# Width-modulation of Si photonic wires for quasi-phase-matching of four-wave-mixing: experimental and theoretical demonstration

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Abstract: We experimentally demonstrate quasi-phase-matched (QPM) four-wave-mixing (FWM) in silicon (Si) nanowire waveguides with sinusoidally modulated width. We perform discrete wavelength conversion over 250 nm, and observe 12 dB conversion efficiency (CE) enhancement for targeted wavelengths more than 100 nm away from the edge of the 3-dB conversion bandwidth. The QPM process in Si nanowires is rigorously modeled, with results explaining experimental observations. The model is further used to investigate the dependence of the CE on key device parameters, and to introduce devices that facilitate wavelength conversion between the C-band and mid-IR. Devices based on a superposition of sinusoidal gratings are investigated theoretically, and are shown to provide CE enhancement over the entire C-band. Width-modulation is further shown to be compatible with zero-dispersion-wavelength pumping for broadband wavelength conversion. The results indicate that QPM via widthmodulation is an effective technique for extending the spectral domain of efficient FWM in Si waveguides.

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#### 1. Introduction

Four-wave-mixing (FWM) in silicon (Si) nanowire waveguides (SiNWGs) has been investigated as a potential means to alleviate the ever-increasing demand on electronics in future information networks, by facilitating all-optical signal processing near the C-band (1530 nm – 1565 nm) [1–3]. Significant work has been reported in this area, including wavelength conversion [1–4], format conversion [5], signal regeneration [6], signal multicasting [7], time-division-demultiplexing [8, 9], modulation instability [10], and tunable delays [11]; however, FWM in SiNWGs is also starting to emerge as a promising building block for a diverse set of applications well outside the C-band.

One new and particularly exciting arena for Si photonics is the mid-infrared (mid-IR) (by which we here include from 2  $\mu$ m – 20  $\mu$ m) [12], where Si is nearly an ideal nonlinear material due to its strong third-order susceptibility and lack of parasitic two-photon-absorption (TPA) and subsequent free-carrier-absorption (FCA) above  $\lambda = 2200$  nm. In fact, in just the past few years, FWM [13–20], self-phase-modulation [21], supercontinuum generation [22], modulation instability [18, 22], and optical parametric amplifiers [13] and oscillators [23], have been experimentally demonstrated at mid-IR wavelengths in SiNWGs. These results show that SiNWGs can be employed for applications in mid-IR systems, including those for chemical sensing and free-space communications [12]. These and other systems applications can be enhanced *via* an effective means to interconvert between the telecommunications bands and mid-IR wavelengths, opening the possibility of leveraging the large device and materials infrastructure at C-band wavelengths [15, 19, 20, 24]. In this

connection, FWM provides a useful approach to interconvert between these two spectrally distant frequency domains.

Four-wave mixing is a parametric process sensitive to the relative phase between copropagating waves, hence its wavelength conversion bandwidth is intrinsically limited by dispersion [25]. SiNWGs exhibit tight modal confinement as a result of the high-index contrast inherent to the Si-on-insulator (SOI) platform. This contrast both enhances the effective nonlinearity and induces substantial waveguide dispersion [26–29], which has a detrimental effect on the FWM conversion bandwidth and effectively limits the spectral reach of FWM. It has been shown that dispersion can be controlled through the SiNWG geometry, and engineered such that the waveguide-dispersion contribution becomes the main component of the overall device dispersion [26–29]. This approach has facilitated broadband, continuously tunable FWM devices [16], however, given the high sensitivity of these devices to SiNWG geometry, and the recent interest in converting signals over spectral spans larger than 1000 nm, it is important to seek alternative solutions.

Here, we explore quasi-phase-matching (QPM) by sinusoidally modulating the width of a SiNWG (termed *w*-modulation) so as to reduce the relative phase mismatch between copropagating waves [30], and consequently increase the FWM conversion efficiency (CE) for signals outside the conversion bandwidth of uniform waveguides (Fig. 1). We experimentally demonstrate QPM *via* a *w*-modulated SiNWG fabricated using CMOS-compatible processes, typically by amounts of only tens of nanometers, and record a 12 dB CE enhancement for signals covering a 15 nm sideband, more than 100 nm beyond the edge of the conversion bandwidth. We rigorously model the FWM QPM process and determine that the modulation strength and grating period can be used as an effective means to tune the bandwidth and spectral location of the enhanced sideband. We use the model to theoretically explore devices designed to facilitate wavelength conversion between the C-band and mid-IR, including devices based on the superposition of sinusoidal gratings, and devices used in conjunction with zero-dispersion-wavelength (ZDWL) pumping, and conclude that QPM *via w*-modulation is an effective technique for increasing the spectral reach for FWM.



Fig. 1. Illustration of QPM FWM in a *w*-modulated SiNWG. Input pump and signal waves interact *via* FWM to generate an idler. The conversion efficiency of this FWM process is enhanced for specific wavelengths due to the periodic modulation of waveguide parameters.

