On the MIMO Capacity with Imperfect CSI

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Abstract: Multiple-input multiple-output (MIMO) antenna systems provide high capacity gain over a single-antenna system. This gain is maximised when channel state information (CSI) is available at the transmitter. In practice, however, only partial information could be obtained. In this paper, we shall present a study of the MIMO system capacity in the case of imperfect CSI. It is found that in the case of partial or imperfect CSI the capacity depends on the statistical properties of the error in the CSI. Based on the knowledge of statistical distributions of the deviations in CSI knowledge, a pre-coding strategy which maximises a capacity upper bound is proposed and evaluated.

1 Introduction

Multiple-input multiple-output (MIMO) antenna systems promise high performance improvement for the next generation wireless communications. Theoretic capacity limits of a MIMO channel have also been known and widely acknowledged in the last decade [1-2]. It has been illustrated that the system capacity is maximised when there is full knowledge of channel state information (CSI) at both the receiver and the transmitter [2]. In practice, CSI may be learned at the receiver by transmitting (known) training sequences and this CSI could also be fed back to the transmitter for exploiting the full potential of a MIMO channel. However, the feedback capacity is limited and this will affect the resolution and quality of the CSI at the transmitter, resulting in partial knowledge of CSI (or imperfect CSI) at the transmitter [4] although studies have shown that a substantial capacity improvement is possible even with a small amount of feedback [3]. It is possible that the system performance may degrade severely if the system is optimised based on an inaccurate CSI [4]. Other factors such as Doppler will definitely cause more imperfection on the CSI at the transmitter if it is not updated frequently enough.

In this paper, a study of MIMO system with imperfect CSI is presented. In particular, we assume that the statistical knowledge of the CSI error is known in addition to the partial CSI and our aim is to devise an efficient MIMO pre-coding scheme robust to the CSI error for capacity maximisation. This will be done by maximising a newly derived capacity upper bound.

The rest of this correspondence is organized as follows. Section 2 lays out the model of the system under investigation. Section 3 derives a new capacity upper bound with the imperfect CSI. Section 4 presents a MIMO pre-coding technique that uses the statistical properties of the CSI error for capacity maximisation. Numerical results will be given in Section 5 and finally, we conclude the paper in Section 6.

2 System Model

Consider an ergodic flat fading Rayleigh channel with N_t transmit antennas and N_r receive antennas as shown in Figure 1. This system can be described in a vector form as [5]

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where **y**, **H**, **x** and **n** are, respectively, the received signal vector, the MIMO channel matrix, the transmit symbol vector and the noise vector. To maximise the mutual information of the system, it is assumed that **x** is circularly symmetric Gaussian [2] while the short-term power is constrained by *P* so that we have

$$\mathtt{tr}(\mathbf{R}_x) \equiv \mathtt{tr}(\mathbf{x}\mathbf{x}^\dagger) \leq P. \tag{2}$$

On the other hand, the channel matrix **H** is assumed to have independent and identically distributed (i.i.d.) zero-mean complex Gaussian entries (so that the channel amplitude is in Rayleigh fading). Likewise, **n** has zero-mean complex Gaussian i.i.d. entries, each with variance of σ^2.

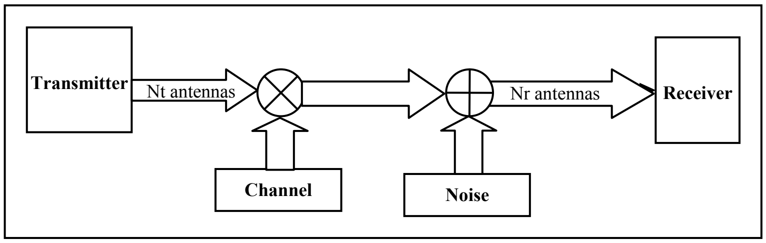

Figure 1: General MIMO system.

3 A New Capacity Upper Bound for MIMO with Imperfect CSI

In this paper, we assume that the transmitter has some knowledge of the channel $\tilde{\mathbf{H}}$ which is different from the actual channel matrix **H** by

$$\mathbf{H} = \tilde{\mathbf{H}} + \Delta\mathbf{H} \tag{3}$$

where $\Delta\mathbf{H}$ denotes the error matrix in CSI, which we assume has i.i.d. complex Gaussian entries.

