Implementation of a M-Sequence Pseudo Random Binary Sequence audio measurement system based on the Hadamard transform

Marcus Perrett, University College London

Abstract

This paper details the implementation of an industrial audio measurement system by the application of pseudo random binary sequences; where the system under test transfer function is computed using a Hadamard Transform. The procedure and benefits from this method are discussed and result in terms of computational complexity and on-line performance, are presented.

1 Introduction

In a manufacturing environment it is important to adhere to daily quotas to achieve profitability. When testing of a complex system is involved, as part of the manufacturing process, often a large initial outlay is required to build facilities which can provide both functionality and efficient manufacturing. Presented here is a computer based system which requires no test equipment and can therefore offer the lowest initial cost while performing required functionality by utilising advanced frequency characterisation methods.

2 Linear Time Invariant systems

The measurement of a system frequency response over its usable or specified bandwidth is known as the systems frequency characteristics. If a systems impulse response, h(t) that is of Linear Time Invariant (LTI) type the convolution of its input x(t) and its output y(t) will contain the characteristics of the system, so long as the input x(t) is suitably random. In discrete terms [1]

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$$
(1)

In terms of cross correlation, using (m) and (n) as time shifting variables:

$$\Phi_{xy}[m] = \mathbf{E} x[n]y[n+m] \tag{2}$$

where \mathbf{E} is the expectation operator. Using Eq. (1) gives:

$$\phi_{xy}[m] = \sum_{k=-\infty}^{\infty} \phi_{xx}[m-k]h[k]$$
(3)

The result indicates that the input-output cross correlation is equal to the convolution of the input with the impulse response of the system. By utilising flat wideband characteristic signals as an input the relationship in time is thus [2]:

$$R_{yx}(\tau) = h(\tau) * R_{xx}(\tau) \tag{4}$$

where $R_{xx}(\tau)$ is the autocorrelation function of the input signal. The equivalent representation in the frequency domain is:

$$S_{yx}(f) = H(f).S_{xx}(f) \tag{5}$$

As $R_{xx}(\tau)$, the input signal, moves from deterministic to random its autocorrelation function approaches an impulse $(\delta(t))$ function and its power spectrum characteristics $S_{xx}(f)$ becomes flat over the bandwidth. The cross correlation function therefore becomes equal to the system impulse response and the cross spectrum become equal to the frequency response of the channel. Signals which are random only over a specified period, so called pseudo random signals such as Pseudo Random Binary Sequences (PRBS), can also be used for this purpose [7]. The generation of these signals is discussed in the following section.

3 Binary Sequence generation techniques

A shift register with feedback, termed a Linear Feedback Shift Register (LFSR) is used to generate a PRBS signal, the two main types being Galois and Fibonacci [8]. The initial value in the registers is known as the *seed* and the positions of the feedback known as the *taps*. The taps determines the characteristics of the sequence, and the seed need only be non zero. A set of feedback Galois taps are written as:

$$[f_1, f_2, \dots f_n]_g \tag{6}$$

where, f is the tap number and g is the Galois field. Although any tap position will result in a binary sequence output, some may repeat before the maximum length pattern and cannot be said to be random during the

entire period. Only sequences with the maximum number of binary symbols before repeating can be called an M-Sequence. An LFSR can be represented as a polynomial of variable X [8], as shown in Eq. (7):

$$G(X) = g_m \cdot X^m + g_{m-1} \cdot X^{m-1} + g_{m-2} \cdot X^{m-2} + \dots + g_2 \cdot X^2 + g_1 \cdot X + g_0$$
(7)

The coefficients g_n represent the tap weights, with '1' being connected and '0' being unconnected. The order m represents the number of stages. For example a 4-bit LFSR can have the form:

$$G(X) = X^4 + X^2 + X^1 \tag{8}$$

written using the Galois field representation:

$$[4,2]_g \tag{9}$$

The method of determining if a particular G(X) will produce an M-sequence is by determining if G(X) is of primitive form and that it is a factor of $X^L + 1$, where L is the length determined by $2^N - 1$ and N the number of stages. Using Eq. (8), it can be seen that factorisation is possible as shown in Eq. (10) Therefore we can say that the tap positions in the example would not be suitable for M-Sequence generation.