#### 2. Principle of operation

We consider degenerate FWM, in which case conservation of energy and momentum can be expressed as

$$2\omega_p = \omega_s + \omega_i \tag{1}$$

and

$$\Delta\beta = 2\gamma P_p - \Delta\beta_{lin} = 2\gamma P_p - (2\beta_p - \beta_s - \beta_i)$$
<sup>(2)</sup>

where  $\omega_{p,s,i}$  and  $\beta_{p,s,i}$  are the respective pump/signal/idler angular frequency and propagation constant, respectively, with  $\Delta\beta_{lin}$  the linear wave vector mismatch and  $\Delta\beta$  the net wave vector mismatch. The term  $2\gamma P_p$  represents the nonlinear wave vector shift originating from the Kerr effect, where  $\gamma$  is the effective nonlinearity and  $P_p$  the pump power. Note that we assume the

powers of the signal and idler are much smaller than  $P_p$  (the undepleted pump approximation), such that the Kerr nonlinear shift of the wave vector mismatch is entirely induced by the pump beam. Frequency dispersion results in a phase mismatch, which generally increases as the pump, signal, and idler become increasingly separated spectrally. In the presence of a grating, the phase matching condition becomes  $2\gamma P_p - (2\beta_p - \beta_s - \beta_i) - G = 0$ , where  $G = 2\pi m/\Lambda$ , is the reciprocal lattice vector, *m* is an integer, and  $\Lambda$  is the grating period. Thus, the FWM phase mismatch can be compensated by a grating *via* QPM [30–32] when  $G = \Delta\beta$ , such as a refractive index modulation of period

$$\Lambda = 2\pi m / \Delta \beta. \tag{3}$$

There have been a number of theoretical investigations centered on how to realize QPM with such a periodic modulation in SiNWGs, including using the intrinsic anisotropic nonlinearity of Si itself to induce a nonlinear index grating in a ring resonator or cascaded waveguide bends [33, 34], or using *w*-modulation to periodically modulate waveguide properties responsible for the linear and nonlinear phase-shifts [35, 36]. QPM  $\chi^{(2)}$  processes in Si has also been explored by using periodic regions of high-stress Si<sub>3</sub>N<sub>4</sub> to both induce a  $\chi^{(2)}$  nonlinearity in Si by breaking its centro-symmetry, while also serving to phase match the corresponding second-order processes such as difference frequency generation [37] and second-harmonic generation [38]. Here, we facilitate FWM QPM through a modulation formed *via* sinusoidal *w*-modulation, as illustrated in Fig. 1. For such a structure, the SiNWG width can be described along the length of the waveguide (*z*-direction) by

$$w(z) = \Delta w \sin\left(\frac{2\pi z}{\Lambda}\right) + w_{\rm DC},\tag{4}$$

where  $\Delta w = (w_2 - w_1)/2$ ,  $w_{DC} = (w_1 + w_2)/2$ , and  $w_{I,2}$  are the minimum and maximum width, as illustrated in Fig. 2(e).

QPM via w-modulation has similarities to an index-grating, except the variation in the index of refraction amounts to a variation in effective-index ( $n_{eff}$ ) resulting from the modulated waveguide cross-section. However, w-modulation induces more complex grating effects than a simple index grating, in that the modulated cross-section also strongly modulates the optical confinement factor and consequently the overlap between the optical mode and Si core. This modulated overlap results in a modulation of both the grating nonlinearity,  $\gamma$ , and the strength of TPA (as described in detail in Sec. 5), which, through the nonlinear Kerr phase shift and free-carrier-dispersion effects, respectively, results in an additional contribution to the net modulated phase-shift with period  $\Lambda$ . In addition, the periodic change in the SiNWG geometry results in significant modulation of all higher-order-dispersion terms, thus leading to complex dynamics of the interacting optical beams.

As an example of the magnitude of  $\Lambda$  needed to phase match a practical device, we consider a 250 nm × 600 nm (height × width) SiNWG with SiO<sub>2</sub> cladding. For a 20 dBm pump at  $\lambda_p = 1543$  nm, and signal at  $\lambda_s = 1687$  nm, FWM results in an idler at  $\lambda_i = 1421$  nm, with a corresponding  $\Delta\beta = 6280$  m<sup>-1</sup>. As can be determined by Eq. (2), the main part of this large wave vector mismatch originates from the linear mismatch  $\Delta\beta_{lin}$ , with the Kerr nonlinear shift constituting ~1% of the total  $\Delta\beta$ . Using Eq. (3), this phase mismatch can be compensated (in a first-order process, m = 1) by using a grating of period  $\Lambda = 1$  mm.

# 3. Width-modulated waveguides: design and fabrication

We experimentally investigate the example presented at the end of Sec. 2, by fabricating both a *w*-modulated waveguide, and a straight waveguide for comparison. Both devices have a height of 250 nm and a length of 5 mm and are fabricated on an SOI wafer with 3  $\mu$ m buried-oxide layer at the Center for Functional Nanomaterials at Brookhaven National Laboratories.

The straight waveguide has a constant width of 600 nm, while the *w*-modulated waveguide has a sinusoidally varying width as defined by Eq. (4), with  $\Delta w = 30$  nm,  $w_{DC} = 600$  nm, and  $\Lambda = 1$  mm. Both devices have a parabolic inverse-taper mode-converter at each facet, as illustrated in Fig. 2(e), for efficient coupling between the on-chip devices and input/output lensed-tapered fibers (LTFs) [39]. The waveguides were defined *via* 100 keV e-beam lithography using a ~110 nm thick HSQ negative e-beam resist, and subsequent inductivelycoupled-plasma Si etching using HBr/Cl chemistry. The HSQ hardmask was left in place, and the devices were covered with 3 µm of SiO<sub>2</sub> *via* plasma-enhanced-chemical-vapor-deposition (PECVD). Finally, the devices were cleaved near the tip of the inverse-tapers. No postprocessing procedures were attempted to smooth the waveguide sidewalls.