Lemma 1: When there is partial knowledge of CSI at the transmitter, the capacity *C* becomes dependent on the statistical error distribution and is given by

$$C = \frac{1}{\ln 2}\mathtt{tr}\left\{\mathtt{E}\left[\ln\left(\mathbf{I} + \frac{1}{\sigma^2}\mathbf{R}_x(\tilde{\mathbf{H}} + \Delta\mathbf{H})^\dagger(\tilde{\mathbf{H}} + \Delta\mathbf{H})\right)\right]\right\}. \tag{4}$$

Proof: Equation (4) can be easily obtained from the original log-det capacity formula in [2] by substituting Equation (3) into the formula. □

Noting that ln(.) is a concave function, a capacity upper bound can be derived using Jensen's inequality [6], which gives

$$C \le \frac{1}{\ln 2}\mathtt{tr}\left\{\ln\left[\mathbf{I} + \frac{1}{\sigma^2}\mathbf{R}_x\left(\tilde{\mathbf{H}}^\dagger\tilde{\mathbf{H}} + \tilde{\mathbf{H}}^\dagger\mathtt{E}[\Delta\mathbf{H}] + \mathtt{E}[\Delta\mathbf{H}^\dagger]\tilde{\mathbf{H}} + \mathtt{E}[\Delta\mathbf{H}^\dagger\Delta\mathbf{H}]\right)\right]\right\}. \tag{5}$$

It is clear from (4) that the capacity depends on the statistical properties of the error. It suggests that if the mean and covariance of the channel error are known at the transmitter, then a better capacity maximisation be possible and this can be realised by maximising the capacity upper bound (5).

With the model (3), it is customary to consider the case where the channel error matrix $\Delta\mathbf{H}$ has zero mean and i.i.d. Gaussian entries, each with variance of σ_e^2, which fits well with conventional channel estimation scenarios. As a result, (5) yields

$$C \le \frac{1}{\ln 2}\ln\left(\mathtt{det}\left[\mathbf{I} + \frac{1}{\sigma^2}\mathbf{R}_x\left(\tilde{\mathbf{H}}^\dagger\tilde{\mathbf{H}} + \sigma_e^2\mathbf{I}\right)\right]\right). \tag{6}$$

4 Robust MIMO Pre-Coding

Given the capacity upper bound in (6), a proper pre-coding design that can make the best use of the available CSI with reference to its (statistical) confidence level to maximise the capacity can be devised. In particular, this is achieved by maximising the capacity upper bound and the solution of $\mathbf{R}_x$ follows the same way as the well known MIMO decomposition and water-filling power allocation in [1] but on the effective channel matrix $\tilde{\mathbf{H}}^\dagger\tilde{\mathbf{H}} + \sigma_e^2\mathbf{I}$.

Another possible way is that we can diagonalise the mean channel matrix by singular value decomposition (SVD) and then allocate the power over the sub-channels based on scaling the estimated channel power to the CSI error variance [or referred to as the channel-to-error ratio (CER)] in a water-filling manner. Both schemes will be evaluated in Section 5.

5 Numerical Results

Capacity upper bound provided in Section 3 was simulated. Figure 2 shows the capacity bound and the actual maximum achievable capacity. Results show that for a (2,2) system, the bound is tight especially at high signal-to-noise ratios (SNR). The results for the two proposed pre-coding techniques are provided in Figures 3-5. For modelling purposes, the channel matrix was normalised to provide a specified SNR using Frobenius matrix norm [7]. Results illustrate that optimised MIMO transmission based on imperfect CSI and error statistical properties results in remarkable capacity gain. In addition, results in Figure 3 reveal the performance gain of the first proposed scheme while Figures 4 and 5 demonstrate the performance of the second method for various CER. As can be seen, the proposed methods offer near-maximal capacity under high CER. On the other hand, at low CER, the second method gives significant capacity gain over the conventional MIMO water-filling solution without considering the CSI error statistics.

