

# Effective Capacity (EC) Model in Fixed-length Packet-switching Systems

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**Abstract:** Packet delay performance is a key metric of Quality of Service (QoS) in packet-switching systems. One method of characterising this performance is Effective Capacity (EC) model, which is proved to be an accurate model for predicting packets' delay bound violation probability, but EC model is developed based on the assumption that the packet size is infinitesimally small. In this paper, we extend the EC model into a discrete-time domain in which data is sampled, digitised and grouped into a fixed-length packet before it is being transmitted. The analytical and simulation results imply the correctness of our extension.

## 1. Introduction.

Packet-switching system is the modern form of digital communication systems and it behaves in a manner that all transmitted data are firstly grouped into suitable-length packet(s). Its features of high utilisation and capability of dynamically managing resource enable the system to achieve high throughput and satisfaction, resulting in gradual replacement for the old circuit-switching systems since the early 1960s. Nowadays, fixed-length packet-switching systems are commonly seen in Asynchronous Transfer Mode (ATM), Second Generation (2G), Third Generation (3G) and Long-Term Evolution (LTE) systems.

However, packet-switching system introduces variable packet delay because packets have to be buffered and queued when traversing network adapters, switches, routers and other network nodes. For delay-sensitive applications, it will become a severe issue when packet delay exceeds a certain threshold (for VOIP, this threshold is 20ms), thus Quality of Service (QoS) provisioning is required in order to guarantee the packet delay performance.

Literally, huge amount of work has been put in the analysis of packet delay performance and packet delay can be further divided into packet transmission delay, radio propagation delay, signal processing delay and queueing delay. Our work focuses on the queueing delay considering other delays are either fixed or negligible, and continues the research of Effective Capacity (EC) theory. EC model, a cross-layer analytical model of delay performance in single-hop wireless communications, was proposed by Wu and Negi [1] in 2003 and it is virtually the dual model of Effective Bandwidth (EB) model, developed by Chang and Thomas [2] in 1995. Both EB and EC models are derived based on the fluid model, which assumes the traffic is infinitely divisible. Fluid traffic model is widely used in research for its simplicity and generality [3], but no literature has tried to adjust EC theory and make use of this theory in real packet-switching digital communications.

In this paper, we extend Wu's EC model and derive a new formula that describes the delay performances in fixed-length packet-switching systems. Our model is verified by a cross-layer simulation platform that adopts MAC layer (including a buffer) on the top of wireless channel. The work that is presented essentially answers the questions of if it is possible for us to predict delay performance and how the characteristics of wireless channel affects packet delay performance.

The rest of the paper is structured as follows. Section 2 explains the system model that we study. The analysis of delay performance in single-hop fixed-length packet switching system is given in Section 3. Simulation results are then compared and discussed in Section 4. Finally, Section 5 concludes the paper.

## 2. System Model.

Figure 1 shows the system model of a single-hop queueing system. The data source is constantly generating fixed-length packets, which is considered as Constant Bit Rate (CBR) traffic model (traffic pattern of voice data and periodic traffic in sensor networks). Packets have to be first buffered in an

infinite-capacity queue (hence packet loss is eliminated) and will be served in the First-In-First-Out (FIFO) discipline. Wireless channel between the pair of transmitter and receiver by nature introduces transmission errors. We assume that in a certain time period, this channel is stationary and successful transmissions of packets are independent, identically distributed (i.i.d.) random variables, which conform to Bernoulli distribution with a Packet Delivery Ratio (PDR),  $p$ . Moreover, there is a feedback mechanism that guarantees 100% reliability of communications, specifically, the failed packets will be retransmitted (by receiver sending negative feedback to the transmitter) until it is received successfully. Channel behaviour and feedback mechanism are modelled as a Markov Chain with transitions from one state (successful transmission with positive ACK) to another (failed transmission with negative ACK), as shown in Figure 2.

In general, the queuing system that we are studying in this paper is  $C/Geo/1/\infty/FIFO$  in terms of Kendal's notation.

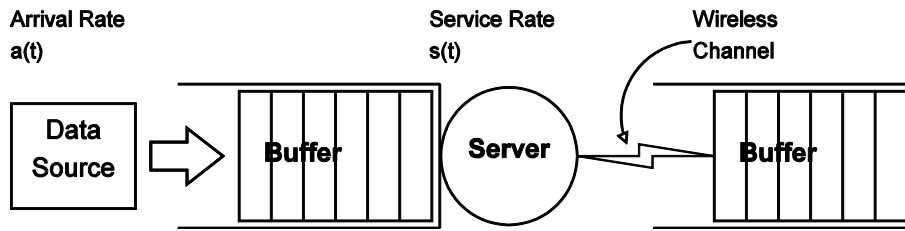


Figure 1: System Model of Single-hop Communication Systems

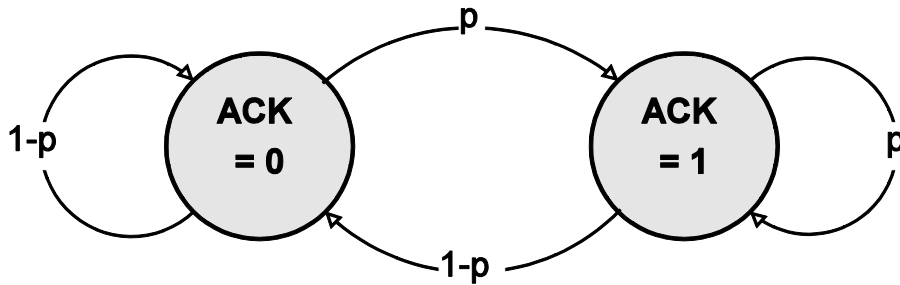


Figure 2: Bernoulli Model for Wireless Channel and Generating ACK

### 3. EC Model in Digital Communication Systems.

Below is the expression that Wu gives in [1] for modelling the delay bound violation probability (DBVP),

$$\sup_t \Pr(D(t) \geq D_{max}) \approx \gamma e^{-\theta D_{max}}. \quad (1)$$

where  $\gamma$  and  $\theta$  are defined as

$$\gamma = \Pr(D(t) > 0) \quad (2)$$

and

$$\theta = \frac{\gamma \times \mu}{\mu \times \tau_s(\mu) + E(Q(t))} \quad (3)$$

