Optimal Two-Way Beamforming with Perfect CSI: An SOCP Formulation

Haixia Chen, Kai-Kit Wong

Abstract

In this paper, we investigate a two-way relay channel where two single-antenna terminals exchange information with each other via a multi-antenna relay terminal operating in half-duplex manner. Our aim is to optimize the two-way beamforming at the relay terminal for minimizing the required relaying power subject to individual users' signal-to-noise ratio (SNR) constraints by exploiting perfect channel state information (CSI). We show that the optimal two-way beamforming solution can be obtained by a second-order cone programming (SOCP) formulation.

I. Introduction

Two-way relay channel (TWRC) is a class of bidirectional channels that has received enormous attention recently, due to its high spectral efficiency of exchanging information between two terminals with the aid of an intermediate relaying terminal. It is also a successful example of using physical-layer network coding [1]–[3]. There have been recent attempts to address the achievable rate region for the TWRC [4]–[12] where some also considered the design of multi-antenna relaying terminal.

In particular, Zhang *et. al* [12] provided a thorough design for two-way beamforming for the TWRC in which a single multi-antenna relaying terminal forwards the array of received noisy signals from the senders to the destination terminals in an amplify-and-forward (AF) fashion. However, the approach relies on a semi-definite programming (SDP) formulation with rank relaxation and uses a linear program to find the best rank-one beamforming vector from the SDP. The optimal two-way beamforming solution is therefore not guaranteed.

In this letter, we readdress the signal-to-noise ratio (SNR) balancing problem for the TWRC in [12]. Our main contribution is a second-order cone-programming (SOCP) formulation, which we show can be used to obtain the exact optimal two-way beamforming solution. Another advantage of the SOCP over the approach in [12] is that SOCP requires much less computational complexity than SDP, let alone the additional complexity needed for the required linear programs.

II. TWRC Model and Problem Formulation

Consider the TWRC as shown in Fig. 1 in which we have two transceivers, labeled as S_1 and S_2, communicating with each other via an M-antenna relaying terminal, labeled as R. It is assumed that there is no direct link between S_1 and S_2. Denote the vector channels from S_1 and S_2 to R, respectively, as $\mathbf{h}$ and $\mathbf{g}$, which represent the flat-fading complex channel coefficients. Communications is achieved by two time slots.

During the first time slot, both S_1 and S_2 simultaneously transmit their messages to R. The signals received at R can be represented in vector form as

$$\mathbf{x} = \sqrt{P_1}\mathbf{h}s_1 + \sqrt{P_2}\mathbf{g}s_2 + \mathbf{v}, \tag{1}$$

where P_1 and P_2 are the respective transmit power of S_1 and S_2, s_1 and s_2 denote the symbols transmitted by S_1 and S_2, respectively, and $\mathbf{v} \in \mathbb{C}^M$ is the complex noise vector at R with independent and identically distributed (i.i.d.) zero-mean

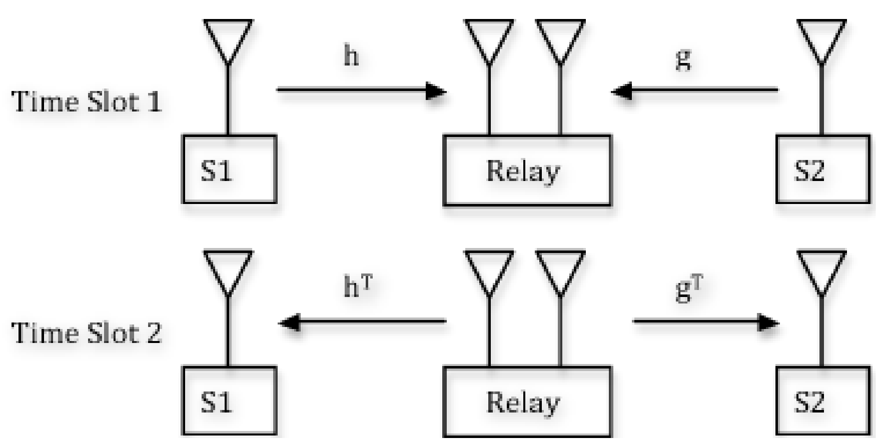

Fig. 1. The TWRC model.

