Interest point analysis as a model for the Poggendorff illusion

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ABSTRACT

This paper describes a recognition mechanism based on the relationships between interest points and their properties that is applied to the problem of modelling the Poggendorff illusion. The recognition mechanism is shown to perform in the same manner as human vision on the standard illusion and reduced effects are modelled on a variant without parallels. The model shows that the effect can be explained as a perceptual compromise between alignment of the elements in the oblique axis and their displacement from each other in the vertical. In addition an explanation is offered how obtuse angled variants of the Poggendorff figures yield stronger effects than the acute angled variants.

Keywords: Pattern recognition, Image recognition, Poggendorff, Human vision

1. INTRODUCTION

The Poggendorff illusion in Figure 1 is seen as an apparent misalignment of two collinear diagonal lines that are separated by a parallel sided inducing contour region. Many explanations have been proposed for this effect and some key papers are referenced here. Spehar & Gillam[1] suggest that the illusion arises from the perception of depth in the figure. Morgan[2] proposes a two-stage filter to model the Poggendorff effect, but does not consider the case when no parallel contours are present. Yu *et al*[3] describe a model based on lateral inhibition as applied to a specific variation of the illusion. Bouma *et al*[4] reported a maximum effect when the transversal was inclined at an angle of about 45° to the verticals.

Day[5] and Westheimer *et al*[6] detected a weak effect when no parallels were present in the inducing image. Day[7] found that this effect was maximal when the transversal was tilted at an angle of 45°. Day *et al*[8] later proposed that the Poggendorff effect is the outcome of a perceptual compromise between alignment of the elements in the oblique axis and their displacement from each other in the vertical.

This paper applies a recent method for colour image recognition[9] based upon the relationships between interest points, to black and white figures derived from the Poggendorff illusion and demonstrates how the recognition performs in the same manner as human vision.

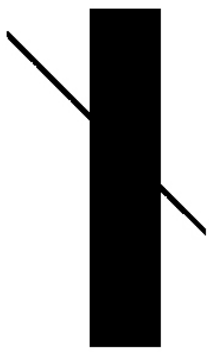


Figure 1. Poggendorff figure with 45° line.

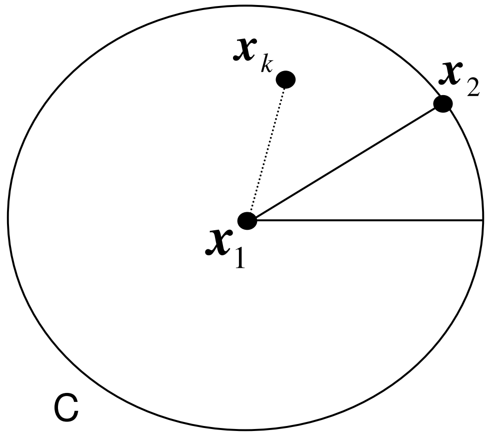


Figure 2. Potential region of interest around $(\boldsymbol{x}_1, \boldsymbol{x}_2)$.

2. RECOGNITION MECHANISM

Interest point generation begins with the selection of a number *n* of ordered pairs of black or white pixels $(\boldsymbol{x}_1, \boldsymbol{x}_2)$ that differ in brightness in an image. Pixels $\boldsymbol{x}_k$ within a circle C centre $\boldsymbol{x}_1$, radius $r_1 = |\boldsymbol{x}_1 - \boldsymbol{x}_2|$, are compared with the pixel $\boldsymbol{x}_2$ (Fig. 2). The *significance* S of $(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is given by

$$S = \left| \sum_{x_k \in C} \boldsymbol{x}_1 - \boldsymbol{x}_k \right| \Big/ \left| \boldsymbol{x}_1 - \boldsymbol{x}_2 \right|^2 \qquad (1)$$

where $\left| u^{x_k} - u^{x_2} \right| = 0$ and u^{x_k} is the brightness value (0,1) at pixel $\boldsymbol{x}_k$.

The orientation of the local gradient is given by

$$\theta = \tan^{-1} \left\{ \left(\sum_{x_k \in C} \boldsymbol{x}_1 - \boldsymbol{x}_k \right)_y \Big/ \left(\sum_{x_k \in C} \boldsymbol{x}_1 - \boldsymbol{x}_k \right)_x \right\} \qquad (2)$$

Let $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ be interest points with orientations θ_i and θ_j. Let $d_{ij} = \left| u^{x_i} - u^{x_j} \right|$.

$$\boldsymbol{x}_i \text{ and } \boldsymbol{x}_j \text{ match if } d_{ij} = 0 \text{ and } \left| \theta_i - \theta_j \right| \le \varepsilon_1 \qquad (3)$$

where ε_1 is a threshold on gradient orientation difference.