Figure 2(e) shows a sketch of the *w*-modulated device, including the inverse-taper modeconverters. For  $\Lambda = 1$  mm, 5 periods fit along the 5-mm waveguide. Figures 2(a) and 2(b) show typical scanning-electron-microscope (SEM) images of the maximum and minimum waveguide widths taken from a separate sample without top-cladding. These images illustrate the relatively moderate  $\Delta w$ , compared to the total SiNWG width, needed for this device; this is especially important for reducing back-reflections associated with a strong periodic perturbation. Section 5.1 discusses the effect of varying  $\Delta w$  on the CE spectrum in more detail. Figure 2(c) shows a tilted-view SEM image of a typical fabricated device, exhibiting smooth sidewalls, and Fig. 2(d) shows a cross-section of the tip of the inverse-taper, revealing undercutting as a result of inadvertently over-etching the SOI device layer.



Fig. 2. (a) SEM image of the narrowest region. (b) SEM image of the widest region. (c) SEM image of SiNWG sidewall. (d) SEM of the tip of the inverse-taper. (e) Sketch of a *w*-modulated SiNWG.

## 4. Experimental setup and results

The experimental setup is shown in Fig. 3. The continuous-wave (CW) pump is formed from a C-band source tuned to 1543 nm and amplified with an erbium-doped fiber-amplifier (EDFA). Either a tunable C-band, or U/L-band laser serves as the CW signal. The state-of-polarization (SOP) of the pump and signal are controlled by polarization controllers (PCs), and multiplexed onto one fiber by a wavelength-division-multiplexer (WDM). The output from the WDM is passed through a linear polarizer to ensure both the pump and signal share the same SOP. The pump and probe beams are then launched on-chip using an LTF, while another PC is used to selectively excite either the quasi-transverse-electric (QTE) or the quasi-transverse-magnetic (QTM) guided mode of the SiNWG. Another LTF is used to collect the output light, and the output optical beam is sent to an optical spectrum analyzer (OSA). A -20 dB tap and -10 dB tap are used to monitor the launch power and output power, respectively.



Fig. 3. FWM experimental set-up: PC = polarization controller, WDM = wavelength division multiplexer, LP = linear polarizer, LTF = lensed tapered fiber,  $P_{RX}$  = power meter, OSA = optical spectrum analyzer.

We first investigate FWM in the straight waveguide. The pump and signal power immediately before the input LTF was measured to be 22.4 dBm and 5 dBm, respectively, and the fiber-to-fiber insertion loss was measured to be 10.1 dB. By switching the pump on and off, we find the nonlinear loss originating from TPA and FCA to be ~0.8 dB for the power levels used. The remaining contributions to the 10.1 dB insertion loss are estimated to be ~3 dB/facet of coupling loss (based on finite-difference time-domain studies of the coupler geometry) and ~6.6 dB/cm of propagation loss. The signal is tuned between 1544 nm and 1690 nm, generating an idler between 1419 nm and 1541 nm. Figure 4(a) shows the output spectra for varying  $\lambda_s$  and fixed  $\lambda_p = 1543$  nm. As the signal is detuned to longer wavelengths, the generated idler becomes weaker, a direct result of increased phase mismatch between the pump, signal, and idler for larger detuning. The CE for different signal wavelengths, measured under the convention that the CE is equal to the ratio between the output idler power ( $P_{i,out}$ ) and the output signal power ( $P_{s,out}$ ) [1, 2, 4, 7, 15, 17, 20],

$$CE = \frac{P_{i,out}}{P_{s,out}},$$
(5)

is shown in Fig. 5(a). The result shows a 3-dB conversion bandwidth of ~70 nm, with a peak CE of -25.5 dB. The CE outside the conversion bandwidth drops significantly, *i.e.* for  $\lambda_s = 1676$  nm, the CE is found to be ~-48 dB.

In the case of the *w*-modulated SiNWG, the input pump and signal powers are measured to be the same as those for the straight device. The fiber-to-fiber insertion loss was measured to be 10.4 dB, indicating the *w*-modulated device has similar losses to the straight device. As in the uniform SiNWG FWM experiment,  $\lambda_s$  is again tuned between 1544 nm and 1690 nm and the resulting idlers are shown in Fig. 4(b). The spectra follow the same trend as the straight SiNWG; however, for signal wavelengths near 1660 nm, an increase in the power of the generated idler is observed. This enhancement is a result of QPM originating from *w*modulation, and occurs near signal wavelengths calculated in Sec. 2, for  $\Lambda = 1$  mm. Differences between the calculated and measured signal wavelengths occur primarily due to deviations from a perfectly rectangular cross-section for the fabricated devices as a result of inadvertently over-etching the SOI device layer [as shown in Fig. 2(d)], thus affecting the dispersion parameters. These ideas are discussed in more detail in Sec. 5.1.



Fig. 4. FWM spectra for  $\lambda_p = 1543$  nm and varying  $\lambda_s$ , in the case of (a) a 250 nm × 600 nm uniform waveguide and (b) a *w*-modulated waveguide with  $w_{DC} = 600$  nm,  $\Delta w = 30$  nm,  $\Lambda = 1$  mm, h = 250 nm. The background noise level is raised for C-band wavelengths since a different laser was used to access this spectral region, as illustrated in Fig. 3. The generated idlers also interact with the pump *via* cascaded FWM to produce higher order idlers, which can be observed as artifacts on the red side of the pump.

Figure 5(a) shows the measured CE, exhibiting an enhanced sideband centered around 100 nm beyond the edge of the 3-dB CE bandwidth, at  $\lambda_s = 1660$  nm, with a peak CE of ~-38 dB. In fact, the enhanced sideband is relatively broadband itself, with a 3-dB bandwidth of ~15 nm, which is more than sufficient to support FWM of multiple dense WDM channels at high data-rates. Figure 5(b) shows a plot of the CE enhancement, calculated by finding the ratio between the measured CE exhibited by the *w*-modulated to the straight SiNWG, revealing that a maximum CE enhancement of ~12 dB is observed for  $\lambda_s = 1668$  nm.



