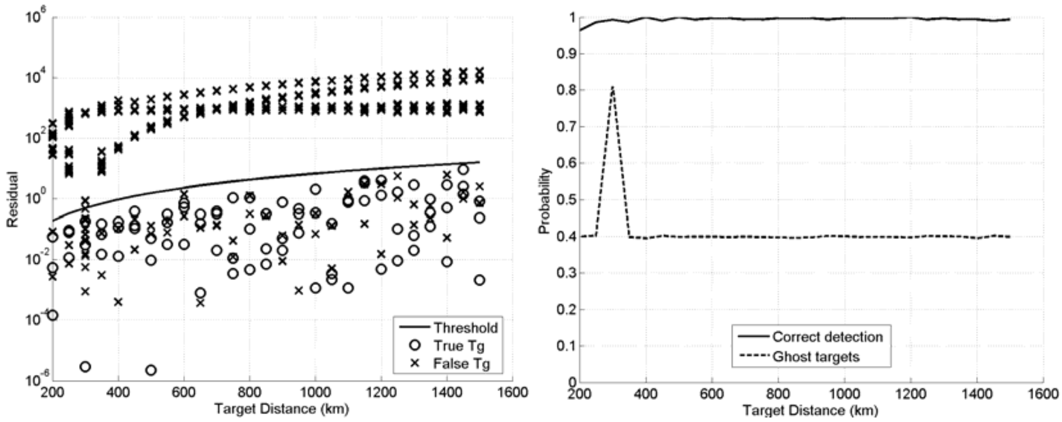

Figure 2: The left hand plot shows the algorithm residuals alongside the threshold. The right hand plot shows the algorithm performance

4. Conclusions

In this paper a single-target and a multiple-target positioning algorithm have been presented for a TOA system. The single target algorithm is robust and meets the CRLB, whilst the multiple target has a high probability of detection for a low probability of false alarm. Both algorithms only use a single snapshot of data and these algorithms would work in conjunction with a tracker. The combination would provide additional positioning accuracy and an additional element of ghost target removal. Further work will study the gains to be made with this integration.

5. References

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