Design and Performance Analysis of Enhanced Receivers for Spectrally Efficient FDM System

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Abstract: Spectrally Efficient FDM systems (SEFDM) promise increased spectral utilization at the cost of receiver complexity. In this paper, new reception arrangement for SEFDM system is proposed and analyzed. The signal demodulation is based on the subcarrier structure of the signal. Simple implementation based on the Discrete Fourier Transform (DFT) is introduced, thus significantly reducing the complexity of the receiver. Furthermore, the optimum detector for the system is derived. Numerical evaluation based on the error performance confirms identical performance with the originally proposed SEFDM receivers.

1 Introduction

The proposal of Spectrally Efficient FDM System in [1] has opened a new door for the exploration of enhanced spectral utilization based on defying the orthogonality principle defined for Orthogonal Frequency Division Multiplexing Systems. Despite the attractiveness of the proposal, the loss of orthogonality results in increased system complexity. Generation of the signal has been efficiently designed via standard Inverse Fast Fourier Transform (IFFT) [2] and is realized in FPGA [3] and ASIC [4] hardware. Furthermore, great attention has been dedicated to develop efficient detectors for the SEFDM signal and many alternatives examined supported variable error performance and complexity [5]. In this work the front end of the receiver is re-designed to reduce complexity. In particular, the demodulation of the signal is transformed to be based on the subcarrier structure of the signal, thus eliminating the need to perform separate orthonormalization as in the original SEFDM proposal. Furthermore, the optimal solution for this demodulation arrangement is derived. Numerical results confirm that the same error performance is achieved with the benefit of the reduced complexity.

2 SEFDM Signal Model

In the SEFDM system of [1] the incoming complex symbols, s, are multiplexed onto the non-orthogonal sub-carriers to generate an SEFDM symbol. The sub-carriers are placed at closer spacing compared to OFDM system. For a system of N carriers, the complex baseband SEFDM signal $x(t)$ is expressed as

$$x\left(t\right)=\frac{1}{\sqrt{T}}\sum_{l=-\infty}^{\infty}\sum_{n=0}^{N-1}s_{l,n}\exp\left(j2\pi n\alpha\left(t-lT\right)/T\right) \tag{1}$$

where $\alpha = \Delta f T$, Δf is the frequency distance between the sub-carriers, T is the SEFDM symbol duration, N is number of sub-carriers and $s_{l,n}$ denotes the symbol modulated on the n^{th} sub-carrier in the l^{th} SEFDM symbol. SEFDM carriers deliberately violate the orthogonality condition of OFDM systems where the spacing is equal to the inverse of the OFDM symbol duration. The spectral efficiency improvement of the SEFDM signal over the OFDM approaches $1/\alpha$ with the increase in N.

The discrete SEFDM signal is expressed as

$$X = \mathbf{\Phi} S, \tag{2}$$

where $X = [x_0, \cdots, x_{Q-1}]'$ is a vector of time samples of $x(t)$ in (1), $S = [s_0, \cdots, s_{N-1}]'$ is a vector of input symbols, $[\cdot]'$ denoting a vector or matrix transpose operation and $\mathbf{\Phi}$ is the sampled carriers matrix. $\mathbf{\Phi}$ is a $Q \times N$ matrix whose elements are $\phi_{k,n} = 1/\sqrt{Q} e^{(j2\pi\alpha nk/Q)}$, for $0 \leq n < N$, $0 \leq k < Q$.

3 The Enhanced SEFDM Receiver

The SEFDM receiver comprises two main stages: the demodulator and the detector. The demodulator generates the statistics of the incoming signal and the detector estimates the originally transmitted symbols based on the collected statistics. Previously, the sufficient statistics were generated by correlating the incoming signal with orthonormal bases obtained through the orthonormalization of the non-orthogonal carriers using Gram Schmidt (GS) technique, its modified versions or other techniques [5]. The work described in the sections below removes the need for separate orthonormalization processes and describes a new reduced complexity demodulator with an associated optimal detector.

