Unleashing the Full Potential of Relaying

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Abstract: Understanding the relay channel is a key to developing efficient cooperative schemes, which could significantly enhance the efficiency of a wireless network. In this study, we consider a decode-and-forward (DF) relay channel and address the resource allocation for optimal relaying.

1 Introduction

To mitigate the random behavior of wireless channels, multiple-input multiple-output (MIMO) antenna technologies have emerged as an attractive means due to their extraordinary capacity without the need of bandwidth expansion and increase in transmit power. Recently, it is proposed that users cooperate to form a virtual MIMO (V-MIMO) system for enhancing the system capacity [1]–[4] while the cost of equipping multiple antennas may be amortized by the users.

Forming a V-MIMO requires a node (or user) to act as a relay for forwarding the information from the source to the destination in a wireless network. Understanding the theoretical limits of a relay channel therefore paves the way to the design of efficient cooperation schemes [5]. In this paper, we study a decode-and-forward (DF) relay channel[1] with three nodes (a source, relay and destination) and aim to optimize the bandwidth (in time) allocation for relaying, in contrast to the previous works using fixed-allocation relaying. The most related works go to [6, 7] where optimum power allocation was considered.

2 System Model

We consider a three-node wireless network, as shown in Figure 1(a), which consists of a source node, S, a destination node, D, and a relay node, R. The network is given one unit of spectral resource to accomplish the communication from S to R, which takes place in two stages in two non-overlapping time slots [see Figures 1(a)&(b)]. First of all, the source takes $1-\tau$ units of bandwidth to broadcast the intended message w to both the relay and the destination nodes. Upon successful decoding of w, the relay then takes τ units of the bandwidth to forward the re-encoded message to the destination. The two independently received copies of the message will be maximally combined for optimal decoding at the destination. Note that conventionally, e.g., in [4], $\tau = 0.5$ was considered and our novelty here is to optimize τ for the full benefit of relaying.

All the channels are assumed to be in frequency-flat Rayleigh-fading so that the channel power coefficients, g_{IJ}, are exponential distributed where $\mathsf{I} \in \{\mathsf{S}, \mathsf{R}\}$ and $\mathsf{J} \in \{\mathsf{R}, \mathsf{D}\}$. We also assume that the noise at the nodes is independent and identically distributed (i.i.d.) zero-mean complex Gaussian with variance of N_0. Denoting the average transmit power from the source and the relay, respectively, as p_{S} and p_{R}, the received signal-to-noise ratio (SNR) at R or D is given by

$$\gamma_{\mathsf{IJ}} = \frac{p_{\mathsf{I}} g_{\mathsf{IJ}}}{B N_0} \tag{1}$$

where B is the channel bandwidth. The average SNR is therefore $\mathtt{E}[\gamma_{\mathsf{IJ}}] = (\frac{p_{\mathsf{I}}}{BN_0})\mathtt{E}[g_{\mathsf{IJ}}]$.

[1] In a DF relay channel, the relay will first decode the data received from the source and then forward the re-encoded data to the destination.

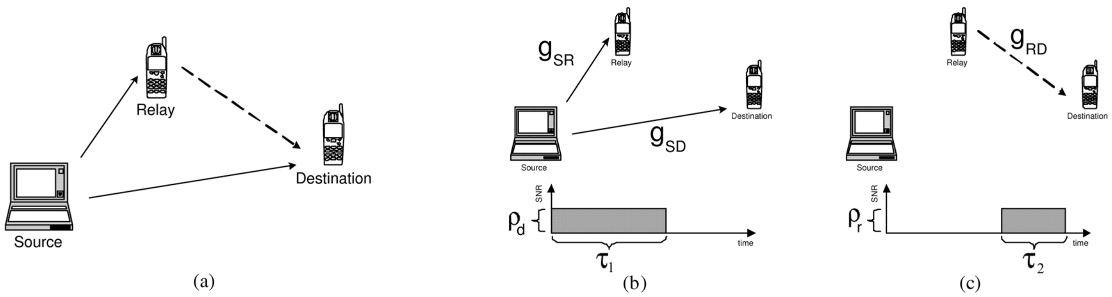

Figure 1: (a) A 3-node network in (b) the direct transmission phase and (c) the relaying phase.

