Performance Evaluation of Linear Detectors for LDPC-Coded MIMO Systems

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Abstract: This paper investigates the performance of linear detectors namely the Minimum Mean-Squared-Error (MMSE) and the Zero-Forcing (ZF) suboptimal detection schemes for Multiple Input Multiple Output (MIMO) signals over an additive white Gaussian noise (AWGN) channel. Unlike the ZF detector which suffers from noise enhancement, it will be shown that the MMSE detector which takes noise into account performs very well when MIMO signals are coded with Low Density Parity Check (LDPC) particularly in symmetrical MIMO setups where the signal-to-interference and noise ratios are exponentially distributed. However, the MMSE detector does not generally approach the generalized maximum-likelihood detector as the noise power vanishes due to increased interference enhancement. It does, however, approach the Successive Interference Cancellation (SIC) detector which cancels out the multi-access interference (MAI). The main objective of this paper is to set the ground work for future study of more powerful MIMO detection strategies such as the Sphere Decoder, where the MMSE detector will be used to compute the initial sphere radius.

1. Introduction

The explosive increase in demand for higher data rates and higher performance of wireless communication networks in recent years has now become a continuous process which calls for the design of more sophisticated Digital Signal Processing (DSP) techniques which allow for effective utilization of spectrum. Spectrum, which is subject to physical constraints and regulation, is a precious and limited resource. Hence, it is justifiable investing more effort in designing systems which relaxes this scarce, valuable and vulnerable resource.

One of the candidates DSP technologies which have provided solutions to the constraints and technical burden placed on spectrum by exploiting the spatial domain of the transmission medium is Multiple-Input Multiple Output (MIMO) systems [1-3]. Equipping both the transmitter and the receiver with multiple antennas can result in significant increase in spectral efficiency, link range and reliability without additional bandwidth and transmit power [4]. However, these advantages come at a potentially high computational cost of the receiver [5].

Several powerful MIMO detection schemes have been proposed in the literature. Examples of such detectors include the Maximum Likelihood (ML) detector and the Sphere Decoder (SD) [3]. The ML detector yields the optimal solution at the expense of high computational complexity, hence cannot be implemented in practice. Currently, the SD has emerged as a powerful and promising means of finding the ML solution. However, its complexity depends on the initial radius of the hyper-sphere.

This paper focuses on the performance analysis of the Minimum Mean-Squared-Error (MMSE) and the Zero-Forcing (ZF) suboptimal linear detection schemes. To boost the performances of these detection schemes, the state-of-the-art Low Density Parity Check (LDPC) codes will be used to encode the MIMO signals. These performance-enhanced detection schemes will be used as precoding schemes for the SD in future work.

The rest of the paper is organized as follows: Section 2 provides a description of the MIMO system. This will be followed by a brief review of the linear detection schemes in Section 3. Particular attention will be paid on the advantages and drawbacks of each detection scheme. Simulation results and discussion will be provided in section 4. This paper will be closed with conclusion and a map for the future direction of this study.

2. MIMO-System Description

Consider a Low Density Parity Check (LDPC) coded MIMO system model with $\boldsymbol{N_T}$ transmit and $\boldsymbol{N_R}$ receive antennas, see Figure 1. The received signal vector at each instant of time is given by:

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n} \tag{1}$$

where $\boldsymbol{x} \in \mathbb{C}^{N_T} = \left[x_1, x_2, \dots, x_{\mathrm{N_T}}\right]^T$ and $\boldsymbol{y} \in \mathbb{C}^{N_R} = \left[y_1, y_2, \dots, y_{\mathrm{N_T}}\right]^T$ are the respective N_T-dimensional and N_R-dimensional transmitted and received complex vectors whose entries have real and imaginary parts that are integers, $\boldsymbol{H} \in \mathbb{C}^{N_R \times N_T}$ denotes the $N_R \times N_T$ flat-fading MIMO channel matrix whose entries $h_{j,i}$ describe the coupling between the i^{th} transmit antenna and the j^{th} receive antenna, i.e., the eigenmodes of the MIMO channel, $\boldsymbol{n} \in \mathbb{C}^{M} = \left[n_1, n_2, \dots, n_{\mathrm{N_T}}\right]^T$ is the independent and identically distributed (i.i.d) circularly symmetric, complex additive white Gaussian noise (AWGN) vector with zero-mean and covariance matrix ${\sigma_n}^2\boldsymbol{I}$ and $[\cdot]^T$is the transpose operator. It is assumed that the lattice generating matrix $\mathbf{H}$ is known at the receiver.