Let $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ be interest points from image 1 and $\boldsymbol{x}'_m$ and $\boldsymbol{x}'_n$ be interest points from image 2 where

$$\left| \boldsymbol{x}_i - \boldsymbol{x}_j \right| < R \text{ and } \left| \boldsymbol{x}'_m - \boldsymbol{x}'_n \right| < R \qquad (4)$$

The interest point pair $(\boldsymbol{x}_i, \boldsymbol{x}_j)$ matches the pair $(\boldsymbol{x}'_m, \boldsymbol{x}'_n)$ if $\boldsymbol{x}_i$ matches $\boldsymbol{x}'_m$ and $\boldsymbol{x}_j$ matches $\boldsymbol{x}'_n$ and

$$\frac{(\boldsymbol{x}_j - \boldsymbol{x}_i) \bullet (\boldsymbol{x}'_n - \boldsymbol{x}'_m)}{\left| \boldsymbol{x}_j - \boldsymbol{x}_i \right| * \left| \boldsymbol{x}'_n - \boldsymbol{x}'_m \right|} \ge \lambda \qquad (5)$$

The inner product in equation (5) constrains the difference in slopes between the pairs of points in each image to be less than a certain angle ε_2

$$\left| \varphi_{ij} - \varphi'_{mn} \right| < \varepsilon_2 \text{ where } \lambda = \cos \varepsilon_2 \qquad (6)$$

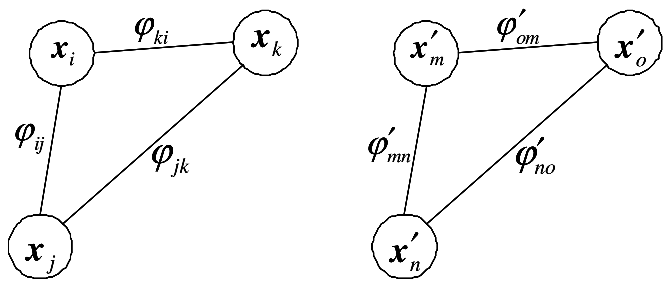


Figure 3. Matching cliques of size 3.

The matching of the pairs of interest points $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ and $\boldsymbol{x}_j$ and $\boldsymbol{x}_k$ has greater reliability if the pair $\boldsymbol{x}_k$ and $\boldsymbol{x}_i$ also match as this shows that all three matching points are in the same relative angular position in each image (Fig. 3). More generally the detection of *N* such points in each image that bear the same angular relationship to a high percentage *P* of the other points in the group will provide greater recognition reliability for larger values of *P* and *N*. The greater reliability is obtained by means of a trade off between the precision of the thresholds $\varepsilon_1, \varepsilon_2$ and the associative power of larger cliques. Figure 4 shows 6 interest points in two images which all match in pairs in both images and form a clique[9]. The cyan lines connect matching interest point pairs, the red lines show the orientation of the local gradient and the circles define the area over which the gradient is calculated. The measure of similarity of image 1 to image 2 used in this paper is defined as the number of matching pairs of interest points $(\boldsymbol{x}_i, \boldsymbol{x}_j)$ in image 1.

The relationship between points is not dependent upon their separation or absolute position and therefore the similarity measure is translation and scale invariant. However, the relationship between points is only partially orientation invariant.

3. RESULTS

A series of Poggendorff figures (320x320 pixels) are analysed by measuring the similarity between a continuous transversal line and a range of figures in which the right-hand line segment is shifted to a series of 10 positions in 6 pixel steps on either side of the collinear position (Fig. 5). The line segments are collinear in position 4 in all cases. Transversals at angles of 30°, 45° and 60° are analysed and in all experiments $(\varepsilon_1, \varepsilon_2, R) =$ (27°,11°,200).

Table 1. Similarity scores for 30°, 45° and 60° transversals in each of 10 positions (4 = collinear position).

	1	2	3	4	5	6	7	8	9	10
60	204	180	190	294	342	372	446	410	284	306
45	72	110	206	266	156	132	264	294	156	14
30	4	8	8	44	102	132	112	332	140	142

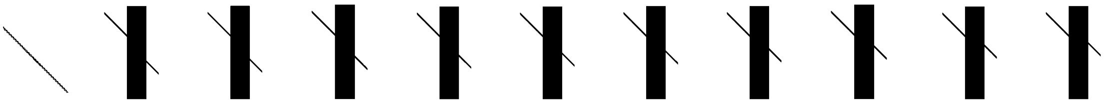
Similarity measures are shown in Table 1. Peaks are seen at position 7 for 60°, 8 for 45° and 8 for 30°. There is also a subsidiary peak at the collinear position 4 for the 45° transversal (Fig. 6). Figure 7 shows the interest point pairs that match interest point pairs on the 45° traversal corresponding to the maximum value in Figure 6. Transversals at 75° and 90° were found to produce no significant peak.

