Modelling and performance assessment of bandwidth efficient FDM communication systems

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Abstract

This project involves the modelling and simulation in MATLAB of the bandwidth efficient Frequency Division Multiplexing (FDM) system proposed by I. Darwazeh and M. R. D. Rodrigues in [2]. The main objectives of the project were to investigate the performance of bandwidth efficient FDM systems and, where applicable, compare it with Orthogonal Frequency Division Multiplexing (OFDM) systems, in order to discover their advantages and limitations. This involved gaining a better understanding of the proposed model and creating a flexible simulation model that can be used to measure the performance and the complexity of the system under different simulation conditions.

I. INTRODUCTION

The increasing popularity of multi-carrier systems can be attributed to their ability to deliver high data rates in adverse channel conditions such as multi-path fading, noise, and interference, which are common characteristics present in many practical wireless channels [1]. The evolution of new technologies, standards and services that demand high data rates, coupled with the fact that the available spectrum is always limited and exceedingly valuable, means that any system that can minimise the effects of the limiting channel constraints has a clear advantage. Consequently, multi-carrier techniques such as OFDM [5] and Multi-Carrier Code Division Multiple Access (MC-CDMA) [6] techniques are widely recognised as promising candidates for the 4th generation (4G) mobile communications systems [9].

OFDM systems are already used in a number of applications. Audio and video broadcasting systems such as Digital Audio Broadcasting (DAB) and the digital terrestrial video broadcasting system (DVB-T) [3] are implemented using OFDM techniques. They are also implemented in High-Speed WLAN systems like IEEE 802.11a/g [7] and the European Standard HIPERLAN/2 [4].

The bandwidth efficient FDM model proposed by I.Darwazeh and M.R.D.Rodrigues in [2], is similar to OFDM systems in that it is a multi-carrier system. The principal difference between the two is that the former has a reduced frequency separation between the sub-carriers. The resulting loss of orthogonality means that the FDM receiver needs a bank of correlators to extract sufficient statistics for detection, followed by a Maximum-Likelihood (ML) detector. The main drawback for this system is the complexity associated with ML detection.

This paper evaluates the performance of this system in different channel conditions and system specifications in the context of its higher complexity. A model of the system was designed and implemented in MATLAB along with an OFDM model. It was verified through simulation that the proposed bandwidth efficient FDM receiver could be used to decode signals sent on non-orthogonal carriers. Additionally, the performance of the FDM system and the OFDM system was analysed in terms of its BER performance in AWGN noise and with frequency and timing offsets.

II. OFDM AND BANDWIDTH EFFICIENT FDM CONCEPT

In OFDM, the N carriers are chosen to be orthogonal to each other in order to avoid Inter-Channel-Interference (ICI). Orthogonality is achieved by making the frequency separation between the sub-carriers equal to $1/T_{\mathrm{OFDM}}$ Hz, where T_{OFDM} is the duration of the signalling interval in each sub-carrier [5]. The OFDM symbol $x(t)$ can be given by

$$x(t) = \frac{1}{\sqrt{T_{\mathrm{OFDM}}}} \sum_{k=-\infty}^{k=\infty} \sum_{n=0}^{N-1} X_{k,n} g_n(t - kT_{\mathrm{OFDM}}), \tag{1}$$

where the n^{th} orthogonal sub-carrier, $g_n(t)$, is given by

$$g_n(t) = e^{j\frac{2\pi nt}{T_{\mathrm{OFDM}}}} \; 0 < t < T_{\mathrm{OFDM}}. \tag{2}$$

At the receiver, the data symbols carried by each individual sub-carrier is recovered by correlating the received OFDM symbol with the complex conjugate of that carrier. The complex data symbol X_n transmitted on carrier n is then given by

$$X_n = \frac{1}{\sqrt{T}} \int_0^T x(t) e^{-j\frac{2\pi nt}{T}} dt. \tag{3}$$

The bandwidth efficient FDM model is similar to OFDM systems in that it is a multi-carrier system, but the frequency separation between the sub-carriers is made smaller than that in OFDM systems. The resulting loss of orthogonality between

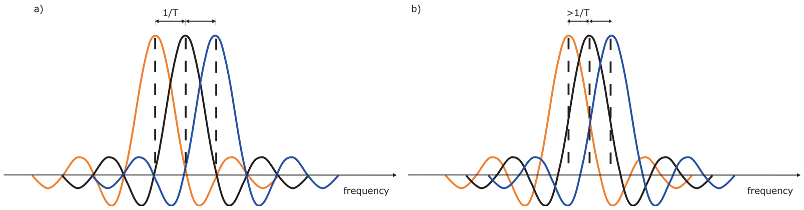

Figure 1: OFDM spectrum (left) and FDM spectrum (right).

the sub-carriers means the overlapping sub-carrier signals are distorted by Inter-Carrier-Interference (ICI). The spectrum of OFDM (right) and bandwidth efficient FDM (left) is shown in Figure 1. It is evident from this figure that such a system experiences ICI.