entries and $\mathsf{E}[\mathbf{v}\mathbf{v}^\dagger]=\sigma^2\mathbf{I}$ where $(\cdot)^\dagger$ denotes the complex conjugate transposition. At the second time slot, R transforms $\mathbf{x}$ by a complex weight matrix $\mathbf{W}\in\mathbb{C}^{M\times M}$ to give $\mathbf{W}\mathbf{x}$ and forwards it back to the transceivers S_1 and S_2. As such, the signals received at S_1 and S_2 are given by

$$y_1=\mathbf{h}^T\mathbf{W}\mathbf{x}+\eta_1, \tag{2}$$
$$y_2=\mathbf{g}^T\mathbf{W}\mathbf{x}+\eta_2, \tag{3}$$

in which $(\cdot)^T$ denotes the transposition, η_1 and η_2 denote the respective noise at S_1 and S_2 and they are assumed to be i.i.d. with zero mean and variance of σ^2.

In TWRCs, s_2 is intended for S_1. As s_1 is known for S_1 and with perfect knowledge of channel state information (CSI), the term carrying s_1 can be removed from y_1 to give

$$\tilde{y}_1=\sqrt{P_2}\mathbf{h}^T\mathbf{W}\mathbf{g}s_2+\mathbf{h}^T\mathbf{W}\mathbf{v}+\eta_1. \tag{4}$$

Similarly for S_2, we have

$$\tilde{y}_2=\sqrt{P_1}\mathbf{g}^T\mathbf{W}\mathbf{h}s_2+\mathbf{g}^T\mathbf{W}\mathbf{v}+\eta_2. \tag{5}$$

As a result, the SNRs at the terminals S_1 and S_2 are given by

$$\text{SNR at } \mathsf{S}_1\equiv\gamma_1=\frac{P_2|\mathbf{h}^T\mathbf{W}\mathbf{g}|^2}{\sigma^2\left(\|\mathbf{h}^T\mathbf{W}\|^2+1\right)}, \tag{6}$$
$$\text{SNR at } \mathsf{S}_2\equiv\gamma_2=\frac{P_1|\mathbf{g}^T\mathbf{W}\mathbf{h}|^2}{\sigma^2\left(\|\mathbf{g}^T\mathbf{W}\|^2+1\right)}, \tag{7}$$

where $\|\cdot\|$ returns the Euclidean norm of a vector.

Our objective is to solve the SNR balancing problem which due to quasi-convexity can be tackled by a bisection search via solving the following problem [13]

$$\min_{\mathbf{W}}\ P_1\|\mathbf{W}\mathbf{h}\|^2+P_2\|\mathbf{W}\mathbf{g}\|^2+\mathsf{trace}(\mathbf{W}\mathbf{W}^\dagger)\sigma^2 \tag{8a}$$
$$\text{s.t.}\ \begin{cases}\gamma_1\ge\Gamma_1,\\ \gamma_2\ge\Gamma_2,\end{cases} \tag{8b}$$

where Γ_1 and Γ_2 are the respective target SNRs at S_1 and S_2.

III. An SOCP Formulation

In this section, we present an SOCP formulation to obtain the optimal solution to (8). To do so, we set $\mathbf{A}=[\mathbf{h}\ \mathbf{g}]\in\mathbb{C}^{M\times 2}$ and as in [12] write the singular-value-decomposition (SVD) of $\mathbf{A}=\mathbf{U}\mathbf{\Sigma}\mathbf{V}^\dagger$, where $\mathbf{U}=[\mathbf{U}^\parallel\ \mathbf{U}^\perp]\in\mathbb{C}^{M\times M}$ is a unitary matrix with $\mathbf{U}^\parallel\in\mathbb{C}^{M\times 2}$ and $\mathbf{U}^\perp\in\mathbb{C}^{M\times(M-2)}$, $\mathbf{V}\in\mathbb{C}^{2\times 2}$ is another unitary matrix and

$$\mathbf{\Sigma}=\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\\ 0 & 0\\ \vdots & \end{bmatrix}\in\mathbb{C}^{M\times 2}, \tag{9}$$