The analysis was also carried out on 45° oblique lines without the vertical parallels (Fig. 8). A peak occurs at position 7, but other values are relatively high (Fig. 9, Table 2) and the peak is not as prominent *c.f.* Figure 6.

Table 2. Similarity scores corresponding to Figure 8.

	1	2	3	4	5	6	7	8	9	10
45	778	760	762	760	760	788	832	810	786	734

Figure 4. Matching interest points (N =6, P = 100%) [9].



1 2 3 4 5 6 7 8 9 10

Figure 5. 45° transversal and shifted versions of Poggendorff figure

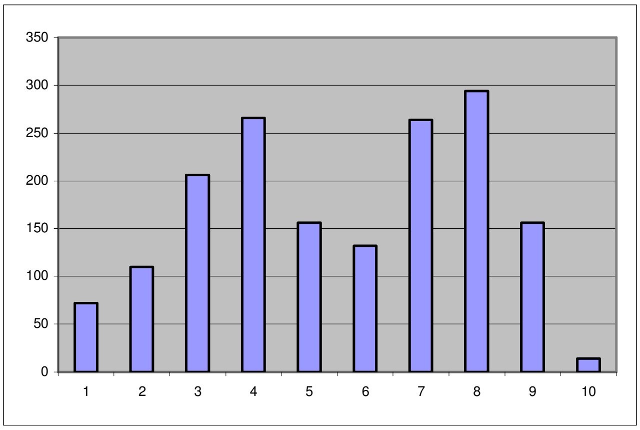


Figure 6. Similarity scores of 45° Poggendorff figures to 45° transversal.

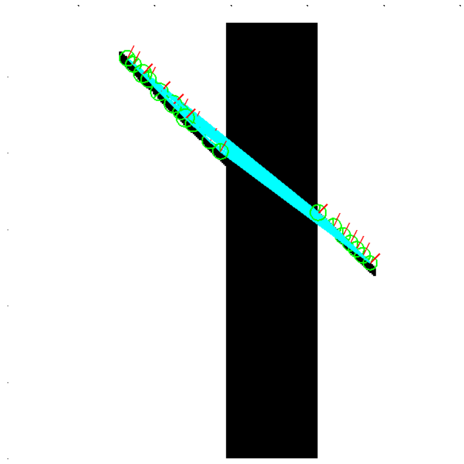


Figure 7. Matching interest point pairs corresponding to the peak score in Figure 6.

1 2 3 4 5 6 7 8 9 10

Figure 8. 45° transversal and shifted versions of segmented oblique.

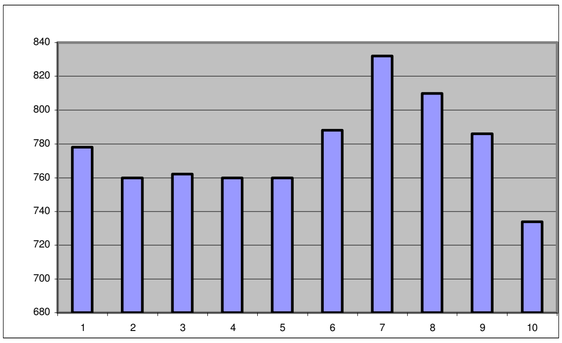


Figure 9. Similarity scores of 45° oblique line segments to 45° transversal. (4 = collinear position).

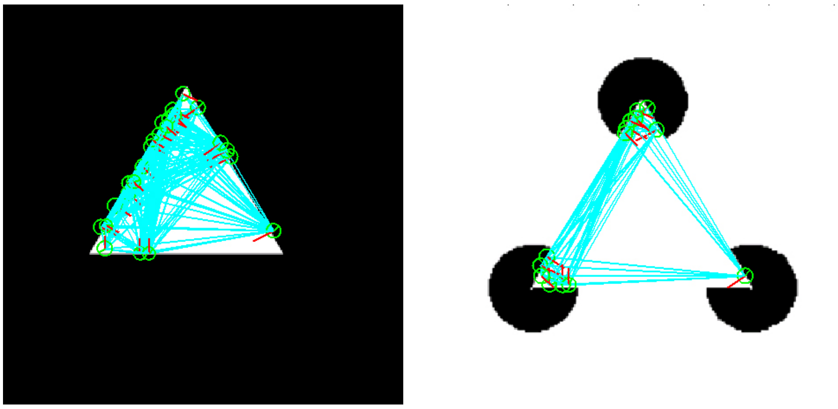


Figure 10. Matching interest point pairs between a white triangle and the Kanizsa figure.