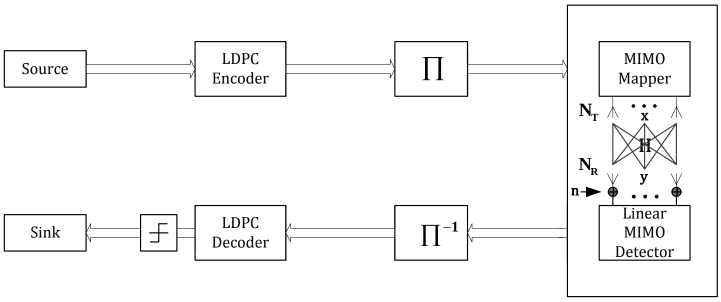

Figure 1: LDPC coded MIMO system model

The system model works as follows: The binary input signals at the source in Figure 1are encoded in the LDPC encoder. The encoded codeword is interleaved before being mapped to the multiple antennas and finally transmitted over the Additive White Gaussian Noise channel. At the receiver, the received signal is processed by linear detectors namely the Minimum Mean-Squared-Error (MMSE) and the Zero-Forcing (ZF) suboptimal detection schemes. The processed signals will be passed on to the LDPC decoder via the deinterleaver for decoding. Finally, decisions will be made by the decision circuit before the information is transferred to the information sink or destination.

3. Overview of Linear Detection Schemes for MIMO Systems

The Zero-Forcing detector: A straightforward solution to the MIMO detection problem is to suppress the interference among the layers, i.e. the received data blocks. The Zero-Forcing (ZF) detector solves the unconstrained least-squares problem by multiplying the received signal by the Moor-Penrose pseudo-inverse $\mathbf{H}^\dagger$ of the channel matrix to obtain equation (2). Since the entries of $\widehat{\boldsymbol{x}}$ are not necessarily integers, they can be rounded off to the closest integer, a process referred to as slicing [6], to obtain:

$$\widehat{\boldsymbol{x}}_B = [\mathbf{H}^\dagger \boldsymbol{y}]\boldsymbol{\mathcal{A}}^m \tag{2}$$

where $\widehat{\boldsymbol{x}}_B$ is the Babai estimate and $\boldsymbol{\mathcal{A}}$ is the set of all constellation or lattice points. This strategy is also referred to as decorrelating detection [7] and is attractive where performance degradation due to noise enhancement can be accepted in order to achieve very low receiver complexity. The advantage of this detector is that it eliminates interference completely. Unlike the Maximum Likelihood detector whose computational complexity per symbol rises exponentially with the number of users, the decorrelating detector has a linear complexity per symbol. The receiver filter matrix $\mathbf{G_{ZF}}$ can be expressed as [8]:

$$\mathbf{G_{ZF}} = (\mathbf{H^T H})^{-1}\mathbf{H^T} = \mathbf{H}^\dagger \tag{3}$$

where $\mathbf{H^T H} = \mathbf{G}$ is the Gram matrix and $\mathbf{H^T}$ is the Hermitian Transpose of the channel matrix $\mathbf{H}$. Multiplying equation (3) with the received signal $\mathbf{y} = \mathbf{Hx} + \mathbf{n}$ yields:

$$\tilde{\mathbf{y}} = \mathbf{G_{ZF}}\mathbf{y} = \mathbf{G_{ZF}}\mathbf{Hx} + \mathbf{G_{ZF}}\mathbf{n} = \boldsymbol{\Psi}\mathbf{x} + \tilde{\mathbf{n}} \tag{4}$$

where $\boldsymbol{\Psi}$ is the residual interference among the layers, $\tilde{\mathbf{n}}$ is the correlated noise at the ZF detector output. However, the ZF linear based equalization shows poor performance particularly in symmetrical MIMO setups where the signal-to-interference and noise ratios are exponentially distributed and the system suffers frequently from strong noise enhancement. This problem can be solved by taking the receiver noise into account in the design of the filter matrix, i.e. the design of the Minimum Mean Squared Error Detector (MMSE) detector.