The transmitted FDM symbol can be expressed as

$$x(t) = \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} X_{k,n} g_n(t - KT), \tag{4}$$

where $g_n(t)$ is the n^{th} sub-carrier,

$$g_n(t) = \frac{1}{\sqrt{T}} e^{j2\pi n \Delta f t}, \ 0 < t < T, \tag{5}$$

and Δf is the frequency separation, (which is less than $1/T$ where T is the signaling interval). N is the number of sub-carriers and $X_{(k,n)}$ is the complex data symbol transmitted on the n^{th} sub-carrier and the k^{th} time slot [2].

At the FDM receiver, a set of sufficient statistics need to be extracted from the received FDM symbol so that ML detection can be used to decode the data. The sufficient statistics are obtained by the orthonormal expansion of the received symbol, using a set of orthonormal basis functions that span the signal space of the FDM symbol. Since the FDM system is multi-dimensional, obtaining the basis functions that will span the signal space for such a symbol is complicated. To achieve this, the Gram-Schmidt orthonormalisation procedure is employed at the receiver. The n^{th} orthonormal basis function, $b_n(t)$, is given by

$$b_n(t) = \frac{1}{\sqrt{\xi_n}} \left[g_n(t) - \sum_{a=0}^{n-1} \left(\int_{-\infty}^{\infty} g_n(t) b_a^*(t) dt \right) b_a(t) \right], \tag{6}$$

where $g_n(t)$ is the n^{th} non-orthogonal sub-carrier, and ξ_n is chosen such that the energy of the basis functions is normalised to one [2].

The received FDM symbol is correlated with each of the N basis functions, b_n, to give a set of N sufficient statistics, $\mathbf{R} = [R_{k,1}, R_{k,2}, ..., R_{k,N}]$. The n^{th} statistic at the k^{th} time slot, $R_{k,n}$, is given by

$$\begin{aligned} R_{k,n} &= \int_{kT}^{(k+1)T} r(t) b_n^*(t - kT) dt \\ &= \int_{kT}^{(k+1)T} x(t) b_n^*(t - kT) dt + \int_{kT}^{(k+1)T} n(t) b_n^*(t - kT) dt \\ &= S_{k,n} + N_{k,n}. \end{aligned} \tag{7}$$

Additionally, if there are N channels and the constellation size is M, then the number of possible transmitted FDM symbols, $x'_i(t), \ i = \{1, 2, ..., (M^N)\}$, is equal to M^N. For ML detection, each of these M^N symbols, $x'_i(t)$, need to be orthonormally expanded in addition to the received symbol. This is because ML detection reduces to minimum distance detection when the signals are assumed to be equally probable *a priori*. For minimum distance detection all the signals need to be expressed in terms of the same orthonormal axis, so that the Euclidean distance between the signals can be calculated. These expansions give M^N statistics vectors, $\mathbf{S}' = [\mathbf{S}'_1, \mathbf{S}'_2, ..., \mathbf{S}'_{M^N}]$. Each vector, $\mathbf{S}'_i$, has N elements as a result of correlations with N basis functions:

$$\mathbf{S}'_i = [S'_{i,1}, S'_{i,2}, ...S'_{i,N}], \tag{8}$$

where

$$S'_{i,n} = \int_0^T x'_i(t) b_n^*(t) dt. \tag{9}$$

Maximum Likelihood detection estimates the transmitted symbols using the set of sufficient statistics with the aim of minimizing the probability of error. The probability of error is given by

$$P(e) = 1 - \sum_{i=0}^{M^N-1} p_i \int_{\text{Region } i} p(\mathbf{R}|\mathbf{S'}_i) d\mathbf{R}, \tag{10}$$

where p_i is the probability of transmission of the i^{th} symbol, which is given by $1/M^N$ if we assume the signals to be equally probable a priori [2]. $p(\mathbf{R}|\mathbf{S'}_i)$, where $i = \{1, 2, .., (M^N)\}$, is the conditional probability density function for the i^{th} FDM symbol, $x'_i(t)$.