4. DISCUSSION

The potential research interest in a visual recognition mechanism is increased if the performance demonstrates parallels with human visual behaviour. An early experiment employing maximal matching cliques measured the similarity of a white triangle with the Kanizsa triangle. A strong similarity was indicated in which interest point pairs in the Kanizsa figure matched those along the sides of the white triangle (Fig. 10). It might therefore be inferred that such features present in the Kanizsa figure stimulate the perception of the triangle edges in the human visual system. However, more objective results are not easily produced in this case. In contrast the Poggendorff illusion presents a specific geometric distortion concerning straight lines that can be tested.

The results in this paper show that the recognition mechanism produces high similarity scores between continuous transversal lines and Poggendorff figures where the line segments are offset from the collinear position. The peaks occur in the same direction of shift as reported in psychophysical investigations of human vision[4]. The most prominent peaks occur with transversals inclined at the smallest angles to the vertical and decrease towards the perpendicular position (90°). Bouma *et al*[4] found a peak at 45°, but was measuring angular displacement rather than shift which is larger for smaller angles.

The highest similarity scores occur when sufficient matching interest point pairs lie within a distance R of each other whilst at the same time subtend an angle of less than ε_2 with a point pair on the continuous transversal. In the collinear position all interest points on the oblique line segments subtend a small or zero angle with the transversal, but many pairs are separated by distances greater than R. Large shifts can bring point pairs closer but subtend large angles with the transversal. This means that maxima occur at intermediate compromise positions as suggested by Day[8] (see Fig. 7).

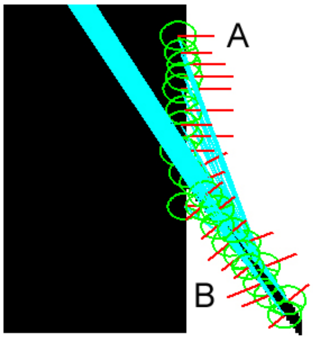


Figure 11. Matching interest point pairs that include interest points on the vertical edge. (Magnified section shown here.)

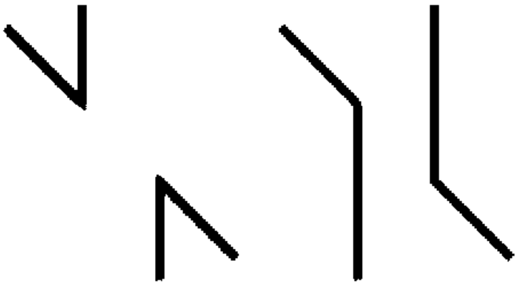


Figure 12. Acute angled and obtuse angled variants of the Poggendorff illusion.

The analysis of figures with no parallels also yielded a peak although this was not as prominent as with the conventional Poggendorff figures. This was also in agreement with earlier behavioural experiments[5,6,7].

It was observed that some matching point pairs included points on the verticals and nearly always within the obtuse angled sector A (Fig. 11). This occurs because the threshold $\varepsilon_1 = 27°$ is large enough to allow gradient orientations along the continuous transversal to match those along the vertical whilst still possessing a slope sufficiently close to the slope of the matching point pairs in the transversal ($\varepsilon_2 < 11°$). Fewer interest points along the vertical within the acute angled sector B are available for matching point pairs because these are closer to those in the line segment and most therefore fail to satisfy $\varepsilon_2 < 11°$. This aspect of the analysis is consistent with psychophysical findings by Day[5] who found that obtuse angled variants of the Poggendorff figures yielded stronger effects than the acute angled variants (Fig. 12).

The extra matching of interest points along the verticals is also consistent with the relatively weaker effect experienced when the verticals are absent in the case of oblique lines alone (Fig. 8).

The interest points defined in this model could correspond to simple or complex cells in the primary visual cortex that are tuned to the same orientation as the interest points. It might also be conjectured that the relative orientation ε_2 of pairs of interest points is represented by a grouping of cortical connections in which relative orientation is reflected by the same cortical architecture as with the simple and complex cells. Maximal cliques of matching interest points would then correspond to the selection of the largest groups of cells in the visual cortex firing simultaneously and thereby signifying a recognition event. This interpretation is analogous to the notion of Neural Darwinism by Edelman[10] in which he points out that neurons that fire together wire together and can form a mechanism for recording and recalling experience.

5. CONCLUSIONS

This paper has described a pattern recognition method and applied it to the analysis of Poggendorff figures. The results indicate that several of the illusory effects are modeled and are consistent with earlier psychophysical investigations. The results also lend support to this approach to image recognition which is seen to reflect several aspects of human vision. This work does not model effects arising from the rotation of the Poggendorff figure which could arise from other independent factors.

Some parallels are highlighted between the identification of maximal cliques of matching interest points and the selection of groups of neurons operating synchronously.

Future work will include a sensitivity analysis of the model parameters and the investigation of related visual illusions.

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