Fig. 5. (a) Experimentally determined FWM CE spectrum demonstrating a CE enhancement near  $\lambda = 1668$  nm for the *w*-modulated device (•) compared to the straight device (•). (b) Measured CE enhancement as a function of signal wavelength, indicating a ~12 dB enhancement for  $\lambda_s = 1668$  nm. (b) is found by taking the difference (in dB) between the two curves in (a).

## 5. Theoretical analysis of QPM FWM in Si nanowires

#### 5.1 QPM FWM model

We model the QPM FWM process using a set of coupled nonlinear Schrödinger equations, which takes into account TPA, FCA, free-carrier-dispersion (FCD), linear losses, and Kerr effects, based on the methods outlined in [3, 27, 29, 40]. The three coupled equations can be described by the rapidly varying beam amplitudes,  $A_j = a_j \exp(i\beta_j z)$ , which change with propagation as

$$\frac{dA_j}{dz} = \frac{da_j}{dz} e^{i\beta_j z} + i\beta_{0,j}A_j,$$
(6)

where j = p, s, i for the pump, signal, and idler, respectively, with the slowly varying amplitudes  $a_p$ ,  $a_s$ , and  $a_i$ , normalized such that they are measured in units of the square root of power ( $\sqrt{W}$ ) described by:

$$\frac{da_{p}}{dz} = -\frac{c\kappa_{p}(z)}{2nv_{gp}(z)} \left(\alpha_{p}^{in} + \alpha_{p}^{FC}(z)\right)a_{p} + i\frac{\omega\kappa_{p}(z)}{nv_{gp}(z)}\delta n_{p}^{FC}(z)a_{p} + i\frac{3\omega_{p}}{4\varepsilon_{0}A_{0}(z)v_{gp}(z)} \left(\frac{\Gamma_{pppp}(z)}{v_{gp}(z)}\left|a_{p}\right|^{2}\right)a_{p},$$
(7)

for the pump, and

$$\frac{da_{s,i}}{dz} = -\frac{c\kappa_{s,i}(z)}{2nv_{g(s,i)}(z)} \left( \alpha_{s,i}^{in} + \alpha_{s,i}^{FC}(z) \right) a_{s,i} + i \frac{\omega\kappa_{s,i}(z)}{nv_{g(s,i)}(z)} \delta n_{s,i}^{FC}(z) a_{s,i} 
+ i \frac{3\omega_{s,i}}{4\varepsilon_0 A_0(z)v_{g(s,i)}(z)} \left( 2 \frac{\Gamma_{(s,i)pp(s,i)}(z)}{v_{g,p}(z)} |a_p|^2 \right) a_{s,i} 
+ i \frac{3\omega_{i,s}}{4\varepsilon_0 A_0(z)v_{g(i,s)}(z)} \frac{\Gamma_{(s,i)p(i,s)p}(z)}{v_{g,p}(z)} a_p^2 a_{i,s}^* e^{i(\Delta\beta_{lin}(z))},$$
(8)

for the signal or idler. Here,  $\kappa$  is the confinement factor,  $\alpha^{in}$  the intrinsic loss coefficient,  $A_0 = h \times w$  the area of the SiNWG core,  $v_g$  the group velocity,  $\Delta \beta_{in} \approx -\beta_{2,p} \Delta \omega^2 - (1/12)\beta_{4,p} \Delta \omega^4$ the linear phase mismatch, and  $\Delta \omega = |\omega_p - \omega_s|$  the frequency spacing between the pump and signal. The dispersion coefficients  $\beta_2$  and  $\beta_4$  are the coefficients used in the Taylor expansion of  $\beta$  about the pump frequency, and are found by:

$$\beta_n = \frac{\partial^n \beta}{\partial \omega^n}.$$
(9)

The introduction of TPA-induced free-carriers results in a change in the material index of Si due to the free-carrier plasma dispersion effect, and an increase in absorptive loss *via* FCA. The carrier-induced index change can be described through a modified Drude model fit to experimental data [41] given by

$$\delta n_{FC} = \frac{-e^2}{2\varepsilon_0 n\omega^2} \left( \frac{N}{m_{ce}^*} + \frac{N^{0.8}}{m_{ch}^*} \right),\tag{10}$$

assuming an equal number of electrons and holes, and the FCA loss is given by [41]

$$\alpha_{FC} = \frac{e^3 N}{\varepsilon_0 c n \omega^2} \left( \frac{1}{\mu_e m_{ce}^{*2}} + \frac{1}{\mu_h m_{ch}^{*2}} \right), \tag{11}$$

with c the speed of light in vacuum, e the charge of electrons,  $\mu_e(\mu_h)$  the electron (hole) mobility, and  $m_{ce}^*(m_{ch}^*)$  the electron (hole) effective mass. The TPA-induced free-carrier-density in steady-state operation can be found, assuming that the majority of carriers are generated by degenerate TPA, by

$$N = \frac{6\tau_c \Gamma_p}{8\varepsilon_0 \hbar A_0^2 v_{gp}^2} \left| A_p \right|^4, \tag{12}$$

where  $\tau_c$  is the free-carrier-lifetime.  $\Gamma$  is the effective susceptibility,

$$\Gamma_{klmn} = \frac{\mathbf{A}_0 \int_{\mathbf{A}_0} \boldsymbol{e}_k * (\mathbf{r}_\perp; \boldsymbol{\omega}_k) \cdot \boldsymbol{\chi}^{(3)}(\mathbf{r}_\perp) : \boldsymbol{e}_l(\mathbf{r}_\perp; \boldsymbol{\omega}_l) \boldsymbol{e}_m^*(\mathbf{r}_\perp; \boldsymbol{\omega}_m) \boldsymbol{e}_n(\mathbf{r}_\perp; \boldsymbol{\omega}_n) d\mathbf{A}}{\zeta_k \zeta_l \zeta_m \zeta_n}, \quad (13)$$

with ( $\vdots$ ) denoting the tensor product between the  $\chi^{(3)}$  susceptibility tensor and three modal electric field vectors, *e*. The indices *k*,*l*,*m*,*n* refer to *i*,*p*,*s* where appropriate in Eq. (7) and Eq. (8).