The MMSE detector: The MMSE detector can be considered as the ZF detector which takes background noise into account and utilize the knowledge of received signal energies to improve detection. Unlike the ZF detector, the minimum mean-squared error was designed to suppress noise enhancement and at the same time eliminate the residual interference. The linear mapping which incorporates noise minimizes the mean-squared error between the actual data and the soft output of the conventional detector by applying a partial or modified inverse of the correlation matrix. The MMSE optimization problem can be modelled as [8]:

$$\mathbf{G}_{\mathrm{MMSE}} = \left(\mathbf{H}^{\mathrm{T}}\mathbf{H} + \sigma_{\mathrm{n}}{}^{2}\mathbf{I}_{\mathrm{N_T}}\right)^{-1}\mathbf{H}^{\mathrm{T}} \tag{5}$$

where $\sigma_{\mathrm{n}}{}^{2}\mathbf{I}_{\mathrm{N_T}}$ is the $\mathrm{N_T} \times \mathrm{N_T}$ noise covariance matrix. The estimate for the transmitted signal can be obtained by applying the MMSE linear filter as follows:

$$\ddot{\mathbf{y}} = \mathbf{G}_{\mathrm{MMSE}}\mathbf{y} \tag{6}$$

The MMSE detector has been proposed for centralized receivers in AWGN and known fading channel. The amount of modification increases with increase in the background noise. It provides better bit-error rate than the ZF detector, however, the performance of the MMSE detector approaches that of a ZF as the noise goes to zero [6]. The reduction of noise enhancement can be achieved at the expense of increased interference between layers. In addition to this problem, some decision errors in multi-level modulation techniques can be made due to the biased nature of the MMSE estimator. This drawback can be overcome by the use of an unbiased MMSE filter [9]. Another important disadvantage of this detector is that, unlike the decorrelating detector, it requires estimation of the received signal power. Like the decorrelating detector, the MMSE detector faces the task of implementing matrix inversion [6].

4. Simulation Results

The results for the performance of uncoded and LDPC coded ZF and MMSE-MIMO signals in AWGN channel are presented in Figure 2. BPSK modulation scheme was used for all scenarios presented in Figure 2. The uncoded ZF linear based equalization shows poor performance in an AWGN channel. The poor performance is attributed to strong noise enhancement. It can be seen clearly that the problem can be alleviated by applying the Minimum Mean Squared Error (MMSE) filter which takes the receiver noise into account, though at the expense of increased interference between the received signal layers. The MMSE yields a performance gain of a few dBs over the ZF. Encoding the MMSE-MIMO signals with the state-of-the art LDPC coding scheme has the effect of reducing the SNR required to achieve a BER of 1×10^{-2} by about 5dB, i.e., a coding gain of about 5dB thereby boosting the performance of linear detectors. However, coding has the effect of reducing the capacity of the MIMO system. Therefore, a trade-off between capacity, performance and complexity has to be made in practical systems.

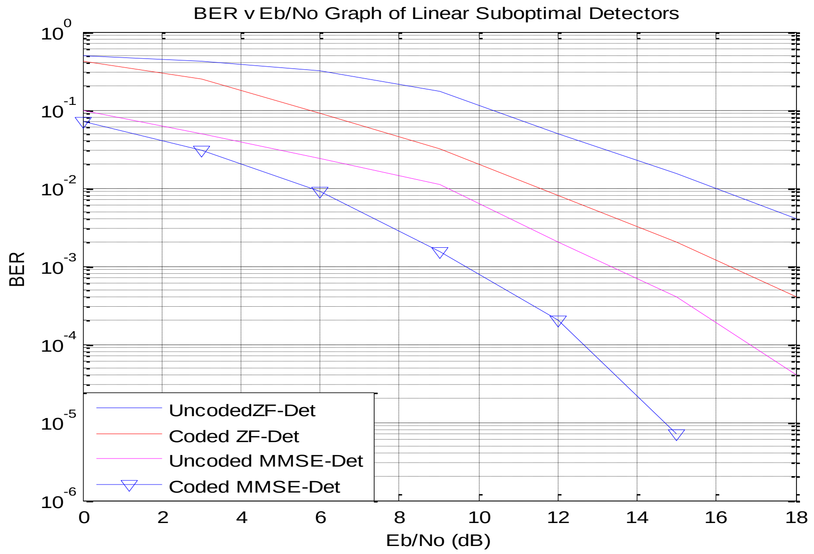

Figure 2: Simulation results for coded and uncoded MIMO signals

5. Conclusions

The performance of linear suboptimal detection strategies have been explored in this paper. It has been shown that the MMSE detector performs very well in the detection of MIMO signals and will therefore, be considered in future work as a strong candidate for precoding schemes for more powerful MIMO detection schemes such as the Sphere Decoder (SD). In such powerful detection schemes, the MMSE will be used to calculate the initial radius for the SD in order to reduce the complexity of the SD and at the same time achieve good performance. It has also been shown that LDPC coding can significantly improve the performance of MIMO detection schemes, thus laying the foundation for a bright future for high capacity MIMO systems.

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