Outage Probability Analysis for Systems Using Amplify-and-Forward Relays with Direct Link

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Abstract: In this paper, we analyze the outage probability of the system where a cluster of amplify-and-forward (AF) relays are employed to assist the reception at the destination terminal. In particular, our model assumes that the channels between the source and the relay terminals and that between the source and destination terminals have the same statistics, and the relay nodes only have the statistical knowledge of the first hop channel. Our main contribution is the derivation of the exact closed-form expression for the outage probability of the system. In addition, the diversity order of the system is investigated.

1. Introduction

Cooperative communication is a new paradigm that can improve the link quality by exploiting other user terminals to relay the source information to its intended destination [1,2]. A variety of protocols on how relays can be operated have been proposed. For instance, the most straightforward method is orthogonal relaying where the relay nodes transmit in an orthogonal fashion, i.e., occupying different time slots or frequency bands, and the performance has also been well studied [3–5].

While orthogonal relaying obviously takes full advantage of the channel diversity of the cooperating nodes, it comes at a cost of more spectrum resources and is thus expensive. Nonorthogonal approaches, where the relay nodes transmit simultaneously in a single band, therefore, emerge as a more attractive cooperative solution [6–9]. In these systems, nevertheless, the availability of channel state information (CSI) at the relays plays a key role on the system performance. While it is of importance to understand the performance of such systems where the relays have perfect first-hop and second-hop CSIs [6–8], recent research also addressed an important, more practical, scenario where only the incoming channel (or the first-hop CSI) is available at the relays. In [9], an exact outage probability expression for such system with no direct link between the source and destination terminals was derived.

In this paper, we generalize the outage analysis of [9] to include the direct link between the source and destination nodes. In particular, it is assumed that the channel statistics for the source-to-destination link and the source-to-relay links are independent and identical, as is typical in the scenarios where the relays are located around the destination terminal. The resulting system can be interpreted as the communication system where a local cluster of relays assist in the reception of the destination terminal. By obtaining an exact closed-form expression for the system outage probability, we take a finer look at the performance in terms of diversity order and coding gain.

2. System Model

We consider a wireless relaying system as shown in Fig. 1. The source terminal, S, communicate with the destination terminal, D, with the help of a cluster of N relay terminals located around the destination terminal, labeled by $\mathsf{R}_1, \ldots, \mathsf{R}_N$. We assume that the relays operate in the half-duplex amplify-and-forward (AF) fashion. It is also assumed that the channel between S and R_i, denoted by h_i, and the channel between S and D, denoted by h_{sd}, are independently and identically distributed (i.i.d.) zero-mean Gaussian random variables with variance of σ_1^2,[1] while the channel between R_i and D, denoted by g_i, are i.i.d. zero-mean Gaussian random variables with variance of σ_2^2. The noise power at all the terminals are assumed to be N_0.

[1] A more general setting arises when h_{sd} and $\{h_i\}$ are not identically distributed. However, the analysis will become intractable in that case, which is beyond the scope of this paper.

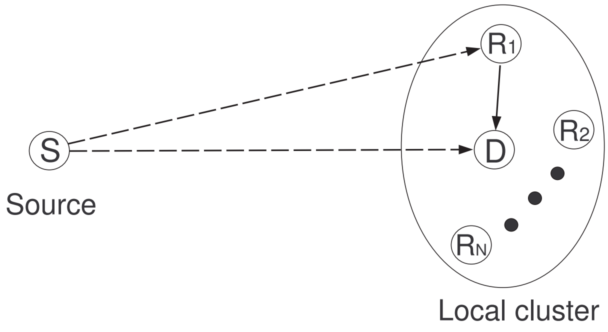

Figure 1. An illustration of the communication systems.

In the first time slot, S transmits the signal, $x[1]$, while all the relay nodes $\{\mathsf{R}_i\}$ and the destination node, D, listen. The received signal at the ith relay node, R_i, can be expressed as

$$y_i[1] = h_i[1]x[1] + n_i[1], \quad \text{for } i = 1, \dots, N, \tag{1}$$

where $n_i[1]$ is the additive white Gaussian noise (AWGN) and the bracket [1] specifies that the signal is for the first time slot. On the other hand, the received signal at D is given by

$$y_d[1] = h_{sd}[1]x[1] + n_d[1], \tag{2}$$

with $n_d[1]$ the AWGN at the destination terminal, D. In the second time slot, the source keeps silent, and the ith relay forwards a scaled version of its received signal $y_i[1]$ (acquired during the first phase of the protocol) as in [9], i.e. $\sqrt{\frac{P_i}{\mathtt{E}\{|y_i[1]|^2\}}}y_i[1]$, which gives the received signal at D as