in which $\lambda_1\ge\lambda_2\ge 0$ are the singular values of $\mathbf{A}$. As a result, $\mathbf{W}$ can be expressed as [12]

$$\mathbf{W}=[(\mathbf{U}^\parallel)^*\ (\mathbf{U}^\perp)^*]\begin{bmatrix}\mathbf{B} & \mathbf{C}\\ \mathbf{D} & \mathbf{E}\end{bmatrix}[\mathbf{U}^\parallel\ \mathbf{U}^\perp]^\dagger, \tag{10}$$

where $(\cdot)^*$ represent the complex conjugate operation, and $\mathbf{B}\in\mathbb{C}^{2\times 2}$, $\mathbf{C}\in\mathbb{C}^{2\times(M-2)}$, $\mathbf{D}\in\mathbb{C}^{(M-2)\times 2}$ and $\mathbf{E}\in\mathbb{C}^{(M-2)\times(M-2)}$ are matrices of appropriate sizes.

Substituting this structure into the SNR constraints, it can be easily seen that γ_1 and γ_2 are not related to $\mathbf{D}$ and $\mathbf{E}$ and furthermore for minimizing the relaying power, the matrices $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{E}$ should all be set to zeros. Hence,

$$\mathbf{W}=(\mathbf{U}^\parallel)^*\mathbf{B}(\mathbf{U}^\parallel)^\dagger. \tag{11}$$

In what follows, (8) can be rewritten as

$$\min_{\mathbf{B}}\ P_1\|\mathbf{B}\mathbf{h}_1\|^2+P_2\|\mathbf{B}\mathbf{g}_1\|^2+\mathsf{trace}(\mathbf{B}\mathbf{B}^\dagger)\sigma^2 \tag{12a}$$
$$\text{s.t.}\ \begin{cases}P_2|\mathbf{h}_1^T\mathbf{B}\mathbf{g}_1|^2\ge\Gamma_1\left[\sigma^2\left(\|\mathbf{h}_1^T\mathbf{B}\|^2+1\right)\right],\\ P_1|\mathbf{g}_1^T\mathbf{B}\mathbf{h}_1|^2\ge\Gamma_2\left[\sigma^2\left(\|\mathbf{g}_1^T\mathbf{B}\|^2+1\right)\right],\end{cases} \tag{12b}$$

where $\mathbf{h}_1 \triangleq (\mathbf{U}^{\|})^{\dagger}\mathbf{h} \in \mathbb{C}^{2\times 1}$ and $\mathbf{g}_1 \triangleq (\mathbf{U}^{\|})^{\dagger}\mathbf{g} \in \mathbb{C}^{2\times 1}$.

Now, define $\mathbf{w} \triangleq \mathsf{vec}(\mathbf{B}) \in \mathbb{C}^{4\times 1}$, which stacks all the elements of $\mathbf{B}$ to form a column vector, $\mathbf{f}_1 \triangleq \mathsf{vec}(\mathbf{h}_1\mathbf{g}_1^T)$, $\mathbf{f}_2 \triangleq \mathsf{vec}(\mathbf{g}_1\mathbf{h}_1^T)$, $\mathbf{U}_1 \triangleq P_1\mathbf{h}_1\mathbf{h}_1^{\dagger} + P_2\mathbf{g}_1\mathbf{g}_1^{\dagger} + \sigma^2\mathbf{I}$, $\mathbf{U}_2 \triangleq [\mathsf{diag}(\mathbf{U}_1^T, \mathbf{U}_1^T)]^{\frac{1}{2}}$, $\mathbf{H} \triangleq \begin{bmatrix} [\mathbf{h}_1]_1 & 0 & [\mathbf{h}_1]_2 & 0 \\ 0 & [\mathbf{h}_1]_1 & 0 & [\mathbf{h}_1]_2 \end{bmatrix}$, and $\mathbf{G} \triangleq \begin{bmatrix} [\mathbf{g}_1]_1 & 0 & [\mathbf{g}_1]_2 & 0 \\ 0 & [\mathbf{g}_1]_1 & 0 & [\mathbf{g}_1]_2 \end{bmatrix}$, where the notation "$[\mathbf{a}]_n$" returns the nth entry of $\mathbf{a}$ (a similar notation is also used for denoting the entry of a matrix). Thus, (8) becomes