3 Maximizing the Instantaneous Mutual Information

Following the model described, the rate achievable at the destination node given only the transmission from the source is given by

$$\mathcal{I}_{\mathsf{SD}}(\tau) = (1-\tau)\log_2(1+\gamma_{\mathsf{SD}}). \tag{2}$$

The relay in the proximity also receives the same message and has the mutual information of

$$\mathcal{I}_{\mathsf{SR}}(\tau) = (1-\tau)\log_2(1+\gamma_{\mathsf{SR}}). \tag{3}$$

When the transmission code-rate from the source is smaller than $\mathcal{I}_{\mathsf{SR}}$, the relay can successfully decode the message and can then forward the re-encoded copy to the destination, which then combines the two independently received copies for maximum-likelihood decoding. The resultant mutual information at the destination node for $\tau > 0$ can be shown to be [8]

$$\mathcal{I}_{\mathsf{relay}}(\tau) = \min\{\mathcal{I}_{\mathsf{SR}}(\tau), \mathcal{I}_0(\tau)\} \tag{4}$$

where

$$\mathcal{I}_0(\tau) = (1-\tau)\log_2\left(\gamma_{\mathsf{SD}} + (1+\gamma_{\mathsf{RD}})^{\frac{\tau}{1-\tau}}\right). \tag{5}$$

For $\tau = 0$, clearly, $\mathcal{I}_{\mathsf{relay}}(0) = \mathcal{I}_{\mathsf{SD}}(0)$. As a result, we have

$$\mathcal{I}_{\mathsf{relay}}(\tau) = \begin{cases} \min\{\mathcal{I}_{\mathsf{SR}}(\tau), \mathcal{I}_0(\tau)\} & \text{if } 0 < \tau < 1, \\ \mathcal{I}_{\mathsf{SD}}(0) & \text{if } \tau = 0. \end{cases} \tag{6}$$

Given that the channel state information is available at all the participating nodes (i.e., CSIN), it is proposed to maximize the mutual information $\mathcal{I}_{\mathsf{relay}}$ by optimizing τ, i.e.,

$$\mathbb{P}: \max_{0\le\tau<1} \mathcal{I}_{\mathsf{relay}}(\tau). \tag{7}$$

To proceed further, the following useful facts are noted (see [8] for details):

- Both the functions $\mathcal{I}_0(\tau)$ and $\mathcal{I}_{\mathsf{SR}}(\tau)$ are convex over τ and hence, their maxima are located at the endpoints, i.e., when $\tau = 0$ or $\tau = 1$.

- If $\gamma_{\mathsf{SD}} < \gamma_{\mathsf{SR}}$, then there exists an intersection point which appears at $\tau = \varsigma$ given by

$$\varsigma = \frac{\log_2(1+\gamma_{\mathsf{SR}}-\gamma_{\mathsf{SD}})}{\log_2(1+\gamma_{\mathsf{SR}}-\gamma_{\mathsf{SD}}) + \log_2(1+\gamma_{\mathsf{RD}})}. \tag{8}$$

Based on that, it can be easily shown that the optimal relaying time τ is given by

$$\tau_{\mathsf{opt}} = \begin{cases} \varsigma & \text{if } \gamma_{\mathsf{SD}} < (1+\gamma_{\mathsf{SR}})^{1-\varsigma} - 1, \\ 0 & \text{if } \gamma_{\mathsf{SD}} \geq (1+\gamma_{\mathsf{SR}})^{1-\varsigma} - 1, \\ 0 & \text{if } \varsigma \text{ is not defined or } \gamma_{\mathsf{SD}} > \gamma_{\mathsf{SR}}, \end{cases} \tag{9}$$

and the corresponding maximum mutual information is expressed as

$$\mathcal{I}_{\mathsf{relay}}(\tau_{\mathsf{opt}}) = \begin{cases} (1-\varsigma)\log_2(1+\gamma_{\mathsf{SR}}) & \text{if } \gamma_{\mathsf{SD}} < (1+\gamma_{\mathsf{SR}})^{1-\varsigma} - 1, \\ \log_2(1+\gamma_{\mathsf{SD}}) & \text{if } \gamma_{\mathsf{SD}} \geq (1+\gamma_{\mathsf{SR}})^{1-\varsigma} - 1 \text{ or } \varsigma \text{ is not defined.} \end{cases} \tag{10}$$

4 Minimizing the Outage Probability

For a highly mobile network, it may be impractical to gather CSIN. Instead, it would be more possible to have the statistical channel information at the nodes (SCIN), which can be exploited for improving the wireless network performance. In this case, τ can be optimized for minimizing the probability of outage (which occurs when a given code-rate, say R_0, is not met). That is,

$$\mathbb{Q}: \min_{0\leq\tau<1} \mathcal{P}\left(\{\mathcal{I}_{\mathsf{relay}}(\tau) < R_0\}\right). \tag{11}$$