$$\zeta = \int_{\mathbf{A}_{\infty}} n^2(\mathbf{r}_{\perp};\omega) \left| \boldsymbol{e}(\mathbf{r}_{\perp};\omega) \right|^2 d\mathbf{A}$$
(14)

is the electric field intensity integrated over an infinite area,  $A_{\infty}$ , and index of refraction profile,  $n(\mathbf{r}_{\perp};\omega)$ .  $\Gamma_{klmn} = \Gamma_{klmn}' + i\Gamma_{klmn}''$  is complex-valued, where  $\Gamma_{klmn}'$  describes the Kerr nonlinearity, and  $\Gamma_{klmn}''$  describes TPA [27, 29]. The effective nonlinear coefficient,  $\gamma$ , is related to the effective susceptibility by  $\gamma = 3\omega\Gamma' / (4\varepsilon_0 A_0 v_g^2)$ . Note that all electric field components are included in the waveguide modes in Eq. (13), including the longitudinal component [42]. For more information on this general method, please see [3, 27, 29, 40].

It is clear from Eq. (13) that  $\Gamma$  is sensitive to waveguide geometry by means of the normalized overlap between the waveguide modes and the Si core, and therefore *w*-modulation affects both the Kerr effect and TPA (and similarly FCA and FCD). As Eq. (8) shows, this periodic *z*-dependence of the main parameters of the SiNWG, and in particular that of  $\Gamma$ , provides the basic mechanism by which the linear component of the wave vector mismatch,  $\Delta\beta_{lin}$ , is compensated by the grating. Note, however, that we did not include in our model the linear grating-induced coupling between the co-propagating beams, as this wave interaction is not phase-matched for the grating parameters used in our study.

Material dispersion for Si and SiO<sub>2</sub> is included through Sellmeier equations [43], and mode profiles and effective indices are determined through the finite element method (FEM). Dispersion parameters are found by fitting  $n_{eff}(\lambda)$ , as determined by FEM, to 7th-order polynomials, and then calculating the corresponding derivatives with respect to frequency. To take into account sinusoidal w-modulation, we functionalize parameters sensitive to the waveguide geometry, including  $A_0$ ,  $v_g$ ,  $\Gamma$ , and  $\beta_i$  (i = 2, 4), to polynomials as a function of width. Once these parameters are functionalized to w, they can then be easily written as a function of z for any w-modulation profile w(z) and included in the model described by Eqs. (6-14). Note that when  $\Delta w \sim w$  or  $\Lambda \sim \lambda$ , it becomes inaccurate to describe the optical modes of w-modulated waveguides by adiabatically varying modes, as in Eqs. (6-14), and wave backscattering effects can no longer be neglected. Instead, one must include the full Bloch modes of the deeply-modulated grating [44]. As a side note, one possible advantage of deeply-

modulated gratings is that the corresponding interacting Bloch modes can be engineered to have low group-velocities, resulting in a slow-light-enhanced FWM processes. This fact suggests that strongly *w*-modulated SiNWGs merit additional theoretical and experimental investigations; however for the remainder of this paper we concentrate on rigorously exploring the weakly modulated regime to achieve efficient FWM.

As an example of how waveguide parameters change with waveguide width, Fig. 6 shows  $\gamma(z)$  and  $\beta_{2,p}(z)$ , for w(z) described by Eq. (2) with  $w_{DC} = 600$  nm,  $\Delta w = 30$  nm,  $\Lambda = 1$  mm,  $\lambda = 1543$  nm, and h = 250 nm. For this particular modulation  $\gamma$  varies by ~14 W<sup>-1</sup>m<sup>-1</sup> and  $\beta_{2,p}$  by ~1.82 × 10<sup>-25</sup> s<sup>2</sup>m<sup>-1</sup>, demonstrating that even for modest width variations, important waveguide parameters can vary substantially. For example, in this case,  $\beta_2$  varies by ~30% for a width variation of ~5%, suggesting that strong gratings can be formed by weak w-modulations.



Fig. 6. Variation of two important waveguide parameters,  $\beta_2$  and  $\gamma$ , along the length of a *w*-modulated waveguide for  $w_{DC} = 600$  nm,  $\Delta w = 30$  nm, and  $\Lambda = 1$  mm.

## 5.2 Modeling with experimental parameters

We use the method described in Sec. 5.1 to model QPM FWM, and solve the system of Eqs. (6)-(14) using a fourth-order Runge-Kutta method. For the calculations, we use the same parameters as in the experiment, including an estimated coupled pump and signal power of 19.4 dBm and 2 dBm, respectively, intrinsic loss of 6.6 dB/cm, h = 250 nm, and  $\lambda_p = 1543$  nm. We assume  $\tau_c = 3$  ns based on values measured for devices with similar geometry and claddings [45]. We vary  $\lambda_s$  between 1543 nm and 1690 nm, and calculate the CE for both the *w*-modulated and uniform waveguide. In order to accurately capture the dependence of the waveguide parameters on width for the actual device tested, we use the cross-sectional profile outlined in the Fig. 7(a) inset in our calculations to approximate the complex sidewall profile measured in the SEM image of the taper cross-section [Fig. 2(d)], resulting from the Si overetch. The group-velocity dispersion parameter,  $D_{\lambda}$ , for this waveguide profile is calculated by

$$D_{\lambda} = -\frac{2\pi c}{\lambda^2} \beta_2, \qquad (15)$$

and the resulting dispersion curve is shown in Fig. 7(a). For  $\lambda_p = 1543$ , the waveguide considerably dispersive, with  $D_{\lambda} = 265 \text{ ps nm}^{-1} \text{ km}^{-1}$ .