$$\mathbb{P} \mapsto \begin{cases} \min\limits_{\mathbf{w}} & \|\mathbf{U}_2\mathbf{w}\|^2 \\ \text{s.t.} & \begin{cases} P_2|\mathbf{f}_1^T\mathbf{w}|^2 \geq \Gamma_1\left[\sigma^2\left(\|\mathbf{H}\mathbf{w}\|^2+1\right)\right], \\ P_1|\mathbf{f}_2^T\mathbf{w}|^2 \geq \Gamma_2\left[\sigma^2\left(\|\mathbf{G}\mathbf{w}\|^2+1\right)\right]. \end{cases} \end{cases} \tag{13}$$

The rest of this section is devoted to show that (13) has an SOCP solution and such solution is optimal.

To do so, we consider the following SOCP problem:

$$\mathbb{P}_{\mathsf{SOCP}} \mapsto \begin{cases} \min\limits_{\mathbf{w}} & \|\mathbf{U}_2\mathbf{w}\|^2 \\ \text{s.t.} & \begin{cases} P_2(\mathbf{f}_1^T\mathbf{w})^2 \geq \Gamma_1\left[\sigma^2\left(\|\mathbf{H}\mathbf{w}\|^2+1\right)\right], \\ P_1(\mathbf{f}_2^T\mathbf{w})^2 \geq \Gamma_2\left[\sigma^2\left(\|\mathbf{G}\mathbf{w}\|^2+1\right)\right], \\ \mathsf{Im}(\mathbf{f}_1^T\mathbf{w}) = \mathsf{Im}(\mathbf{f}_2^T\mathbf{w}) = 0. \end{cases} \end{cases} \tag{14}$$

The additional constraints in (14) can be rewritten as

$$\frac{\mathsf{Re}([\mathbf{W}]_{1,2}) - \mathsf{Re}([\mathbf{W}]_{2,1})}{\mathsf{Im}([\mathbf{W}]_{1,2}) - \mathsf{Im}([\mathbf{W}]_{2,1})} = -\frac{\mathsf{Re}([\mathbf{h}_1]_2[\mathbf{g}_1]_1) - \mathsf{Re}([\mathbf{h}_1]_1[\mathbf{g}_1]_2)}{\mathsf{Im}([\mathbf{h}_1]_2[\mathbf{g}_1]_1) - \mathsf{Im}([\mathbf{h}_1]_1[\mathbf{g}_1]_2)} = a. \tag{15}$$

To facilitate our analysis for the SOCP problem, we reexpress (14) into the form of real vectors and matrices by defining

$$\tilde{\mathbf{w}} = [\mathsf{Re}(\mathbf{w})^T \ \mathsf{Im}(\mathbf{w})^T]^T, \tag{16}$$

$$\mathbf{F}_1 = \begin{bmatrix} \mathsf{Re}(\mathbf{f}_1) & -\mathsf{Im}(\mathbf{f}_1) \\ \mathsf{Im}(\mathbf{f}_1) & \mathsf{Re}(\mathbf{f}_1) \end{bmatrix}^T \begin{bmatrix} \mathsf{Re}(\mathbf{f}_1) & -\mathsf{Im}(\mathbf{f}_1) \\ \mathsf{Im}(\mathbf{f}_1) & \mathsf{Re}(\mathbf{f}_1) \end{bmatrix}, \tag{17}$$

$$\mathbf{F}_2 = \begin{bmatrix} \mathsf{Re}(\mathbf{f}_2) & -\mathsf{Im}(\mathbf{f}_2) \\ \mathsf{Im}(\mathbf{f}_2) & \mathsf{Re}(\mathbf{f}_2) \end{bmatrix}^T \begin{bmatrix} \mathsf{Re}(\mathbf{f}_2) & -\mathsf{Im}(\mathbf{f}_2) \\ \mathsf{Im}(\mathbf{f}_2) & \mathsf{Re}(\mathbf{f}_2) \end{bmatrix}, \tag{18}$$