From this we see that the probability of error is minimised when the conditional probabilities are maximised. Finding the largest conditional probability is equivalent to finding the vector $\mathbf{S}'_i$ that is closest in distance to the vector $\mathbf{R}$. The Euclidean Distance between the vectors can be expressed as

$$D_i(\mathbf{R}, \mathbf{S}'_i) = \sqrt{(\mathbf{R} - \mathbf{S}'_i)(\mathbf{R} - \mathbf{S}'_i)^T} = ||\mathbf{R} - \mathbf{S}'_i||, \ i = 1, 2, ..., M^N. \tag{11}$$

This effectively gives the distance between the received symbol $r(t)$, and each of the possible transmitted symbols $x'_i(t)$. From Eq. 10 and Eq. 11, we see that the probability of error is minimised when we choose as our estimate for the transmitted signal $x'_j(t)$ such that D_j is the smallest of the Euclidean distances calculated. This is an example of the *minimum distance detection* rule [8].

III. FDM MODEL IMPLEMENTATION

Figure 2 shows a block diagram of the FDM model that was designed and implemented in MATLAB, with N giving the number of sub-channels and M giving the constellation size.

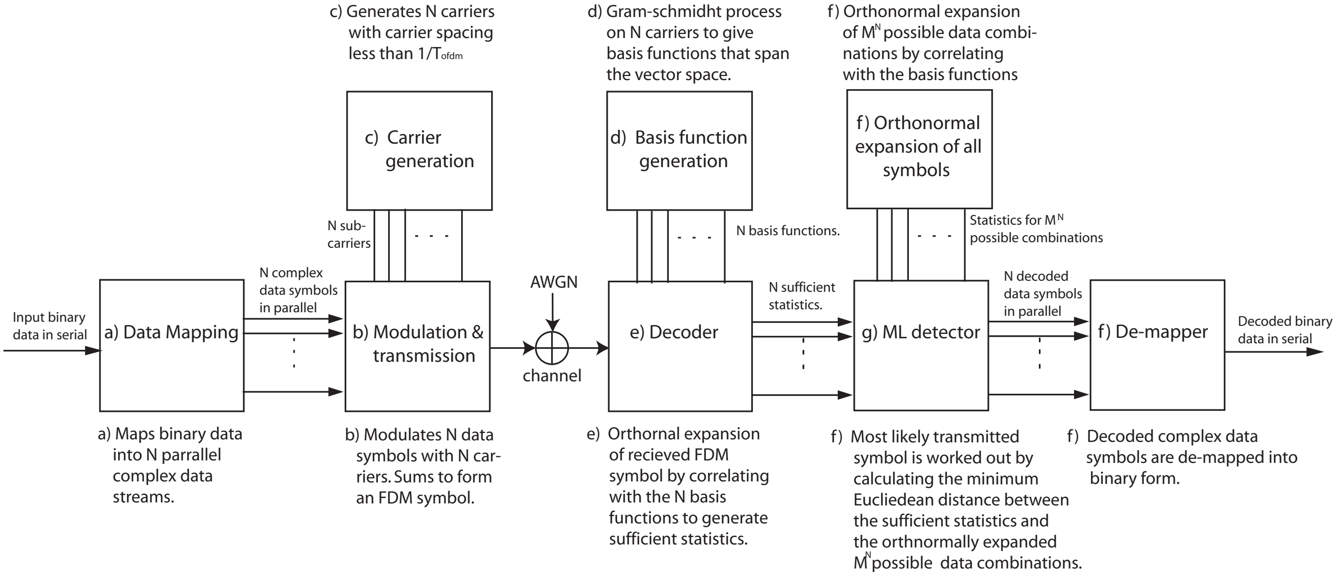

Figure 2: Block diagram of the FDM model

IV. RESULTS AND ANALYSIS

We can see from Figure 3 (a), that for the FDM system, the BER performance with QPSK mapping remained close to the theoretical ideal until the channel separation was reduced to approximately 80% of that of the OFDM system. We also found that the BER performance with BPSK mapping of the FDM system remained close to the theoretical ideal until the channel separation was reduced to to approximately half that of the OFDM system. As the channel separation was reduced further in both cases, the BER performance began to deteriorate rapidly. From this, we can conclude that after the sub-carrier spacing has been reduced to a certain level, further improvements in bandwidth efficiency for the FDM system were obtained at the expense of a deterioration in BER performance for the AWGN channel.

Within the channel separation limits given above, we verified through simulation that the proposed FDM system could be implemented with the same BER performance in an AWGN channel as the OFDM system, even for large constellation sizes

It was found that when small frequency offsets were introduced in the receiver, the BER performance of the FDM system was better than that of the OFDM systems by between 7 and 10 dBs, as seen in Figure 3 (b) . This may have been expected, since FDM systems should not be as vulnerable to the loss of orthogonality of the sub-carriers as OFDM