$$y_d[2] = \sum_{i=1}^{N} g_i[2]\sqrt{\frac{P_i}{\sigma_1^2 P_s + N_0}}y_i[1] + n_d[2], \tag{3}$$

where $\{P_i\}_{i=1}^N$ are the transmit powers for the relays, and $P_s \triangleq \mathtt{E}[|x[\cdot]|]$. Defining

$$\begin{cases} \mathbf{h} \triangleq [h_1[1]\ h_2[1]\ \cdots\ h_N[1]], \\ \mathbf{g} \triangleq [\sqrt{P_1}g_1[2]\ \sqrt{P_2}g_2[2]\ \cdots\ \sqrt{P_N}g_N[2]]^T, \\ \mathbf{n} \triangleq [n_1[1]\ n_2[2]\ \cdots\ n_N[1]], \end{cases} \tag{4}$$

where $(\cdot)^T$ denotes the transposition. Thus, (3) can be rewritten as

$$y_d[2] = \frac{1}{\sqrt{\sigma_1^2 P_s + N_0}}\mathbf{hg}x[1] + \frac{1}{\sqrt{\sigma_1^2 P_s + N_0}}\mathbf{ng} + n_d[2]. \tag{5}$$

Combining $y_d[1]$ and $y_d[2]$, the mutual information for this channel can be expressed as

$$I = \frac{1}{2}\log_2\left[1 + \frac{P_s}{N_0}|h_{sd}[1]|^2 + \frac{P_s}{N_0}\frac{\mathbf{hg}(\mathbf{hg})^\dagger}{\mathbf{g}^\dagger\mathbf{g} + (\sigma_1^2 P_s + N_0)}\right], \tag{6}$$

where $(\cdot)^\dagger$ denotes the conjugate transposition. Hence, the outage probability is given by

$$\mathcal{P}_{\mathsf{out}}(\gamma_{\mathsf{th}}) = \text{Prob}(\gamma_2 < \gamma_{\mathsf{th}}), \quad \text{for some target } \gamma_{\mathsf{th}}, \tag{7}$$

where $\gamma_2 \triangleq \frac{P_s}{N_0}|h_{sd}[1]|^2 + \gamma_1$ and $\gamma_1 \triangleq \frac{P_s}{N_0}\frac{\mathbf{hg}(\mathbf{hg})^\dagger}{\mathbf{g}^\dagger\mathbf{g}+(\sigma_1^2 P_s+N_0)}$.

3. Outage Analysis

In this section, we analyze the outage probability of the system, which is given in the following theorem.

Theorem 1 *The outage probability of the system can be expressed as*

$$\mathcal{P}_{\mathsf{out}}(\gamma_{\mathsf{th}}) = 1 - e^{-\frac{N_0\gamma_{th}}{P_s\sigma_1^2}} - \frac{2N_0}{P_s\sigma_1^2} e^{-\frac{N_0\gamma_{\mathsf{th}}}{P_s\sigma_1^2}} \sum_{i=1}^{\pi(\mathbf{P})}\sum_{j=1}^{\tau_i(\mathbf{P})} \chi_{i,j}(\mathbf{P})\times \left[\frac{jP_sP_{[i]}}{2(\sigma_1^2P_s+N_0)N_0} - \left(\frac{(\sigma_1^2P_s+N_0)N_0}{P_sP_{[i]}}\right)^{\frac{j-1}{2}} \frac{\gamma_{\mathsf{th}}^{\frac{j+1}{2}}}{(j-1)!} K_{j+1}\left(2\sqrt{\frac{(\sigma_1^2P_s+N_0)N_0\gamma_{\mathsf{th}}}{P_sP_{[i]}}}\right)\right]. \quad (8)$$

in which $\mathbf{P} = \mathtt{diag}(P_1\sigma_2^2, \ldots, P_N\sigma_2^2)$, $P_{[1]} > \cdots > P_{[\pi(\mathbf{P})]}$ *are the distinct diagonal elements of* $\mathbf{P}$ *in ascending order,* $\pi(\mathbf{P})$ *is the number distinct diagonal elements,* $\tau_i(\mathbf{P})$ *is the multiplicity of* $P_{[i]}$, $\chi_{i,j}(\mathbf{P})$ *is the* (i,j)*th characteristic coefficient, and* $K_j(\cdot)$ *denotes the Bessel function of the second kind [10].*

Proof: The proof is omitted due to space limits. □

Thereom 1 is general and applicable for arbitrary relay power distribution. In Fig. 2, we plot the outage probability of a two-relay system with $P_1 = 0.4P_s$, and $P_2 = 0.6P_s$. The analytical result in (8) aligns perfectly with the Monte-Carlo simulation results which confirms the correctness of Theorem 1.