$$X^{4} + X^{2} + X^{1} = \pm (X^{2} + X + 1)(X^{2} - X + 1)$$
(10)

In general it is preferable to find all the tap positions for a given number of stages (N), rather than test each out individually. This can be achieved by first finding the Length (L) of the sequence before repetition, in this example L = 15 before finding the primitive factors of $X^{15} + 1$. The prime factors are:

$$(X+1)(X^4 + X + 1)(X^4 + X^3 + 1)(X^4 + X^3 + X^2 + X + 1)$$
(11)

Primitive factors are the prime factors where the order is the same as the register size (N). The primitive factors are therefore:

$$(X^4 + X + 1)(X^4 + X^3 + 1) \tag{12}$$

It can be seen that the tap positions required to generate an M-sequence are one of the following:

$$[4,1]_g, [4,3]_g \tag{13}$$

4 The Fast Hadamard Transform

A method proposed by Hee [9], based on the work of and Cohn and Lempel [11], and Chu [10], uses a Hadamard Matrix and the Fast Hadamard Transform (FHT) [12] to produce the impulse response of a system when an M-Sequence is applied. The FHT is a reduced complexity variant the Hadamard Transform which is itself based around the Walsh Series [13]. Similar to an FFT, the FHT requires only N.Log(N) operations. For a given M-sequence a matrix can be constructed using a sliding window across a hypothetical output sequence. A 7-symbol length M-Sequence would generate a 7x7 matrix as shown in Eq. (14)

Which can be summarised as:

$$[h] = \frac{1}{7} [M_7] [S_0] \tag{15}$$

 $[M_7]$ is the left circularly delayed version of the original M-sequence with a period of seven times the bit period. $[S_o]$ is the output sequence and [h] the cross-correlation. It can be seen that the matrix is recursive in both rows and columns. Since $[M_7]$ only contains ± 1 , the multiplication of $[M_7][S_o]$ requires only addition and subtraction operations. It has been noted by Lempel [11] that the cross-correlation matrix is similar to a Hadamard matrix. If the required permutation matrices are constructed and the Cross-Correlation Matrix are re-ordered to a Hadamard Matrix, then it allows the use of the FHT. In order to realise this translation, the matrix in Eq. (14) must be decomposed into a L x N and a N x L matrix, here called α and β . β is the inverse of the first N rows of the matrix [M], α is found by:

$$\alpha = \beta' \sigma^{-1} \tag{16}$$

 β' is the Prime Transposed Matrix of β and σ^{-1} is the Matrix Inverse of an N x N matrix of β . Using these 'tags' the resulting cross-correlation matrix from the Device Under Test (DUT) can be converted to a format suitable for use with the FHT.

5 Implementation

The method of obtaining an impulse response using the Hadamard Matrix equivalent is thus:

- 1. Generate an M-Sequence and compute the α and β 'tags'.
- 2. Modify the M-Sequence DC offset, $0 \rightarrow 1$, $1 \rightarrow -1$, and apply to the Device Under Test.
- 3. Capture the resulting M-Sequence and re-order according to β tags.
- 4. Apply a FHT to the β permuted data. The output will be sequency ordered.
- 5. Re-order the post FHT data according to the tags of α .
- 6. Perform a FHT on the data. The systems frequency response has been realised.

Such an implementation lends itself to being implemented in a programming language such as C++, due to its direct memory access properties and efficient maths functions.

5.1 System details

The system purpose is to evaluate the frequency characteristics of an automotive audio amplifier. It is designed around a standard PC, with a PCI I/O card controlling a custom switch box to route signals from the 15 outputs of the DUT to a single line-in connector of a sound card which is used here as an ADC. The signals are in the audio frequency range with a bandwidth range of 20Hz to 22KHz. Other interfaces included but not discussed here are Common Automotive Network (CAN) used to control the DUT and UART over USB to read-back messages from the DUT. The block diagram shows the interconnections in Figure 1.