3.1 The Matched Filter DFT Based Demodulator

The received SEFDM signal, denoted as $r(t)$, arrives contaminated with Additive White Gaussian Noise (AWGN), $w(t)$, therefore is related to the transmitted signal as $r(t) = x(t) + w(t)$. $r(t)$ is sampled to give the vector Y of size $N \times 1$. Correlating this signal with the sampled conjugate carriers Φ^*, over a symbol period T results in matched filtering and will produce the statistics vector R such that

$$R \;=\; \mathbf{\Phi}^* Y = \mathbf{\Phi}^* \left(\mathbf{\Phi} S + W \right). \tag{3}$$

Equivalently, $R[k]$ is expressed as

$$R[k] \;= \mathcal{F}_k^{N/\alpha} \quad \{Y\} \;\text{ for } 0 \leq k < N, \tag{4}$$

where $\mathcal{F}^{N/\alpha}\{A\}$ is the N/α point DFT of the N long sequence A. Therefore, the matched filter (MF) based demodulator can be realized by the Discrete Fourier Transform (DFT) or equivalently the Fast Fourier Transform (FFT). The MF based SEFDM demodulator generates statistics that are expressed by

$$R \;=\; \mathbf{C} S + N_{\mathbf{\Phi}^*}, \tag{5}$$

where $\mathbf{C}$ represents the correlation coefficient matrix given by $\mathbf{C} = \mathbf{\Phi}^* \mathbf{\Phi}$ and $N_{\mathbf{\Phi}^*} = \mathbf{\Phi}^* W$. Based on the obtained statistics R, the detection of the sent symbols is then pursued.

3.2 Optimal Detection for Matched Filter

The second stage of the SEFDM receiver is a detector that estimates the originally transmitted symbols based on the collected statistics at the demodulator stage. In this section, the optimal detector for the statistics obtained via the matched filter is derived. The detector is optimal in the sense that it minimizes the probability of error. The detector applies maximum a posteriori (MAP) criterion to estimate the originally sent data symbols.

For a received statistics denoted by the vector R and a sent message denoted by the vector S_m, the a 'posteriori' probability is expressed as [6]

$$p(S_m|R) = \frac{p(R|S_m)\, p(S_m)}{p(R)} \tag{6}$$

Assuming equi-probability of all transmitted messages S_m, the determinant component in (6) becomes $p(R|S_m)$ whereas R have a multivariate normal distribution with mean vector $U_m = \mathbf{C} S_m$ and covariance matrix $\mathbf{\Psi}$ where $\mathbf{\Psi} = \sigma^2 \mathbf{C}$. The pdf of $p(R|S_m)$ for N carriers system may be expressed as:

$$p(R|S_m) = \left((1/2\pi)^{N/2} \left| det(\mathbf{\Psi}) \right|^{1/2} \right) e^{-\frac{1}{2}(R-U_m)^* \mathbf{\Psi}^{-1} (R-U_m)}. \tag{7}$$

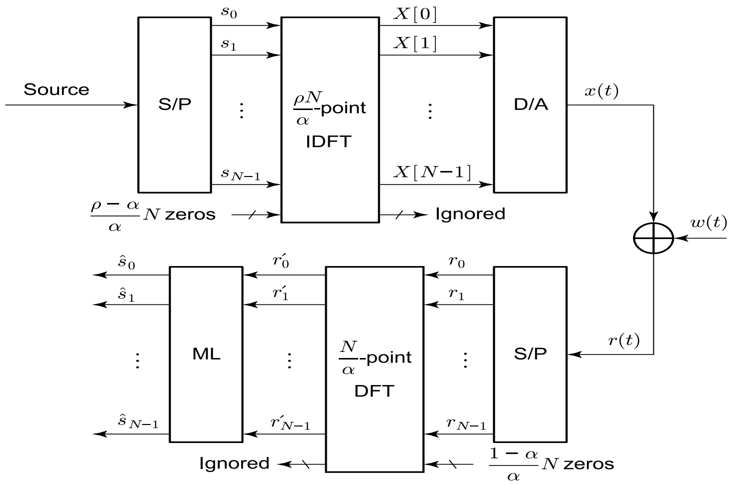

Figure 1: SEFDM block diagram with MF demodulator realized with the DFT block.