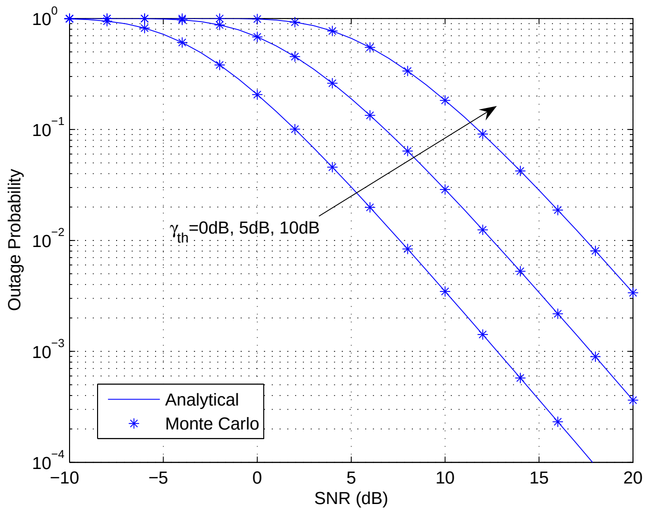

Figure 2. Outage probability of the system when $N = 2$ and $P_1 = 0.4P_s$, $P_2 = 0.6P_s$.

To gain more insight, we consider a special case where the relay powers are the same. In this case, a simpler expression is possible which reveals the diversity order and coding gain of the system.

Corollary 1 *For the case with* $P \equiv P_1 = P_2 = \cdots = P_N$ *and* $P_s = kP$, *for any* $k > 0$, *the outage probability,* $\mathcal{P}_{\mathsf{out}}(\gamma_{\mathsf{th}})$, *for* $\rho \triangleq \frac{P}{N_0} \to \infty$, *has the following asymptotic expansion*

$$\mathcal{P}_{\mathsf{out}}(\gamma_{\mathsf{th}}) = \begin{cases} \frac{1}{2}\left[\frac{1}{(k\sigma_1^2)^2} + \frac{1}{k\sigma_2^2}\left(\psi(1)+\psi(3)-\ln\frac{\gamma_{\mathsf{th}}\sigma_1^2}{\rho\sigma_2^2}\right)\right]\left(\frac{\gamma_{\mathsf{th}}}{\rho}\right)^2 & \text{if } N = 1, \\ \frac{1}{2}\left[\frac{1}{(k\sigma_1^2)^2} + \frac{1}{k(N-1)\sigma_2^2}\right]\left(\frac{\gamma_{\mathsf{th}}}{\rho}\right)^2 & \text{if } N \geq 2, \end{cases} \quad (9)$$

where $\psi(\cdot)$ *denotes the digamma function [10].*

Proof: The proof is omitted due to space limit. □

Note that the simultaneous relaying systems studied in [6–8] are shown to achieve a diversity order of $N+1$ with N relays and the direct link between S and D. However, we can see from (9) that the diversity order achieved by our relay system is only 2. This can be explained by recognizing the fact that the relay terminals do not have access to the CSI for the second hop and therefore, the received signals at D will not add constructively. Further, we observe that the number of relay nodes affects the system coding gain. Fig. 3 plots the diversity order of the system for different number of relay nodes. As shown in this figure, all four configurations achieve diversity order of 2. In addition, it is observed that the benefit of increasing the number of relay nodes is marginal for $N \geq 5$.

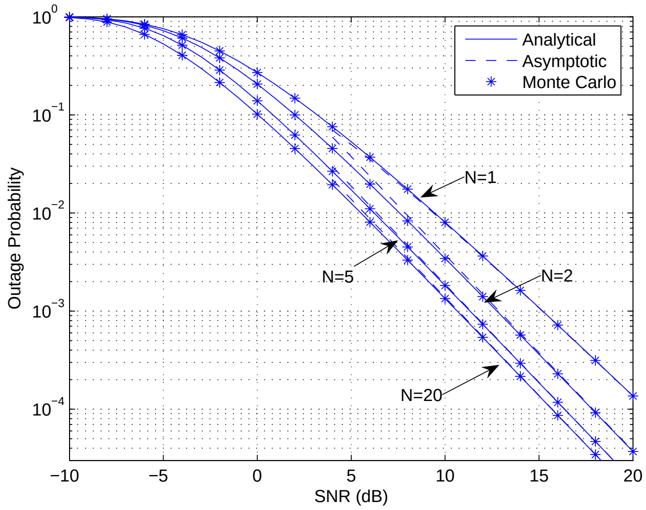

Figure 3. Outage probability of the system for different number of equal power relay nodes.

4. Conclusion

The outage performance of the system with a cluster of AF relays to aid the reception of destination terminal with the direct link has been addressed. In particular, we have derived a closed-form expression for the outage probability, based on which the diversity order is revealed.

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