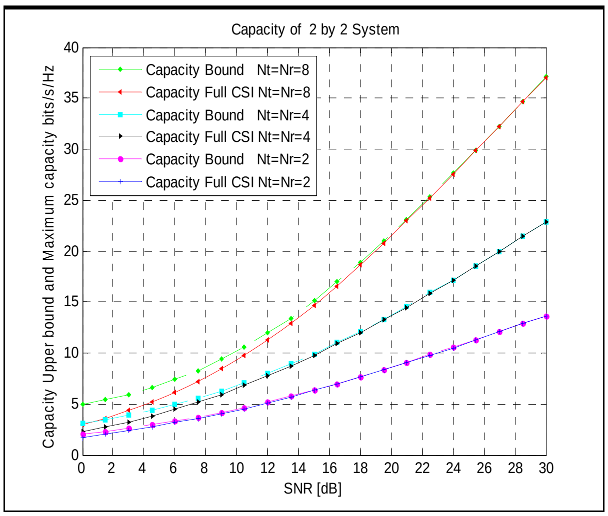

Figure 2: Capacity upper bound

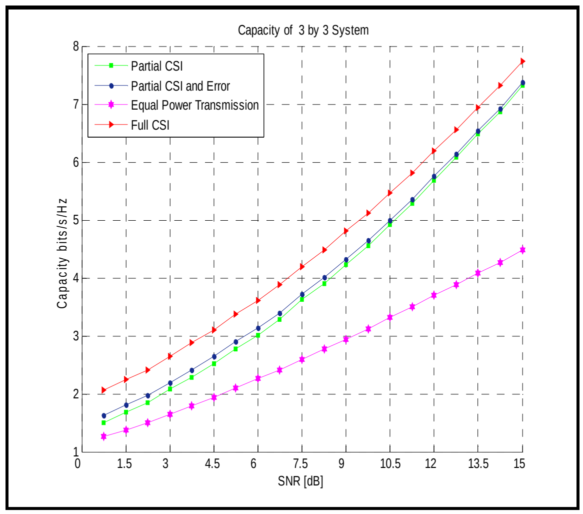

Figure 3: Capacity first scheme ZMG error

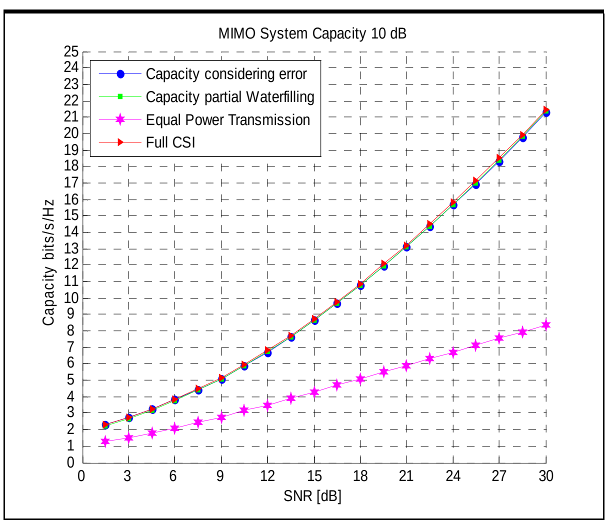

Figure 4: Capacity second scheme CER 10dB

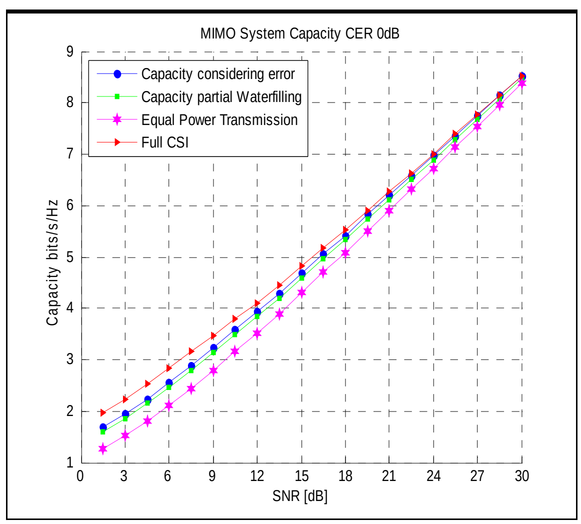

Figure 5: Capacity second scheme CER 0dB

6 Conclusion

In this paper, a study of MIMO system with partial or imperfect CSI at the transmitter has been presented. A new capacity upper bound for a MIMO channel with CSI error has been given using Jensen's inequality and robust MIMO solutions which maximise the channel capacity based on the bound have been devised and evaluated. Numerical results have revealed significant capacity gains over conventional non-robust MIMO systems by the proposed schemes exploiting the CSI error statistical knowledge for a wide range of CER and SNR.

References

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