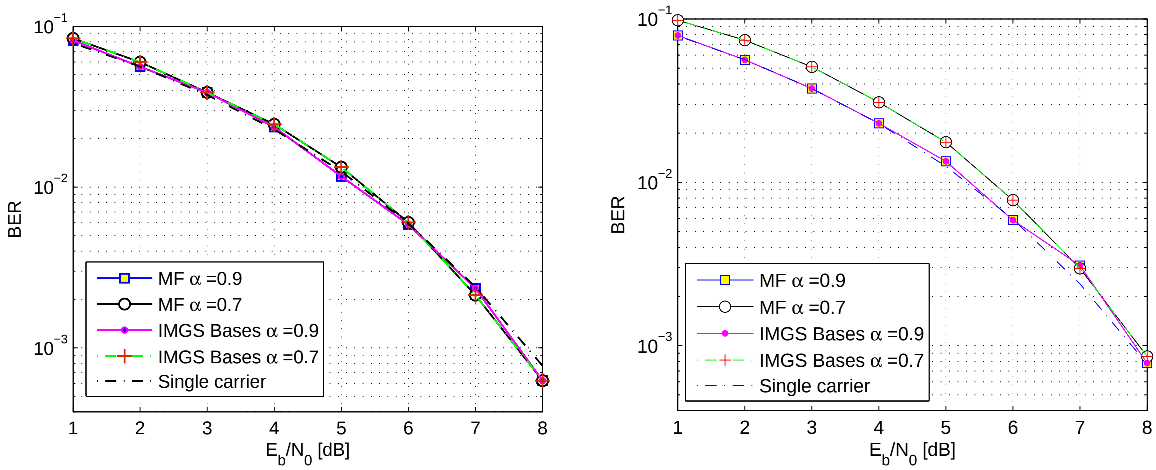

Figure 2: BER of different demodulators and optimal detection for $\alpha = 0.7, 0.9$, 4 subcarrier system for BPSK (left) and QPSK (right) input symbols.

Maximizing the likelihood function is reached by minimizing exponent term $(R - U_m)^* \boldsymbol{\Psi}^{-1} (R - U_m)$, which can be written as $\frac{1}{\sigma^2} \left\| \boldsymbol{\Phi}^{-1} (R - U_m) \right\|$. Formally, this minimization problem is expressed as:

$$\min_{s \in Q^N} \left\| \boldsymbol{\Phi}^{-1} (R - \mathbf{C} S_m) \right\| . \tag{8}$$

Equation (8) shows that the optimal solution for the MF obtained statistics is the one that minimizes the euclidean norm of the cost function. This solution differs from the previously derived optimal solution for the GS based system [5], by the terms Φ^{-1}.

Fig. 1 depicts a block diagram of an SEFDM transceiver. The transmitter is based on the IDFT and is explained in [2, 7]. The newly proposed receiver obtains statistics of the signal by the DFT block and then applies maximum likelihood (ML) criterion to estimate the transmitted symbols as derived in (8).

4 Numerical results

The error performance of the optimal detector derived in section 3.2 is examined by numerical simulations. The optimal solution for the MF based receiver is simulated and its performance compared to the original GS based system of [5]. For these simulations optimal detection is done through an exhaustive search for all possible transmitted combinations of input symbols. Fig. 2 shows the BER performance

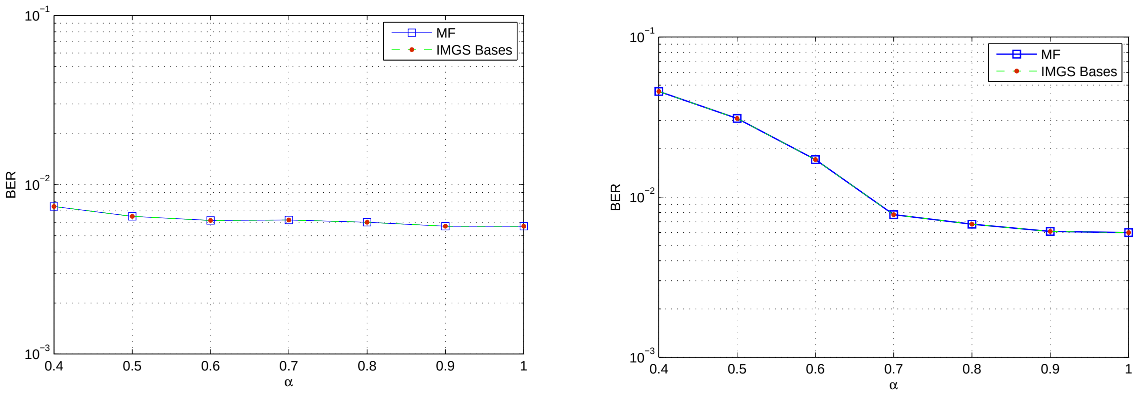

Figure 3: Matched filter detection versus E_b/N_0 for BPSK and QPSK.

for optimal detection for MF and GS demodulator. The figures show that the two detectors have identical performance for different values of E_b/N_0, for BPSK and QPSK input symbols. Furthermore, Fig. 3 confirms that the for all values of α error performance of the optimally detected system is the same for MF demodulation and correlation receivers with orthonormal bases.

5 Conclusions

In this paper, the demodulation of the SEFDM signal based on standard DFT operations is presented. The matched filter based system offers complexity reduction. In addition, the optimal solution in the case of MF is derived. Numerical results confirm that optimal error performance is achieved, thus the reduction complexity comes at no performance penalty.

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