# Extracting light from a scintillator detector using a one dimensional dielectric structure 

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2nd August 2011


#### Abstract

Not all luminescence generated by a scintillator is detected. Photons incident upon the planar surface between the scintillator and the detector beyond the critical angle are trapped by total internal reflection. One method to improve photon extraction by designing a one-dimensional dielectric structure on this surface is explored.


## 1 Introduction

Scintillators are media which convert gamma rays into visible or ultraviolet light by photoluminescence. For this light to be detected it must leave the scintillator and arrive at a detector, typically a photomultiplier tube or avalanche photodiode. At the boundary between the two, a significant source of light loss is reflection due to the refractive index difference between the two media[1]. Significantly the scintillator has a higher refractive index than the attached detector resulting in total reflection beyond a critical angle defined by Snell's law. If a greater proportion of the generated light could arrive at the detector improvements in timing and energy resolution could be achieved. In this work one method to accomplish this is attempted. Layers of dielectric material placed between the scintillator and the detector will alter the reflection and refraction properties of the surface. By appropriate selection of refractive indices and layer composition, the surface can be rendered transparent to a wide range of frequencies and incident angles.

## 2 Theory

### 2.1 Boundary between media

Light incident upon a boundary between two dielectric media will experience reflection and refraction. The properties of these three components can be considered using Maxwell's equations. Incident photons are treated as polarised waves of the form

$$
\begin{equation*}
\vec{E}^{(i)}=E_{0}^{(i)}\left(\vec{A}_{T M}^{(i)}+\vec{A}_{T E}^{(i)}\right) \exp [i(k \vec{r} \cdot \vec{s}-\omega t)] \tag{1}
\end{equation*}
$$

where $E_{0}^{(i)}$ is the total amplitude of the incident wave; $\vec{r}$ is the position vector; $\vec{s}$ is the direction of propagation; $\left|\vec{A}_{T M}\right|$ and $\left|\vec{A}_{T E}\right|$ are the proportions of orthogonal polarisations 'Transverse Magnetic' and 'Transverse Electric' respectively. The superscript $(i)$ refers to incident wave. For non-conducting media the transmitted, $(t)$, and reflected,
$(r)$, waves have the same form as equation 1 Therefore the reflection and transmission amplitude coefficients are defined as

$$
\begin{equation*}
t_{T M}=\frac{A_{T M}^{(t)} E_{0}^{(t)}}{A_{T M}^{(i)} E_{0}^{(i)}}, r_{T M}=\frac{A_{T M}^{(r)} E_{0}^{(r)}}{A_{T M}^{(i)} E_{0}^{(i)}} \tag{2}
\end{equation*}
$$

Transverse electric are similarly defined. For appropriate selection of boundary conditions the Fresnel equations are derived[2]. Note that whilst light propagating through a lossless medium will not experience any attenuation, and thus have a reflectivity of zero, it will experience a phase change determined by the layer thickness. Reflectance is defined as $R=|r|^{2}$. By energy conservation therefore transmittance is defined as $T=1-R$.

### 2.2 Transfer-Matrix Method

For simple structures such as the Fabry-Perot etalon the transmittance and reflectance can be calculated by a geometric summation of propagating terms. For more complex structures this quickly becomes an unwieldy technique[3]. In this work we use the Transfer-Matrix method. In this, each component of a structure is represented by a matrix. Each matrix acts as black box to compare electric field amplitudes before and after a component. The key benefit of such a system is the ability to combine matrices by multiplication, to generate arbitrarily complex structures [4].

The transfer matrix for a lossless system can be defined as

$$
M=\left(\begin{array}{cc}
\frac{1}{t^{*}} & \frac{r}{t}  \tag{3}\\
\frac{r^{*}}{t^{*}} & \frac{1}{t}
\end{array}\right), R=\left|\frac{M[0][1]}{M[1][1]}\right|^{2}
$$

where $t$ and $r$ are the transmission and reflection amplitude coefficients respectively. Structures can be defined as series of propagation and boundary matrices. Furthermore equation 3 also applies to any combination of transfer matrices, allowing extraction of the reflectance and transmittance of any structure.