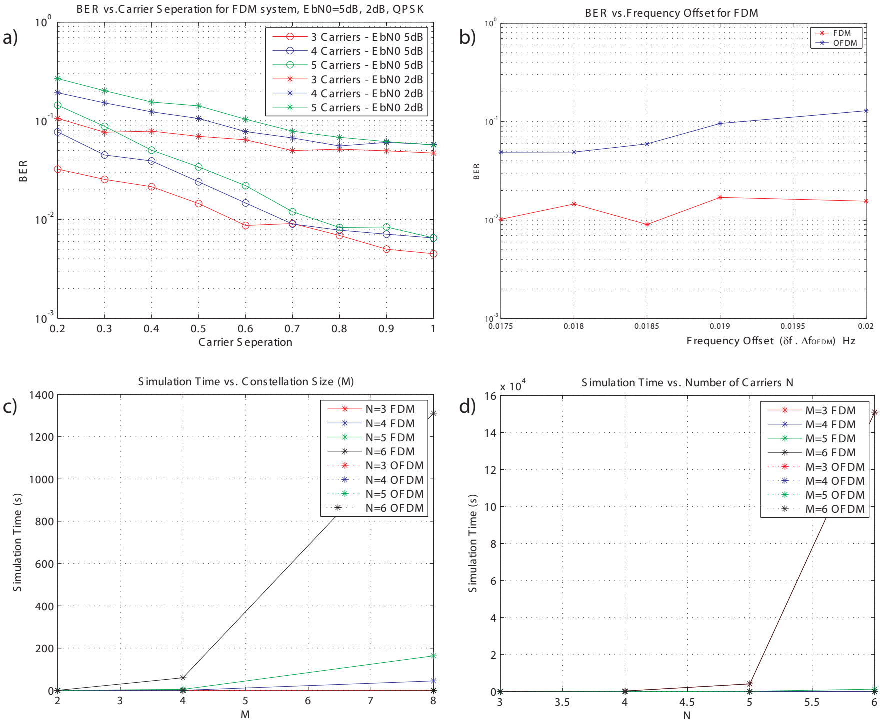

Figure 3: (a) BER of FDM simulation against FDM carrier separation. (b) BER against frequency offsets. (c) Simulation time against constellation size M. (d) Simulation time against number of carriers N.

systems. With timing offsets, which were modelled as phase offsets, no significant improvement in the BER performance was seen compared to OFDM.

To analyse the complexity of both systems, the CPU execution times for the simulations were recorded for different constellation sizes and number of sub-carriers, the results of which can be seen in Figure 3 (c) and (d). We found that broadly, the execution time did follow the trends that were predicted; namely an exponential increase in time as the number of sub-carriers, N, were increased; a polynomial increase of the order N, as M was increased; and a total complexity of order M^N.

In conclusion, although it has been shown that the proposed FDM system has certain performance advantages over OFDM, the complexity of the system is an issue that needs to be addressed to make the system a serious contender for future wireless channels.

V. FURTHER WORK

Based upon the experience and knowledge acquired during the course of this project, several areas of improvement and promising areas of future research have come to light. These include analysing the performance of the FDM model in frequency selective fading channels, and with non-linear distortions. Additionally, the alternative methods suggested by the authors in [2] to reduce the complexity (linear detection and genetic algorithms) can be investigated in more detail. It could also be beneficial to research other receiver techniques that could reduce the complexity of such a bandwidth efficient system, as well as possible methods to reduce the complexity of the ML detection technique in particular.

REFERENCES

[1] BINGHAM, J. Multi-carrier modulation for data transmission: An idea whose time has come. *IEEE Communications Magazine* (May 1990), 5–14.
[2] DARWAZEH, I., AND RODRIGUEZ, M. A spectrally efficient frequency division based communications system. In *Proceedings of the Interntional Symposium on Broadband Communications* (Moscow / St. Petersburg, Russia, Sept. 2006).
[3] ETSI. Digital video broadcasting (DVB); Framing structure, channel coding, and modulation for digital terrestrial television. European Telecommunications Standard, ETS 300-744, Mar. 1997.
[4] ETSI. Broadband radio access networks (BRAN); HIPERLAN type 2; Physical (PHY) layer. European Telecommunications Standard, TS 101-475, Apr. 2000.
[5] HANZO, G., MUNSTER, M., CHOI, B., AND KELLER, T. *OFDM and MC-CDMA*. Artech House, 2000.
[6] HATHI, N. *Design, Characterisation and Performance Assessment Issues of Multi-Carrier CDMA Exploiting Higher Order PSK/QAM Formats for Future Wireless Systems*. PhD thesis, University College London, 2003.
[7] IEEE. Further higher-speed physical layer extension in the 2.4 GHz band. IEEE 801.11g, Jan. 2000.
[8] PROAKIS, J. G. *Digital Communications*. McGraw Hill, 2000.
[9] RODRIGUES, M. *Modelling and Performance Assessment of OFDM Communications Systems in the Presence of Non-Linearities*. PhD thesis, University College London, 2002.