$$\tilde{\mathbf{H}} = \begin{bmatrix} \mathsf{Re}(\mathbf{H}) & -\mathsf{Im}(\mathbf{H}) \\ \mathsf{Im}(\mathbf{H}) & \mathsf{Re}(\mathbf{H}) \end{bmatrix} \begin{bmatrix} \mathsf{Re}(\mathbf{H}) & -\mathsf{Im}(\mathbf{H}) \\ \mathsf{Im}(\mathbf{H}) & \mathsf{Re}(\mathbf{H}) \end{bmatrix}^T, \tag{19}$$

$$\tilde{\mathbf{G}} = \begin{bmatrix} \mathsf{Re}(\mathbf{G}) & -\mathsf{Im}(\mathbf{G}) \\ \mathsf{Im}(\mathbf{G}) & \mathsf{Re}(\mathbf{G}) \end{bmatrix} \begin{bmatrix} \mathsf{Re}(\mathbf{G}) & -\mathsf{Im}(\mathbf{G}) \\ \mathsf{Im}(\mathbf{G}) & \mathsf{Re}(\mathbf{G}) \end{bmatrix}^T, \tag{20}$$

$$\tilde{\mathbf{U}}_2 = \begin{bmatrix} \mathsf{Re}(\mathbf{U}_2) & -\mathsf{Im}(\mathbf{U}_2) \\ \mathsf{Im}(\mathbf{U}_2) & \mathsf{Re}(\mathbf{U}_2) \end{bmatrix} \begin{bmatrix} \mathsf{Re}(\mathbf{U}_2) & -\mathsf{Im}(\mathbf{U}_2) \\ \mathsf{Im}(\mathbf{U}_2) & \mathsf{Re}(\mathbf{U}_2) \end{bmatrix}^T. \tag{21}$$

As a result, $\mathbb{P}_{\mathsf{SOCP}}$ becomes

$$\mathbb{P}_{\mathsf{SOCP}} \mapsto \begin{cases} \min\limits_{\mathbf{w}} & \tilde{\mathbf{w}}^T\tilde{\mathbf{U}}_2\tilde{\mathbf{w}} \\ \text{s.t.} & \begin{cases} \tilde{\mathbf{w}}^T\tilde{\mathbf{A}}_1\tilde{\mathbf{w}} \geq \Gamma_1\sigma^2, \\ \tilde{\mathbf{w}}^T\tilde{\mathbf{A}}_2\tilde{\mathbf{w}} \geq \Gamma_2\sigma^2, \\ \dfrac{[\tilde{\mathbf{w}}]_2 - [\tilde{\mathbf{w}}]_3}{[\tilde{\mathbf{w}}]_6 - [\tilde{\mathbf{w}}]_7} = a, \end{cases} \end{cases} \tag{22}$$

where $\tilde{\mathbf{A}}_1 = P_2\mathbf{F}_1 - \Gamma_1\sigma^2\tilde{\mathbf{H}}$ and $\tilde{\mathbf{A}}_2 = P_1\mathbf{F}_2 - \Gamma_2\sigma^2\tilde{\mathbf{G}}$.

Theorem 3.1: The problem $\mathbb{P}$ in (8) has an SOCP optimal solution which is the optimal solution to $\mathbb{P}_{\mathsf{SOCP}}$ in (14) or (22).

Proof: Given the optimal solution to $\mathbb{P}$, say $\mathbf{x}$, we create a vector, $\mathbf{y}$, which differs only on the second, third, sixth and seventh elements. We prove our result by showing that it is feasible to find a suitable $\mathbf{y}$ that satisfies the SOCP constraints in (22) and achieves the same minimum objective value, i.e., Prove of exist of $\mathbf{y}$: If the original optimal $\mathbf{x}$ and the SOCP optimal