Figure 7(b) shows the result of the modeled QPM FWM process, and matches well with the experimentally determined plot in Fig. 5(a), including a calculated CE enhancement of ~12 dB, verifying that the CE enhancement observed experimentally is a result of QPM. The nonlinear loss originating from TPA and FCA is also found to be 0.5 dB, which matches well with the nonlinear loss of 0.8 dB determined experimentally. The relatively low nonlinear loss indicates that higher conversion efficiencies can be obtained with increased pump powers. The inset of Fig. 7(b) shows the calculated peak conversion efficiency in the enhanced sideband for varying coupled pump powers. Saturation of the CE can be observed, however, for increasing pump power, the corresponding asymptotic value is close to -20 dB. The CE of the enhanced sideband can be further increased by pumping above 2200 nm to avoid TPA, using a *p-i-n* structure to sweep away free-carriers [45], or pumping closer to a ZDWL.

Some of the differences between the calculated and measured CE spectrum are likely attributed to the material dispersion of the PECVD  $SiO_2$  cladding (a Sellmeier equation for thermal oxide is used to approximate the material dispersion of the PECVD oxide [43]), and fabrication variations in the waveguide cross-sectional profile. In general, however, the good comparison between our experimental and theoretical results allows us to use our model in the following sections to explore the impact of important grating parameters, and accurately describe the performance of devices designed to facilitate FWM over larger spectral spans.



Fig. 7. (a) Wavelength dependence of  $D_{\lambda}$  of the waveguide used in the calculations, for w = 600 nm. The inset shows a diagram of the cross-section used to approximate the sidewall etched profile observed in Fig. 2(d). (b) Calculated CE spectrum for the straight (black line), and w-modulated waveguide case (red line), indicating a ~12 dB CE enhancement for the w-modulated device. Results match well with the measured CE spectrum shown in Fig. 5(a). The inset shows how the peak CE of the enhanced sideband varies with pump power.

# 5.3 Exploring the influence of the w-modulation parameters on the CE spectrum

To better understand the dependence of the CE spectrum on the parameters from Eq. (4), we perform additional calculations for varying  $\Delta w$  and  $\Lambda$ . We first use a constant  $w_{DC}$  of 600 nm, height of 250 nm, waveguide length of 5 mm, and  $\lambda_p = 1543$  nm, and consider a rectangular cross-sectional geometry instead of the complex geometry used to fit the experimental results. Figure 8(a) shows how the CE spectrum varies with  $\Lambda$  for  $\Delta w = 30$  nm. We see that as  $\Lambda$  increases, the phase-matched signal wavelength is tuned closer to the pump, since  $\Lambda \propto^{-1} \Delta \beta$ , as can be seen in Eq. (1). In fact, relatively broadband CE enhancement over the uniform waveguide case can be realized for long periodicities, illustrating the potential of using *w*-modulated QPM to enable broadband wavelength conversion.



Fig. 8. Calculated CE spectrum for (a)  $\Delta w = 30$  nm, and  $\Lambda$  varying between 0 and 5 mm (offset 10 dB for clarity), (b)  $\Lambda = 1$  mm, and varying  $\Delta w$  between 0 and 60 nm (offset 10 dB for clarity). (c) A more detailed plot of the enhanced sideband for the  $\Delta w = 20$  nm  $\Lambda = 1$  mm case, demonstrating spectral splitting.

We also perform calculations for  $\Lambda = 1$  mm and varying  $\Delta w$ . We see that as the modulation becomes stronger a spectral splitting in the CE enhanced sideband occurs, which is explained by the fact that for the *w*-modulated grating the waveguide width oscillates between  $w_1$  and  $w_2$ . These two waveguide widths are associated with different  $\Delta\beta$  for a given

set of wavelengths; therefore, different  $\lambda_s$  are compensated by the grating for a given  $\Lambda$ . This can be verified by looking at the  $\Delta w = 20 \text{ nm } \Lambda = 1 \text{ mm } \text{case}$ , as shown in Fig. 8(c), and calculating the phase-matched  $\lambda_s$  for both  $w_1$  and  $w_2$ . We find that for  $w_1 = 580 \text{ nm}$ ,  $\Delta \beta = 2\pi/\Lambda$  is satisfied for  $\lambda_s = 1670 \text{ nm}$ , while for  $w_2 = 620 \text{ nm}$ ,  $\lambda_s = 1700 \text{ nm}$  is compensated. In both cases, the compensated signal wavelength corresponds perfectly with the peaks of the bifurcated sideband, as shown in Fig. 8(c). This splitting characteristic serves as a way to both increase the sideband bandwidth, and to tune the location of the peak CE enhancement.

# 5.4 QPM FWM between C-band and mid-IR

The experiment described in Sec. 4 demonstrates enhanced discrete wavelength conversion over 250 nm, between an idler near 1544 nm and a signal near 1690 nm, as determined by the geometry of the *w*-modulated SiNWG, and  $\lambda_p$ . These targeted wavelengths were chosen based on considerations of observing the FWM CE enhanced sideband with laboratory components available to us; however, this QPM approach can facilitate FWM over even larger spectral spans. In order to illustrate the feasibility of applying QPM in *w*-modulated SiNWGs over broad spectral ranges in future devices, we use the model described in Sec. 5.1 to explore FWM wavelength conversion between the C-band and mid-IR.