## 3 Structures

### 3.1 Fabry-Perot Interferometer

A simple structure we can consider is the Fabry-Perot interferometer. This is composed of a finite slab of material $n_{1}$ in a medium of $n_{2}$ which can be considered as three components; two boundaries and one propagating region. The transfer matrix for the interferometer is defined as $M=M_{21} M_{11} M_{12}$ where the subscripts refer to the refractive indices $n_{1}$ and $n_{2}$.

### 3.2 Dielectric Bragg Grating

The dielectric Bragg grating, as shown in figure 1. is composed of alternating layers of dielectric of equal thickness. The resulting reflectance with wavenumber for normal incidence can be seen in figure 2

The transfer matrix for the grating is found by combining the four components defining a single period of the structure and raising the calculated transfer matrix to the power of the number of periods used. This is written as

$$
\begin{equation*}
M=\left(M_{11} M_{12} M_{22} M_{21}\right)^{N} \tag{4}
\end{equation*}
$$



Figure 1: Finite repetition of two equal thickness dielectric layers $n_{1}$ and $n_{2}$.
where $N$ is the number of periods and subscripts refer to refractive indices.


Figure 2: Reflectance with wavenumber for a dielectric Bragg grating with $n_{1}=1.5$ and $n_{2}=3.5$ at normal incidence is considered. Both TE and TM modes are identical for normal incidence.

## 4 Application

### 4.1 Unmatched Medium



Figure 3: Reflectance maps for transverse electric (TE) and transverse magnetic (TM) polarisations with respect to wavenumber and incident angle. As shown beyond the crtical angle defined by the unmatched media, no light is transmitted. At normal incidence all four maps match.

To apply the transfer matrix method to reality, the top and bottom surfaces of any dielectric structure must take into account the surrounding media. This is so reflection and refraction that would occur in a real example is taken into account. For the dielectric Bragg grating discussed in section 3.2, this results in the reflectance maps shown in figure 3 A matched medium is one where the refractive indices at the surfaces of the dielectric structure match those of the surrounding material. In the unmatched case, light is totally reflected beyond a critical angle defined by the structure. In the case we consider these are the scintillator and a layer of grease. The transfer matrix is therefore

$$
\begin{equation*}
M=M_{\text {Scin }}\left[M_{p h C}\right] M_{\text {Grease }} \tag{5}
\end{equation*}
$$

where $M_{p h C}$ is the transfer matrix of the photonic crystal used. In the case of figure 2 this is the dielectric Bragg grating.

### 4.2 Useful Solutions

Using a structure composed of various refractive indices, layer thicknesses and periods, many possible regions of transmittance and reflectance could be possible. Specifically a structure which would allow total transmittance over all angles and a fixed range of wavelengths would be ideal for improving the photon extraction efficiency of a scintillation detector. To this end we consider an optimisation routine for minimising the reflectance for a specific area of a reflectance map such as those shown in figure 2 .

An arbitrary structure is composed of alternating layers of propagation and boundary. For a significant number of components the number of parameters for a brute force solution quickly becomes unmanageable. Therefore we define the refractive index and thickness of layers in terms of a polynomial such that:

$$
\begin{array}{r}
n(x)=A_{1}+B_{1} x+C_{1} x^{2}+D_{1} x^{3}+\ldots \\
d(x)=A_{2}+B_{2} x+C_{2} x^{2}+D_{2} x^{3}+\ldots \tag{7}
\end{array}
$$

where $x$ is the distance from the scintillator surface. Limits are placed upon the constants limiting range of solutions and physical limits such as $n(x) \neq 0$ and $d>0$. A structure is defined in terms of number of layers, total thickness and periodicity using equations to define parameters for the transfer matrices.

## 5 Discussion

The current work produces several useful results, however the complexity of the system being modelled is unrealistically simple. Firstly, the transfer matrices are currently assumed to be lossless. To overcome this requires redefinition of the transfer matrices using the solution for electromagnetic waves in a conducting medium [5]. Secondly the refractive index is currently constant and not frequency dependent. Furthermore no weighting is currently performed to match the actual incidence angle profile for the scintillator. Inclusion of both these would improve the accuracy of the transfer matrix method.

## 6 Summary

Improving light extraction from a scintillator is key to improving the timing and energy resolution. One method to accomplish this by creating a one-dimensional dielectric structure between the scintillator and detector is discussed.

## Acknowledgements

Thanks to my supervisor Dr Ioannis Papakonstantinou and to the CDT for a wonderful first year.

## References

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