$\mathbf{y}$ are getting the same value for the objective function. We have:

$$\mathbf{x}^T\tilde{\mathbf{U}}_2\mathbf{x} = \mathbf{y}^T\tilde{\mathbf{U}}_2\mathbf{y} \quad (23)$$

$$\mathbf{x}^T\tilde{\mathbf{A}}_1\mathbf{x} \geq r_1\sigma^2, \mathbf{y}^T\tilde{\mathbf{A}}_1\mathbf{y} \geq r_1\sigma^2 \quad (24)$$

$$\mathbf{x}^T\tilde{\mathbf{A}}_2\mathbf{x} \geq r_1\sigma^2, \mathbf{y}^T\tilde{\mathbf{A}}_2\mathbf{y} \geq r_1\sigma^2 \quad (25)$$

$$\frac{\mathbf{y}_2 - \mathbf{y}_3}{\mathbf{y}_6 - \mathbf{y}_7} = -\frac{R(\mathbf{h}_2\mathbf{g}_1) - R(\mathbf{h}_1\mathbf{g}_2)}{I(\mathbf{h}_2\mathbf{g}_1) - I(\mathbf{h}_1\mathbf{g}_2)} = a \quad (26)$$

Set:

$$\mathbf{y} = \mathbf{x} + \mathbf{y}^{'}, \mathbf{x} = \mathbf{y} + \mathbf{x}^{'} \quad (27)$$

In which $\mathbf{x}^{'} = -\mathbf{y}^{'}$ Take (27) into (23), set$\tilde{\mathbf{U}}_2\mathbf{x} = \mathbf{u_1}$,$\tilde{\mathbf{A}}_1\mathbf{x} = \mathbf{a_1}$,$\tilde{\mathbf{A}}_2\mathbf{x} = \mathbf{a_2}$;$\mathbf{q} = [\mathbf{0}, \mathbf{1}, -\mathbf{1}, \mathbf{0}, \mathbf{0}, -\mathbf{a}, \mathbf{a}, \mathbf{0}]^{\mathbf{T}}$,$b = (x_3 - x_2) + a(x_6 - x_7)$,we need to prove there exits $\mathbf{y}^{'}$ that:

$$||\tilde{\mathbf{U}_2}^{\frac{1}{2}}\mathbf{y}^{'} + \tilde{\mathbf{U}_2}^{-\frac{1}{2}}\mathbf{u_1}||^{\mathbf{2}} = \mathbf{u_1^T}\tilde{\mathbf{U}_2^{-1}}\mathbf{u_1} \quad (28)$$

$$||\tilde{\mathbf{A}_1}^{\frac{1}{2}}\mathbf{y}^{'} + \tilde{\mathbf{A}_1}^{-\frac{1}{2}}\mathbf{a_1}||^{\mathbf{2}} \geq \mathbf{a_1^T}\tilde{\mathbf{A}_1^{-1}}\mathbf{a_1} \quad (29)$$

$$||\tilde{\mathbf{A}_2}^{\frac{1}{2}}\mathbf{y}^{'} + \tilde{\mathbf{A}_2}^{-\frac{1}{2}}\mathbf{a_2}||^{\mathbf{2}} \geq \mathbf{a_2^T}\tilde{\mathbf{A}_2^{-1}}\mathbf{a_2} \quad (30)$$

$$\mathbf{q}\mathbf{y}^{'} = \mathbf{b} \quad (31)$$

Due to space limitation, detailed provident is ignored here but will exit in future paper. ■

IV. Numerical Results

Since the unsymmetrical matrix in our case, we use the number of variables to determine the general complexity. The worst case about complexity of SDP and SOCP can be regard as $O(N^{3.5})$ with N is the number of variables. (12) shows change of MIMO antenna number has no influence on the simulation complexity. [12] always have 8×8 symmetric matrix which including 36 unknown variables .In our optimization, we always have a 8×1 vector as veriable. So the complexity for our solution is $O(8^{3.5})$, while which is $O(36^{3.5})$ for [12] . Besides, after SDP relaxation, [12] still need to take 1) Cholesky decomposite $X^* = \sum_{i-1}^{r} x_i x_i^T$ for r is the rank of the SDP optimal beamforming matrix which can be computed in $O(r^3)$ and 2) a rank one solution approaching algorithm in which is linear programming which take computing complexity of $O(M^k)$ for some constant k with M is the number of variables.

V. Conclusion

This paper has introduced a SOCP methods for the relay beamforming matrix based SNR balancing optimization problem in TWRC with one MIMO relay exist in the network . We proved the optimization problem has a set of optimal beamforming solutions and at least one of them is our SOCP solution, which is also proved to have lower complexity compare to existing solutions.

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