We choose a waveguide geometry of 250 nm × 500 nm, which supports a single QTE mode both in the C-band and near  $\lambda = 2000$  nm, as shown by the Fig. 9(a) insets. A *w*-modulated grating with  $w_{DC} = 500$  nm,  $\Delta w = 15$  nm, and  $\Lambda = 280 \ \mu\text{m}$  is employed, which phase-matches FWM between  $\lambda_i = 1558$  nm and  $\lambda_s = 2000$  nm for a pump wavelength of  $\lambda_p = 1750$  nm, as described by Eq. (1). Propagation loss of 3 dB/cm is assumed, along with a coupled pump power of 20 dBm. Figure 9(a) plots the resulting FWM CE spectrum, both on the blue and red side of the pump, and shows a peak CE of ~-20 dB within the conversion bandwidth. A CE sideband enhanced by as much as 23 dB is observed near 2000 nm and 1558 nm, allowing as high as -30 dB FWM CE between the C-band and mid-IR region.



Fig. 9. (a) FWM CE spectrum for a uniform 250 nm × 500 nm SiNWG (black line), and a *w*-modulated SiNWG with  $w_{\rm DC} = 500$  nm,  $\Delta w = 15$  nm, and  $\Lambda = 280 \ \mu m$  (red line). A CE enhancement is observed between the C-band and mid-IR, for the *w*-modulated device. Insets show the  $E_x$  component of the fundamental QTE mode, for a wavelength of 1550 nm and 2000 nm, demonstrating that the mode is confined at both wavelengths. (b) Enhanced sideband on both the blue- and red-side of the pump, for different  $\Delta w$ , showing the ability to tune the sideband bandwidth. Spectra are offset by 10 dB for clarity.

The bandwidth of the enhanced spectral region is about 31 nm (18 nm) on the red (blue) side of the pump, characterized as 3-dB down from the CE at the center of the sideband. The sideband bandwidth can be tuned *via*  $\Delta w$ , as described in Sec. 5.3, and Fig. 9(b) shows the enhanced sidebands for various  $\Delta w$  (plots are offset by 10 dB for clarity). These results illustrate that the bandwidth for  $\Delta w = 15$  nm (31 nm on the red-side of the pump) is more than a factor of 3 greater than that of  $\Delta w = 5$  nm (9 nm on the red-side of the pump) as a result of spectral splitting, while also maintaining the peak sideband CE within 3 dB, at the expense of a ~9 dB decrease in the CE at the center of the sideband.

An alternate approach to tune the spectral shape of the enhanced region, is by using a grating profile, w(z), formed by a linear superposition of N sinusoidal gratings of different periodicity,  $\Lambda_n$  (n = 0, 1, ... N), and weights  $c_n$ :

$$w(z) = \left[\frac{1}{N} \sum_{n=1}^{N} c_n \sin\left(\frac{2\pi z}{\Lambda_n}\right)\right] + w_{\rm DC}.$$
 (16)

In order to provide relatively flat enhancement, we use a superposition of N = 10 sinusoidal gratings, each with  $c_n = 10$  nm, and each with a periodicity targeting specific idler wavelengths throughout the C-band as listed in Table 1. The corresponding w(z) is plotted in the Fig. 10 inset, and the resulting CE spectrum of this w-modulated SiNWG is shown in Fig. 10, demonstrating discrete wavelength conversion over as much as 518 nm between the entire C-band and the spectral region between 1986 nm and 2048 nm. Note that an increase of the CE of more than 20 dB can be easily achieved by using this approach. The flatness of the enhanced sideband can be further optimized by including more sinusoidal gratings in the superposition, and through fine control over each  $c_n$  coefficient and  $\Lambda_n$  in order to compensate for the complex CE response of a straight SiNWG.

 Table 1. Grating periodicities and weights used to for the grating profile plotted in Fig.

 10. The table also shows the idler and signal wavelengths targeted by each periodicity.

n	1	2	3	4	5	6	7	8	9	10
$c_n$ (nm)	10	10	10	10	10	10	10	10	10	10
$\Lambda_n (\mu m)$	194	204	216	229	232	243	257	273	290	308
$\lambda_{i}$ (nm)	1530	1534	1539	1543	1544	1548	1552	1556	1561	1565
$\lambda_{\rm s} ({\rm nm})$	2044	2037	2028	2021	2019	2013	2006	1999	1991	1985



Fig. 10. The FWM CE spectrum resulting from both a uniform 250 nm × 500 nm SiNWG (black line), and a *w*-modulated device with w(z) formed by a superposition of sinusoidal *w*-modulations. A CE enhancement is observed across the entire C-band. The inset shows the waveguide width, w(z), formed by using Eq. (10), with  $c_n$  (n = 1, 2, ..., 10) = 10 nm,  $w_{DC} = 500$  nm, and  $\Lambda_n$  as listed in Table 1.

Generally, complex profiles w(z) can be fabricated, such as chirped or apodized gratings, in order to control the spectral shape of the enhancement region. In fact, this QPM technique is not limited only to *w*-modulation, but can be extended to any grating that modulates phase sensitive waveguide parameters described in Sec. 5.1. For example, binary or sinusoidal gratings formed by modulating the waveguide cladding [46, 47], or modulating the waveguide height [48] could also be used for QPM FWM.