Figure 1: Hardware system block diagram

5.2 Performance Results

In terms of computational efficiency compared to other methods the FHT was compared with traditional crosscorrelation (fast methods such as [14] and [15] exist but are not evaluated here). The values for Cross-Correlation were taken from [14]

	Addition/Subtraction		Multiplication	
Method	Numerator	Denominator	Numerator	Denominator
Cross Correlation	N(M - N + 1)	2N(M-N+1)	N(M-N+1)	N(M-N+1)
Hadamard	$N\log(N)$		0	0

Table 1: Computational analysis of cross correlation and the FHT

It can be seen that the Hadamard transform offers superior computation performance compared to the cross-correlation method. It is important to note that the 'tags' of any N x N matrix must be computed once for any new value of N, and that Table 1 does not include the data permutation as these do not require any operators and can be implemented with little computation time penalty. The system was evaluated through

measurements of the DUT assessed in term of line throughput against a quotation specification of 30 tested devices per day (where a day here is defined as $T_D = 510$ working minutes at 80% efficiency). Each device has 15 outputs (N_O) and 8 inputs (N_I) and required to be fully tested twice. The time per measurement allowed was thus:

$$M_T = \frac{T_D}{(N_O.N_I.2)} \approx 120s \tag{17}$$

A measurement of the time to perform a single measurement on average was under 52s.

6 Conclusion

This paper concerns a method for generating Pseudo Random Binary Signals and resolving a systems frequency response by application of the Fast Hadamard Transform. The aim is to produce a system utilising this method for testing commercial audio systems. In the paper the Pseudo Random Binary Sequence and Maximum Length Sequence generation is discussed and used to introduce the concept of the Fast Hadamard Transform. This is implemented as an alternative to traditional cross-correlation techniques in conjunction with the generation of a permutation matrix for translation into the format required for the Fast Hadamard Transform. Details of the system construction and connection are provided, along with estimated computational complexity and real timing of the system performance in a manufacturing environment. This system now in use in both the UK and China for manufacturing line testing of the target product.

References

- Wolfgang Fuerst Paul A Lynn. Introductory Digital Signal Processing with Computer Applications. John Wiley and Sons, second edition, 2007.
- [2] Zaiton Sharif and Ahmad Zuri Shaíameri. The application of cross correlation technique for estimating impulse response and frequency response of wireless communication channel. The 5th Student Conference on Research and Development ñSCOReD, 2007.
- [3] Naoya Moriya and Yutaka Kaneday. Impulse response measurement that maximizes signal-to-noise ratio against ambient noise. Acoustical Letters, 28, 2007.
- [4] S. Wang Q. Meng, D. Sen and L. Hayes. Impulse response measurement with sine sweeps and amplitude modulation schemes. 2008.
- [5] Angelo Farina. Simultaneous measurement of impulse response and distortion with a swept sine technique. Paris,France, 2000. Presented at the 108th AES Convention.
- [6] ARCHAMBEAU Dominique STAN Guy-Bart, EMBRECHTS Jean-Jacques. Comparison of different impulse response measurement techniques. Sound and Image Department, University of Liege, 2002.
- [7] Ole Herman Bjor. Maximum length sequence. www.norsonic.com.
- [8] Andrew M. Klapper Mark Goresky. Fibonacci and galois representations of feedback-with-carry shift registers. *IEEE Transactions on Information Theory*, 48, 2002.
- [9] J. Hee. Impulse response measurement using mls. pages 193 205, 1990.
- [10] W Chu. Impulse response and reverberation-decay measurements made by using a periodic pseudo random sequence. pages 135 – 137, 1977.
- [11] Martin Cohn and Abraham Lempel. On fast m-sequence transform. IT-23, 23, 2003.
- [12] Henry Y. L. Mar and C. L. Sheng. Fast hadamard transform using the h diagram. *IEEE Transactions On Computers*, October 1973.
- [13] P.M Ralmondo Tallia. The walsh hadamard transform: An alternative means of obtaining phase and amplitude maps. 1984.
- [14] Jae-Chern Yoo and Tae Hee Han. Fast normalized cross-correlation. Circuits Syst Signal Process, 28:819– 843, 2009.
- [15] Shyamalee Mukherji. Fast algorithms for binary cross-correlation. Geoscience and Remote Sensing Symposium, 1, 2005.