Given (6), the outage probability can be evaluated by

$$\mathcal{P}\left(\{\mathcal{I}_{\mathsf{relay}}(\tau) < R_0\}\right) = 1 - (1-\mathcal{P}(\{\mathcal{I}_{\mathsf{SR}}(\tau) < R_0\}))(1-\mathcal{P}(\{\mathcal{I}_0(\tau) < R_0\})), \tag{12}$$

where it can be found that

$$\begin{cases} \mathcal{P}\left(\{\mathcal{I}_{\mathsf{SR}}(\tau) < R_0\}\right) = 1 - e^{-\lambda_{\mathsf{SR}}\left(2^{\frac{R_0}{1-\tau}}-1\right)}, \\ \mathcal{P}\left(\{\mathcal{I}_0(\tau) < R_0\}\right) = 1 - e^{-\lambda_{\mathsf{SD}}\left(2^{\frac{R_0}{1-\tau}}-1\right)} - \lambda_{\mathsf{SD}} \displaystyle\int_0^{2^{\frac{R_0}{1-\tau}}-1} e^{-\lambda_{\mathsf{RD}}\left[\left(2^{\frac{R_0}{1-\tau}}-x\right)^{\frac{1-\tau}{\tau}}-1\right]} e^{-\lambda_{\mathsf{SD}}x} dx, \end{cases} \tag{13}$$

where $\lambda_{\mathsf{SD}} = \frac{1}{\mathsf{E}[\gamma_{\mathsf{SD}}]}$, $\lambda_{\mathsf{SR}} = \frac{1}{\mathsf{E}[\gamma_{\mathsf{SR}}]}$ and $\lambda_{\mathsf{RD}} = \frac{1}{\mathsf{E}[\gamma_{\mathsf{RD}}]}$. Using (13), we can get

$$\begin{aligned} \mathcal{P}\left(\{\mathcal{I}_{\mathsf{relay}}(\tau) < R_0\}\right) = 1 &- e^{-(\lambda_{\mathsf{SD}}+\lambda_{\mathsf{SR}})\left(2^{\frac{R_0}{1-\tau}}-1\right)} \\ &- \lambda_{\mathsf{SD}} e^{-\lambda_{\mathsf{SR}}\left(2^{\frac{R_0}{1-\tau}}-1\right)} \int_0^{2^{\frac{R_0}{1-\tau}}-1} e^{-\lambda_{\mathsf{RD}}\left[\left(2^{\frac{R_0}{1-\tau}}-x\right)^{\frac{1-\tau}{\tau}}-1\right]} e^{-\lambda_{\mathsf{SD}}x} dx. \end{aligned} \tag{14}$$

The minimization (11) can then be performed numerically using (14).

5 Numerical results

In this section, numerical results are presented to demonstrate some of the benefits of optimal relaying as compared to no relaying and equal-time relaying schemes. In Figure 2(a), results for the mutual information are shown assuming that the average channel power gain to the noise ratio (CNR) from the source to the destination is -30 (dB), from the source to the relay is -5 (dB) and from the relay to the destination is -10 (dB). As can be seen, the optimum relaying scheme outperforms significantly both the no relaying and equal-time relaying schemes, and the superior performance is more apparent at low SNR regimes.

In Figure 2(b), results are provided for the outage probability performance for various relaying schemes, and the CNRs from both the source to the relay and from the relay to the destination are set to be -55 (dB) while the CNR from the source to the destination has a value of -15 (dB). Results demonstrate a remarkable reduction of outage probability by optimizing the time allocation as compared to the no relaying and equal-time relaying systems.

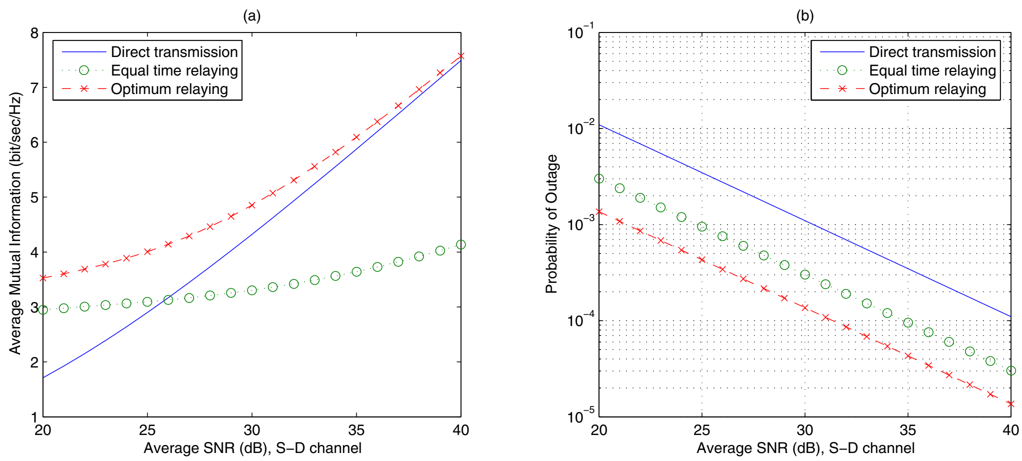

Figure 2: Results for (a) the mutual information with CSIN and (b) the outage probability with SCIN for various schemes of a 3-node relay channel.

6 Conclusions

In this paper, we have addressed the optimal resource allocation for a DF relay channel for both maximizing the instantaneous mutual information with CSIN and minimizing the outage probability with SCIN. Simulation results have revealed that significant performance improvement can be obtained by optimal relaying as compared to no and equal-time relaying systems. The results of this paper could be used to develop more efficient cooperative communication schemes.

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