#### 5.5 QPM for SiNWGs pumped near the zero-dispersion-wavelength

Our experimental and theoretical analysis to this point has focused on realizing QPM in the case of high dispersion; however, another approach to extending the conversion bandwidth of FWM is to pump near a ZDWL in order to minimize phase mismatch over a large spectral range. This technique relies on controlling the spectral location of the ZDWL through careful design of the waveguide geometry. It is therefore important to understand if near-ZDWL-pumping can be used simultaneously with *w*-modulation to both maintain the broadband standard conversion-bandwidth, and realize an enhanced sideband beyond the edge of the conversion bandwidth.

We consider a structure similar to that in [16], which is a waveguide with a 300 nm height and 30 nm slab height, completely clad with SiO<sub>2</sub>. We choose a waveguide width of 1000 nm, which positions the ZDWL near 1855 nm. In Ref [16]. is has been shown that the broadband conversion bandwidth is highly sensitive to waveguide cross-section, so it is important to find a weaker perturbation grating that facilitates QPM while also largely maintaining the same cross-section. We achieve this weak perturbation by using slab-modulation [49]. In particular, we use the same 300 nm × 1000 nm waveguide with 30 nm slab height, except with a sinusoidally varied slab width ( $w_{slab}$ ), as shown in the inset of Fig. 11. In this case, the slab width profile can be defined using a similar equation as Eq. (4), however with  $\Delta w_{slab}$  and  $w_{DC,slab}$  referring to the definitions in the inset of Fig. 11.



Fig. 11. The CE spectrum for a 300 nm × 1000 nm SiNWG with 30 nm slab height and  $\Delta w_{slab}$  = 100 nm,  $w_{DC,slab}$  = 600 nm, and  $\Lambda$  = 3000  $\mu$ m. An enhanced CE sideband is present due to QPM, and facilitates FWM over more than 1000 nm, between 1625 nm and 2695 nm. The peak CE of the sideband is –25 dB, only 8 dB below the CE near the pump wavelength. The inset shows a schematic illustrating a slab-modulated SiNWG.

Figure 11 shows the result of modeling this waveguide structure, with  $\Delta w_{\text{slab}} = 100$  nm,  $w_{\text{DC,slab}} = 600$  nm, and  $\Lambda = 3000 \,\mu\text{m}$ . A coupled pump power of 20 dBm is assumed, along with a 1.5 cm waveguide length, 1 dB/cm propagation loss,  $\lambda_p = 2025$  nm, and  $\tau_c = 3$  ns. Since such a broad wavelength range is considered in this example, we calculate  $\Delta \beta_{lin}$  directly for use in Eq. (8) from  $\beta_{p,s,i} = 2\pi n_{eff}(\lambda_{p,s,i})/\lambda_{p,s,i}$  and Eq. (2), where  $n_{eff}(\lambda_{p,s,i})$  is found from FEM calculations, instead of using a Taylor expansion about the pump wavelength.

The results of these calculations show that the straight waveguide exhibits a broad conversion bandwidth of 600 nm—a direct result of pumping near the ZDWL—and is similar to the results reported in [17]. For the slab-modulated structure, a broadband conversion bandwidth of ~620 nm is found, with a more than 20 dB enhanced CE for a signal wavelength centered at 2695 nm, 310 nm from the edge of the standard conversion bandwidth. The enhanced sideband has a bandwidth of 32 nm (11 nm) on the red (blue) side of the pump. In our calculations we did not include the linear material absorption; however, we note that it is important to consider the device cladding as the pump, signal, and idler wavelengths move

further into the mid-IR. For wavelengths larger than 2600 nm, modal absorption exceeds 2 dB/cm for the SOI platform [50], however other material platforms allow low loss modes beyond 2600 nm, including Si-on-sapphire (1200 nm to 4400 nm), Si-on-nitride (1200 nm to 6700 nm), and suspended Si (1200 nm to 8000 nm) [50].

In general, the results in this section show that no matter how broad the standard conversion bandwidth is made by engineering the waveguide dispersion, *w*-modulation can be employed to further extend the spectral reach of FWM *via* QPM.

### 6. Conclusions

We have reported on an experimental demonstration of QPM FWM in SiNWGs using a grating formed by *w*-modulation. This device allows efficient discrete wavelength conversion over a 250 nm span, and a ~12 dB conversion efficiency enhancement for a targeted sideband more than 100 nm away from the edge of the 3-dB conversion bandwidth. The sideband exhibits a 15 nm bandwidth and peak conversion efficiency of -38 dB.

A rigorous FWM model is described, and confirms the enhanced sideband is a result of QPM. The model is used to theoretical study the influence important device parameters have on the CE spectrum, revealing that the grating period and modulation strength can be used to tune the spectral characteristics of the FWM CE enhanced sideband. The grating profile can be engineered by using a superposition of sinusoidal gratings, and used to define a device that supports FWM between the entire C-band and a wavelength range near 2000 nm. QPM is also shown to be compatible with ZDWL-pumping for broad conversion efficiency bandwidths by employing slab-modulation, and we describe a device that allows FMW over more than 1000 nm, between 1625 nm with a conversion efficiency of -25 dB.

The work presented in this paper shows that QPM FWM by means of *w*-modulation can be used as an enabling technology for realizing efficient wavelength conversion over large spectral spans, including FWM between the telecommunications bands and mid-IR wavelengths. As shown in Sec. 5.5 *w*-modulation can be used in conjunction with other techniques for achieving a broad conversion bandwidth, such as ZDWL-pumping, and therefore serves as a means to extend the spectral reach of FWM. Mid-IR applications, including free-space communication and chemical sensing, can use QPM to interconvert between the mid-IR and telecommunications bands, and leverage telecom sources and detectors in mid-IR systems for increased performance.

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