(1) is derived based on the fluid traffic model and it could not be used directly in real communication systems without any modification.

Hence, we first give a new equation of DBVP in digital communication systems.

**Proposition 1.** The probability of delay,  $D(t)$ , exceeding a pre-defined delay bound,  $D_{max}$ , in a digital communication system is given by the relation below,

$$\sup_t \Pr(D(t) \geq D_{max}) \approx \gamma e^{-\theta \times \lfloor D_{max}/T_s \rfloor \times T_s} \quad (4)$$

where  $T_s$  is the sampling time step in this system.

**Proof**

It is known that the minimum resolution of queueing delay for each packet is  $T_s$ , the following equation always holds,

$$\sup_t \Pr(D(t) \geq D_{max}) = \sup_t \Pr(D(t) \geq \lfloor D_{max}/T_s \rfloor \times T_s)$$

Then the following equation also holds,

$$\sup_t \Pr(D(t) \geq \lfloor D_{max}/T_s \rfloor \times T_s) = \gamma e^{-\theta \times \lfloor D_{max}/T_s \rfloor \times T_s}$$

**Q.E.D.**

Furthermore,  $\theta$  should be adjusted in digital systems and is described in the following proposition.

**Proposition 2.** Given  $\gamma, \mu, \tau_s(\mu)$  and  $E(Q(t))$ ,  $\theta$  are given as follows

$$\mathcal{L} = 1 - \frac{\mu \times \gamma \times T_s}{\mu \times \tau_s(\mu) + E(Q(t))} \quad (5)$$

and

$$\theta = -\frac{\ln(\mathcal{L})}{T_s} \quad (6)$$

**Proof**

From the probability theory, the following equations must be satisfied

$$\sum_{i=0}^{\infty} \gamma \cdot \exp(-\theta \cdot T_s \cdot i) \cdot T_s = \frac{\mu \times \tau_s(\mu) + E(Q(t))}{\mu}$$

By solving the above two equations and use the attribute

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x} \text{ when } |x| < 1$$

we have (6).

**Q.E.D.**

## 4. Analytical and Simulation Results.

### 4.1 Simulation Setting.

Two simulation scenarios with different PDRs are investigated. The rest values of simulation parameters are listed in Table 1.

**Table 1: Parameters**

Parameter	Value
Bit Rate, $R$	250 kbps
Packet Delivery Ratio, $p$	0.65 and 0.70

Packet Length, $L_{packet}$	133 Bytes
Sampling Step, $T_s$	4.256 ms
Simulation Time	60 seconds
Source Generator Type	Constant Bit Rate (CBR)
Traffic Load, $\mu$	125 kbps

## 4.2. Results.

Figure 3 and Figure 4 show the simulation and analysis results (simulation results are shown in red solid lines while the analytical results are shown in blue dashed lines). The X-coordinates are Delay Bounds (the unit is milliseconds), and the Y-coordinates are DBVP. It can be seen that simulation results well match the analytical results, showing the accuracy of (4).

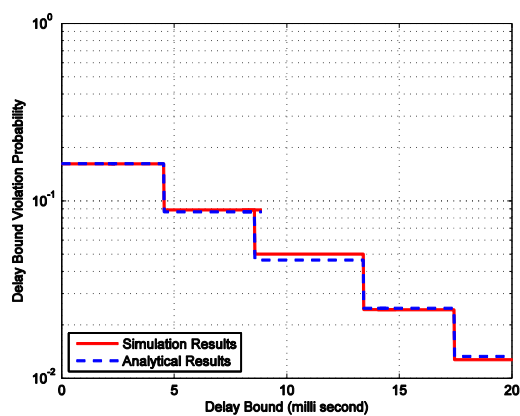


Figure 3:  $p = 0.65$

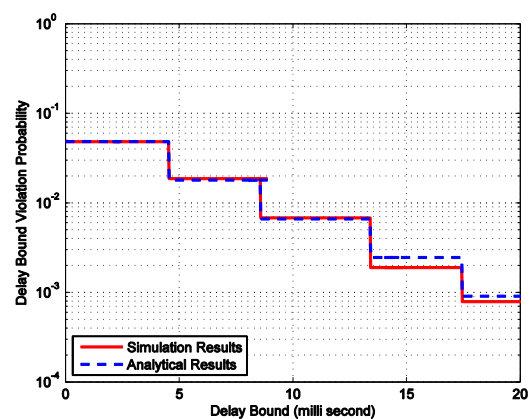


Figure 4:  $p = 0.75$

## 5. Conclusions.

In this paper, we extended the existing single-hop Effective Capacity (EC) model to digital communication systems and derived an adjusted formula for Delay Bound Violation Probability (DBVP). Extensive simulations showed that our mathematical formula of DBVP is accurate and gave a key insight into QoS provisioning in digital communication systems.

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## References.

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